M2REG-TRIG-2425-ASM-SET 2-MATH
Suggested solutions

1. $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}+\dfrac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$

$=\dfrac{(1+\sin\theta-\cos\theta)^2+(1+\sin\theta+\cos\theta)^2}{[(1+\sin\theta)+\cos\theta][(1+\sin\theta)-\cos\theta]}$ 1M

$=\dfrac{(1+\sin\theta)^2-2(1+\sin\theta)\cos\theta+\cos^2\theta+(1+\sin\theta)^2+2(1+\sin\theta)\cos\theta+\cos^2\theta}{(1+\sin\theta)^2-\cos^2\theta}$ 1M

$=\dfrac{2(1+\sin\theta)^2+2\cos^2\theta}{1+2\sin\theta+\sin^2\theta-\cos^2\theta}$

$=\dfrac{2+4\sin\theta+2\sin^2\theta+2\cos^2\theta}{2\sin\theta+2\sin^2\theta}$ 1M

$=\dfrac{4(1+\sin\theta)}{2\sin\theta(1+\sin\theta)}$ 1M

$=\dfrac{2}{\sin\theta}$

$=2\csc\theta$ 1

2. $\sin^2 x\tan x+\cos^2 x\cot x+2\sin x\cos x$

$=(1-\cos^2 x)\tan x+(1-\sin^2 x)\cot x+2\sin x\cos x$ 2M

$=\tan x-\cos^2 x\left(\dfrac{\sin x}{\cos x}\right)+\cot x-\sin^2 x\left(\dfrac{\cos x}{\sin x}\right)+2\sin x\cos x$ 1M

$=\tan x-\cos x\sin x+\cot x-\sin x\cos x+2\sin x\cos x$

$=\tan x+\cot x$ 1

3. $\left[\csc\left(\theta-\dfrac{7\pi}{2}\right)+\tan(3\pi-\theta)\right]\left[\tan\dfrac{5\pi}{4}-\cos\left(\theta-\dfrac{3\pi}{2}\right)\right]$

$=\left[\csc\left(\theta-\dfrac{3\pi}{2}\right)+\tan(\pi-\theta)\right]\left[1-\cos\left(\theta-\dfrac{3\pi}{2}\right)\right]$

$=(\sec\theta-\tan\theta)(1+\sin\theta)$ 1M

$=\left(\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}\right)(1+\sin\theta)$

$=\dfrac{1-\sin^2\theta}{\cos\theta}$ 1M

$=\dfrac{\cos^2\theta}{\cos\theta}$

$=\cos\theta$ 1

4. $\dfrac{\sec(-\theta)+\cos(\pi-\theta)}{\csc(\pi-\theta)-\sin(\pi-\theta)}$

$= \dfrac{\sec\theta-\cos\theta}{\csc\theta-\sin\theta}$ 1M

$= \dfrac{\frac{1}{\cos\theta}-\cos\theta}{\frac{1}{\sin\theta}-\sin\theta}$

$= \dfrac{1-\cos^2\theta}{\cos\theta} \div \dfrac{1-\sin^2\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta} \times \dfrac{\sin\theta}{\cos^2\theta}$ 1M

$= \dfrac{\sin^3\theta}{\cos^3\theta}$

$= \tan^3\theta$ 1

5. $\dfrac{\tan^2(90^\circ+\theta)}{\csc(360^\circ+\theta)-1} - [1-\sec(270^\circ-\theta)]$

$= \dfrac{(-\cot\theta)^2}{\csc\theta-1} - (1+\csc\theta)$ 1M+1M

$= \dfrac{\cot^2\theta-(1+\csc\theta)(\csc\theta-1)}{\csc\theta-1}$

$= \dfrac{\cot^2\theta-(\csc^2\theta-1)}{\csc\theta-1}$

$= \dfrac{\cot^2\theta-\cot^2\theta}{\csc\theta-1}$ 1M

$= 0$

Thus, $\dfrac{\tan^2(90^\circ+\theta)}{\csc(360^\circ+\theta)-1} = 1-\sec(270^\circ-\theta)$. 1

6. $\tan^2 x + 3\sec x - 3 = 0$

$(\sec^2 x - 1) + 3\sec x - 3 = 0$ 1M

$\sec^2 x + 3\sec x - 4 = 0$

$\sec x = 1 \quad \text{or} \quad -4$

$\cos x = 1 \quad \text{or} \quad -\dfrac{1}{4}$ 1M

$\cos x = 1 \qquad \text{or} \quad \cos x = -\dfrac{1}{4}$

$x = 0 \quad \text{or} \quad 2\pi \qquad x \approx 1.82 \quad \text{or} \quad 4.46$

Thus, $x = 0$ or 1.82 or 4.46 or 6.28. 1A

7. $\sec^2\theta + 14\tan\theta + 48 = 0$

$(1+\tan^2\theta) + 14\tan\theta + 48 = 0$ 1M

$\tan^2\theta + 14\tan\theta + 49 = 0$

$\tan\theta = -7$ 1M

$\theta \approx 98.1^\circ \quad \text{or} \quad 278^\circ$ 1A

8. $\tan 195^\circ = \tan(60^\circ + 135^\circ)$

$$= \frac{\tan 60^\circ + \tan 135^\circ}{1 - \tan 60^\circ \tan 135^\circ}$$ 1M

$$= \frac{\sqrt{3} + (-1)}{1 - (\sqrt{3}(-1))}$$ 1M

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$ 1A

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{3 - 2\sqrt{3} + 1}{2}$$

$$= 2 - \sqrt{3}$$ 1A

9. $\sin^2(x - y) - 2\sin x \cos y \sin(x - y)$

$= (\sin x \cos y - \cos x \sin y)^2 - 2\sin x \cos y(\sin x \cos y - \cos x \sin y)$ 1M

$= \sin^2 x \cos^2 y - 2\sin x \cos x \sin y \cos y + \cos^2 x \sin^2 y - 2\sin^2 x \cos^2 y + 2\sin x \cos x \sin y \cos y$

$= \cos^2 x \sin^2 y - \sin^2 x \cos^2 y$

$= \cos^2 x(1 - \cos^2 y) - (1 - \cos^2 x)\cos^2 y$ 1M

$= \cos^2 x - \cos^2 y$ 1

10. $\dfrac{\cos(A - B) - \cos(A + B)}{\cos(A - B) + \cos(A + B)}$

$$= \frac{(\cos A \cos B + \sin A \sin B) - (\cos A \cos B - \sin A \sin B)}{(\cos A \cos B + \sin A \sin B) + (\cos A \cos B - \sin A \sin B)}$$ 1M+1A

$$= \frac{2\sin A \sin B}{2\cos A \cos B}$$

$= \tan A \tan B$ 1

11. $2\cos(A - B) = 3\cos(A + B)$

$2\cos A \cos B + 2\sin A \sin B = 3\cos A \cos B - 3\sin A \sin B$ 2M

$5\sin A \sin B = \cos A \cos B$

$5\tan A = \cot B$ 1

12. $\sin(A + B)\cos C - \sin(A + C)\cos B$

$= (\sin A \cos B + \cos A \sin B)\cos C - (\sin A \cos C + \cos A \sin C)\sin B$ 1M

$= \sin A \cos B \cos C + \cos A \sin B \cos C - \sin A \cos B \cos C - \cos A \cos B \sin C$

$= \cos A(\sin B \cos C - \cos B \sin C)$

$= \sin(B - C)\cos A$ 1M+1

13. $\tan\alpha = \dfrac{12}{\sqrt{13^2-12^2}} = \dfrac{12}{5}$ and $\tan\beta = \dfrac{3}{\sqrt{5^2-3^2}} = \dfrac{3}{4}$ 1M+1A

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} \quad \text{1M}$$

$$= \frac{\frac{12}{5} - \frac{3}{4}}{1 + \left(\frac{12}{5}\right)\left(\frac{3}{4}\right)}$$

$$= \frac{33}{56} \quad \text{1A}$$

14. (a) $\sin(x+\alpha) = 3\cos(x-\alpha)$

$$\sin x\cos\alpha + \cos x\sin\alpha = 3\cos x\cos\alpha + 3\sin x\sin\alpha \quad \text{1M}$$

$$\frac{\sin x}{\cos x} + \frac{\sin\alpha}{\cos\alpha} = 3 + \frac{3\sin x\sin\alpha}{\cos x\cos\alpha}$$

$$\tan x + \tan\alpha = 3 + 3\tan x\tan\alpha \quad \text{1M}$$

$$\tan x(1 - 3\tan\alpha) = 3 - \tan\alpha$$

$$\tan x = \frac{3-\tan\alpha}{1-3\tan\alpha} \quad \text{1}$$

(b) $\sin\left(x+\dfrac{\pi}{4}\right) = 3\cos\left(x-\dfrac{\pi}{4}\right)$

$$\tan x = \frac{3-\tan\frac{\pi}{4}}{1-3\tan\frac{\pi}{4}} \quad \text{1M}$$

$$= \frac{3-1}{1-3(1)}$$

$$= -1$$

$$x = \frac{3\pi}{4} \quad \text{or} \quad \frac{7\pi}{4} \quad \text{1A}$$

15. $\tan(x+y) = \dfrac{\tan x + \tan y}{1-\tan x\tan y}$ 1M

$$= \frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}$$

$$= \frac{7}{9} \quad \text{1M}$$

$$\tan(x+y+z) = \frac{\tan(x+y)+\tan z}{1-\tan(x+y)\tan z}$$

$$= \frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}$$

$$= 1 \quad \text{1A}$$

Note that $\tan(x+y) = \dfrac{7}{9} < 1$. We have $x+y < \dfrac{\pi}{4}$ and $x+y+z < \dfrac{5\pi}{4}$.

Thus, $x+y+z = \dfrac{\pi}{4}$. 1A