M2REG-BT-2425-ASM-SET 2-MATH
Suggested solutions

1. 7th term $= C_6^n(ax^2)^{n-6}\left(\frac{1}{2x}\right)^6$

$= C_6^n a^{n-6}2^{-6}x^{2n-18}$ 1A

Put $2n - 18 = 0$, we have $n = 9$. 1A

$C_6^9 a^{9-6}2^{-6} = \frac{567}{16}$

$a^3 = 27$

$a = 3$ 1A

2. General term $= C_r^{12}(x^5)^{12-r}\left(-\frac{2}{x}\right)^r$

$= C_r^{12}(-2)^r x^{60-6r}$ 1M

Put $60 - 6r = 0$, we have $r = 10$. 1M

Constant term $= C_{10}^{12}(-2)^{10}$

$= 67\,584$ 1A

3. (a) General term $= C_r^9(2x^2)^{9-r}\left(\frac{1}{x}\right)^r$ 1A

$= C_r^9(2)^{9-r}x^{18-3r}$ 1M

When $18 - 3r = 0$, $r = 6$. 1M

Constant term $= C_6^9(2)^{9-6} = 672$. 1A

(b) When $18 - 3r = 2$, $r = \frac{16}{3}$, which is not an integer.

There is no x^2 term. 1M+1A

4. (a) General term $= C_r^8 x^{8-r}\left(\frac{2}{x^3}\right)^r$

$= C_r^8 2^r x^{8-4r}$ 1M

Put $8 - 4r = 0$, we have $r = 2$. 1M

Constant term exists.

Constant term $= C_2^8 2^2$

$= 112$ 1A

(b) Put $8 - 4r = 3$, we have $r = \frac{5}{4}$. 1M

Since r should be a non-negative integer, x^3 term does not exist. 1A

5. General term $= C_r^{12}(x)^{12-r}\left(-\frac{y}{2}\right)^r$

$= C_r^{12}\left(-\frac{1}{2}\right)^r x^{12-r}y^r$ 1M

Put $r = 8$. 1M

Required coefficient $= C_8^{12}\left(-\frac{1}{2}\right)^8$

$= \frac{495}{256}$ 1A

6. (a) $(1-4x)^n = 1 + C_1^n(-4x) + C_2^n(-4x)^2 + \ldots$ 1M

$\frac{n(n-1)}{2}(-4)^2 = 240$ 1M

$n^2 - n - 30 = 0$

$n = 6 \quad \text{or} \quad -5 \text{ (rejected)}$ 1A

(b) $(1-4x)^6 = 4096x^6 - 6144x^5 + 3840x^4 + \ldots$

$\left(1+\frac{2}{x}\right)^5 = 1 + \frac{10}{x} + \frac{40}{x^2} + \ldots$ 1M

Required coefficient $= 4096(40) - 6144(10) + 3840(1)$ 1M

$= 106\,240$ 1A

7. $(x+3)^5 = x^5 + C_1^5(3)x^4 + C_2^5(3)^2x^3 + C_3^5(3)^3x^2 + C_4^5(3)^4x + 3^5$ 1M

$= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$ 1A

$(x+3)^5\left(x-\frac{4}{x}\right)^2 = (x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243)\left(x^2 - 8 + \frac{16}{x^2}\right)$ 1M

Required coefficient $= (1)(16) + (90)(-8) + (405)(1)$ 1M

$= -299$ 1A

8. $(3+x)^6 = x^6 + C_1^6x^5(3) + C_2^6x^4(3)^2 + C_3^6x^3(3)^3 + \ldots$ 1M

$= x^6 + 18x^5 + 135x^4 + 540x^3 + \ldots$

$\left(\frac{2}{x}-1\right)^2(3+x)^6 = \left(\frac{4}{x^2} - \frac{4}{x} + 1\right)(x^6 + 18x^5 + 135x^4 + 540x^3 + \ldots)$

Required coefficient $= 4(18) + (-4)(135) + 1(540)$ 1M

$= 72$ 1A

9. (a) $\left(2x+\frac{1}{x}\right)^3 = (2x)^3 + 3(2x)^2\left(\frac{1}{x}\right) + 3(2x)\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$

$= 8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$ 1A

(b) $(3x^2 - x - 5)\left(2x+\frac{1}{x}\right)^3 = (3x^2 - x - 5)\left(8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}\right)$

Coefficient of $x = (3)(6) + (-5)(12)$ 1M

$= -42$ 1A

10. $\left(2-\frac{x}{2}\right)^7 = 2^7 + C_1^7(2)^6\left(-\frac{x}{2}\right) + C_2^7(2)^5\left(-\frac{x}{2}\right)^2 + C_3^7(2)^4\left(-\frac{x}{2}\right)^3 + \ldots$ 1M

$= 128 - 224x + 168x^2 - 70x^3 + \ldots$

$\left(4x+\frac{9}{x}\right)^2\left(2-\frac{x}{2}\right)^7 = \left(\frac{81}{x^2} + 72 + 16x^2\right)(128 - 224x + 168x^2 - 70x^3 + \ldots)$

Required coefficient $= 81(-70) + 72(-224)$ 1M

$= -21\,798$ 1A

11. $(1+2x)^3 = 1 + 6x + 12x^2 + \ldots$ 1A

$(1+3x)^4 = 1 + 12x + 54x^2 + \ldots$

$(1+2x)^4(1+3x)^4 = (1 + 6x + 12x^2 + \ldots)(1 + 12x + 54x^2 + \ldots)$

$= 1 + 18x + 138x^2 + \ldots$ 1M+1A

12. $(1+x)^m = 1 + C_1^m x + C_2^m x^2 + \ldots$

$= 1 + mx + \frac{m(m-1)}{2}x^2 + \ldots$

$(1-2x)^n = 1 + C_1^n(-2x) + C_1^n(-2x)^2 + \ldots$

$= 1 - 2nx + 2n(n-1)x^2 + \ldots$

Compare the coefficients of x and x^2, we have

$$\begin{cases} m - 2n = 0 \\ \frac{m(m-1)}{2} - 2mn + 2n(n-1) = -9 \end{cases}$$ 1M

$\frac{2n(2n-1)}{2} - 2(2n)n + 2n(n-1) = -9$ 1M

$-3n = -9$

$n = 3$ 1A

When $n = 3$, $m = 2(3) = 6$. 1A

13. $(1+4px)(1+qx)^4 = (1+4px)[1 + 4qx + 6(qx)^2 + \ldots]$ 1M

$= (1+4px)(1 + 4qx + 6q^2x^2 + \ldots)$

$0 = 1(4q) + (4p)(1)$ 1M

$p = -q$

$-40 = 1(6q^2) + 4p(4q)$ 1M

$-40 = 6q^2 + 16(-q)(q)$ 1M

$q^2 = 4$

$q = \pm 2$

Thus, $(p,\ q) = (-2,\ 2)$ or $(2,\ -2)$. 1A+1A

14. $(a+5x)(1+4x)^n = (a+5x)\left[1 + C_1^n(4x) + C_2^n(4x)^2 + \ldots\right]$ 1M

$= (a+5x)[1 + 4nx + 8n(n-1)x^2 + \ldots]$

$= a + (5+4an)x + [20n + 8an(n-1)]x^2 + \ldots$

Compare like terms, we have

$$\begin{cases} a = 11 & \text{1A} \\ 5 + 4an = 313 & \text{1M} \\ 20n + 8an(n-1) = b \end{cases}$$

$5 + 4(11)n = 313$ 1M

$n = 7$ 1A

$20(7) + 8(11)(7)(6) = b$

$b = 3836$ 1A

15. (a) $(1-x)^4 = 1 - 4x + 6x^2 - 4x^3 + x^4$ 1A

(b) $(1+kx)^9(1-x)^4 = (1 + 9kx + 36k^2x^2 + \ldots)(1 - 4x + 6x^2 - 4x^3 + x^4)$ 1A

$-3 = 1(6) + 9k(-4) + 36k^2(1)$ 1M

$0 = 36k^2 - 36k + 9$

$k = \dfrac{1}{2}$ 1A

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