

M2REG-TRIG-2425-ASM-SET 4-MATH

Suggested solutions

1. When $\sin \theta = -\frac{2}{\sqrt{5}}$ and $\pi < \theta < \frac{3\pi}{2}$,

$$\begin{aligned}\cos \theta &= -\frac{\sqrt{(\sqrt{5})^2 - 2^2}}{\sqrt{5}} & 1\text{M} \\ &= -\frac{1}{\sqrt{5}}\end{aligned}$$

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta & 1\text{M} \\ &= 2 \left(-\frac{1}{\sqrt{5}}\right) \left(-\frac{2}{\sqrt{5}}\right) \\ &= \frac{4}{5} & 1\text{A}\end{aligned}$$

$$\begin{aligned}2. \quad \frac{\sin 5\theta}{\cos \theta} + \frac{\cos 5\theta}{\sin \theta} &= \frac{\sin 5\theta \sin \theta + \cos 5\theta \cos \theta}{\sin \theta \cos \theta} \\ &= \frac{\cos(5\theta - \theta)}{\frac{1}{2} \sin 2\theta} & 2\text{M} \\ &= \frac{2 \cos 4\theta}{\sin 2\theta} \\ &= \frac{2(1 - 2 \sin^2 2\theta)}{\sin 2\theta} & 1\text{M} \\ &= 2 \csc 2\theta - 4 \sin 2\theta & 1\end{aligned}$$

$$\begin{aligned}3. \quad (a) \quad \cot A - \cot 2A &= \frac{\cos A}{\sin A} - \frac{\cos 2A}{\sin 2A} & 1\text{M} \\ &= \frac{\cos A \sin 2A - \sin A \cos 2A}{\sin A \sin 2A} \\ &= \frac{\sin(2A - A)}{\sin A \sin 2A} & 1\text{M} \\ &= \frac{\sin A}{\sin A \sin 2A} \\ &= \csc 2A & 1\end{aligned}$$

$$\begin{aligned}(b) \quad \csc 2\theta + \csc 4\theta + \csc 8\theta \\ &= (\cot \theta - \cot 2\theta) + (\cot 2\theta - \cot 4\theta) + (\cot 4\theta - \cot 8\theta) \quad [\text{From (a)}] & 1\text{M} \\ &= \cot \theta - \cot 8\theta & 1\end{aligned}$$

$$\begin{aligned}
4. \quad \frac{1 - 2 \sin^2 x}{1 + \sin 2x} &= \frac{1 - 2 \sin^2 x}{1 + 2 \sin x \cos x} && 1\text{M} \\
&= \frac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2 \sin x \cos x} && 1\text{M} \\
&= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} && 1\text{M} \\
&= \frac{\cos x - \sin x}{\sin x + \cos x} \\
&= \frac{1 - \frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} + 1} \\
&= \frac{1 - \tan x}{1 + \tan x} && 1
\end{aligned}$$

$$\begin{aligned}
 5. \quad (a) \quad \frac{1 - \cos 2\theta}{1 + \cos 2\theta} &= \frac{1 - (1 - 2 \sin^2 \theta)}{1 + (2 \cos^2 \theta - 1)} && 1\text{M}+1\text{A} \\
 &= \frac{2 \sin^2 \theta}{2 \cos^2 \theta} \\
 &= \tan^2 \theta && 1
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \text{Put } \theta &= \frac{\pi}{8}, \\
 \tan^2 \frac{\pi}{8} &= \frac{1 - \cos 2(\frac{\pi}{8})}{1 + \cos 2(\frac{\pi}{8})} && 1\text{M} \\
 &= \frac{1 - \frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} \\
 &= \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} && 1\text{M} \\
 &= \frac{2 - 2\sqrt{2} + 1}{2 - 1} \\
 &= 3 - 2\sqrt{2} && 1\text{A}
 \end{aligned}$$

$$\begin{aligned}
 \text{Put } \theta &= \frac{3\pi}{8}, \\
 \tan^2 \frac{3\pi}{8} &= \frac{1 - \cos 2(\frac{3\pi}{8})}{1 + \cos 2(\frac{3\pi}{8})} && 1\text{M} \\
 &= \frac{1 - (-\frac{1}{\sqrt{2}})}{1 + (-\frac{1}{\sqrt{2}})} \\
 &= \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} && 1\text{M} \\
 &= \frac{2 + 2\sqrt{2} + 1}{2 - 1} \\
 &= 3 + 2\sqrt{2} && 1\text{A}
 \end{aligned}$$

Required equation is

$$\begin{aligned}
 x^2 - \left(\tan^2 \frac{\pi}{8} + \tan^2 \frac{3\pi}{8} \right) x + \left(\tan^2 \frac{\pi}{8} \tan^2 \frac{3\pi}{8} \right) &= 0 && 1\text{M} \\
 x^2 - 6x + 1 &= 0 && 1\text{A}
 \end{aligned}$$

$$\begin{aligned}
6. \quad (a) \quad \sin 2\beta &= \frac{3 \sin 2\alpha}{2} && 1M \\
&= \frac{3(2 \sin \alpha \cos \alpha)}{2} && 1A \\
&= 3 \sin \alpha \cos \alpha \\
\cos 2\beta &= 1 - 2 \sin^2 \beta && 1M \\
&= 1 - (1 - 3 \sin^2 \alpha) && 1A \\
&= 3 \sin^2 \alpha \\
(b) \quad \cos(\alpha + 2\beta) &= \cos \alpha \cos 2\beta - \sin \alpha \sin 2\beta && 1M \\
&= 3 \sin^2 \alpha \cos \alpha - 3 \sin^2 \alpha \cos \alpha \\
&= 0 && 1A
\end{aligned}$$

$$\begin{aligned}
7. \quad \frac{\sin A}{1 - \cos A} &= \frac{\sin(180^\circ - 2B)}{1 - \cos(180^\circ - 2B)} && 1M \\
&= \frac{\sin 2B}{1 + \cos 2B} && 1M \\
&= \frac{2 \sin B \cos B}{1 + 2 \cos^2 B - 1} && 1M \\
&= \tan B && 1
\end{aligned}$$

$$\begin{aligned}
8. \quad \frac{\cos^2 A - \cos^2 B}{\cos^2 A + \cos^2 B} &= \frac{\cos^2 A - \cos^2 \left(\frac{\pi}{2} - A\right)}{\cos^2 A + \cos^2 \left(\frac{\pi}{2} - B\right)} \\
&= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A} && 1M \\
&= \frac{\cos 2A}{1} && 1M \\
&= \cos 2A && 1
\end{aligned}$$

9. (a) $\cos 4\theta = \cos 2(2\theta)$
- $$= 2 \cos^2 2\theta - 1 \quad 1\text{M}$$
- $$= 2(2 \cos^2 \theta - 1)^2 - 1 \quad 1\text{M}$$
- $$= 2(4 \cos^4 \theta - 4 \cos^2 \theta + 1) - 1$$
- $$= 8 \cos^4 \theta - 8 \cos^2 \theta + 1 \quad 1$$
- (b) Put $x = \cos 15^\circ$.
- $$16 \cos^4 15^\circ - 16 \cos^2 15^\circ + 1 = 2(8 \cos^4 15^\circ - 8 \cos^2 15^\circ + 1) - 1$$
- $$= 2 \cos 4(15^\circ) - 1 \quad 1\text{M}$$
- $$= 2 \cos 60^\circ - 1$$
- $$= 1 - 1 \quad 1\text{M}$$
- $$= 0$$
- Thus, $\cos 15^\circ$ is a root of $16x^4 - 16x^2 + 1 = 0$. 1
- (c) $16 \cos^4 15^\circ - 16 \cos^2 15^\circ + 1 = 0$
- $$\cos^2 15^\circ = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(16)(1)}}{2(16)} \quad 1\text{M}$$
- $$= \frac{16 \pm 8\sqrt{3}}{32}$$
- $$= \frac{2 + \sqrt{3}}{4} \text{ or } \frac{2 - \sqrt{3}}{4} \quad 1\text{A}$$
- Since $\cos 15^\circ > \cos 60^\circ > 0$, we have $\cos^2 15^\circ > \cos^2 60^\circ = \frac{1}{4}$ and $\cos^2 15^\circ = \frac{2 + \sqrt{3}}{4}$. 1A
- Thus, $\cos 15^\circ = \frac{\sqrt{2 + \sqrt{3}}}{2}$. 1A

10. (a) $\cos 3\theta = \sin 2\theta$

$$\cos 3\theta = \cos(90^\circ - 2\theta)$$

$$3\theta = 90^\circ - 2\theta$$

$$\theta = 18^\circ$$

1A

(b) $\cos 3\theta = \sin 2\theta$

$$\cos 2\theta \cos \theta - \sin 2\theta \sin \theta = \sin 2\theta$$

1M

$$(2 \cos^2 \theta - 1) \cos \theta - 2 \sin^2 \theta \cos \theta = 2 \sin \theta \cos \theta$$

$$(2 \cos^2 \theta - 1) - 2(1 - \cos^2 \theta) = 2 \sin \theta$$

$$4 \cos^2 \theta - 3 = 2 \sin \theta$$

$$16 \cos^4 \theta - 24 \cos^2 \theta + 9 = 4 \sin^2 \theta$$

1M

$$16 \cos^4 \theta - 24 \cos^2 \theta + 9 = 4 - 4 \cos^2 \theta$$

$$16 \cos^4 \theta - 20 \cos^2 \theta + 5 = 0$$

1

(c) When $\theta = 18^\circ$, $\cos 3\theta = \sin 2\theta$ and

$$16 \cos^4 18^\circ - 20 \cos^2 18^\circ + 5 = 0$$

$$\cos^2 18^\circ = \frac{20 \pm \sqrt{20^2 - 4(16)(5)}}{2(16)}$$

$$= \frac{5 + \sqrt{5}}{8} \quad \text{or} \quad \frac{5 - \sqrt{5}}{8} \text{ (rejected)}$$

1M

$$\sin 54^\circ = \cos 36^\circ$$

$$= 2 \cos^2 18^\circ - 1$$

1M

$$= \frac{1 + \sqrt{5}}{4}$$

1A