M2REG-BT-2425-ASM-SET 1-MATH
Suggested solutions

1. $(x+8)^4 = x^4 + 4(x)^3(8) + 6(x)^2(8)^2 + 4(x)(8)^3 + 8^4$
$= x^4 + 32x^3 + 384x^2 + 2048x + 4096$ 1A

2. $(5-x)^5 = (5)^5 + 5(5)^4(-x) + 10(5)^3(-x)^2 + 10(5)^2(-x)^3 + 5(5)(-x)^4 + (-x)^5$
$= 3125 - 3125x + 1250x^2 - 250x^3 + 25x^4 - x^5$ 1A

3. (a) $\left(y - \frac{1}{y}\right)^3 = y^3 + 3(y)^2\left(-\frac{1}{y}\right) + 3(y)\left(-\frac{1}{y}\right)^2 + \left(-\frac{1}{y}\right)^3$ 1M
$27 = y^3 - \frac{1}{y^3} - 3\left(y - \frac{1}{y}\right)$
$y^3 - \frac{1}{y^3} = 36$ 1A

(b) $\left(y - \frac{1}{y}\right)^5 = (y)^5 + 5(y)^4\left(-\frac{1}{y}\right) + 10(y)^3\left(-\frac{1}{y}\right)^2 + 10(y)^2\left(-\frac{1}{y}\right)^3 + 5(y)\left(-\frac{1}{y}\right)^4 + \left(-\frac{1}{y}\right)^5$ 1M
$243 = y^5 - \frac{1}{y^5} - 5\left(y^3 - \frac{1}{y^3}\right) + 10\left(y - \frac{1}{y}\right)$
$y^5 - \frac{1}{y^5} = 393$ 1A

4. The term containing x^4
$= C_4^{10}(3)^6(2x)^4$ 1M
$= 2\,449\,440x^4$ 1A

5. $(2x+1)^{13} = 1^{13} + C_1^{13}(1)^{12}(2x) + C_2^{13}(1)^{11}(2x)^2 + C_3^{13}(1)^{10}(2x)^3 + \ldots$ 1M
$= 1 + 26x + 312x^2 + 2288x^3 + \ldots$ 1A

6. Required coefficient $= C_{10}^{17}(2)^{10}\left(-\frac{1}{4}\right)^{17-10}$ 1M
$= -\frac{2431}{2}$ 1A

7. The 5th term $= C_4^n (ax^2)^{n-4} \left(\frac{1}{3x}\right)^4$ 1M

$= C_4^n a^{n-4} \left(\frac{1}{81}\right) x^{2n-12}$

Therefore,

$2n - 12 = 0$

$n = 6$ 1A

and

$C_4^6 a^{6-4} \left(\frac{1}{81}\right) = \frac{5}{27}$

$a^2 = 1$

$a = 1 \quad \text{or} \quad -1$ 1A

8. Required coefficient $= C_3^{10}(3)^{10-3}$ 1M

$= -262\,440$ 1A

9. $(2x^3 + 1)^n = 1 + C_1^n(2x^3) + C_2^n(2x^3)^2 + C_3^n(2x^3)^3 + \ldots$ 1M

$= 1 + 2nx^3 + 2n(n-1)x^6 + \frac{n(n-1)(n-2)}{6}(8)x^9 + \ldots$

If $d = 28b$,

$\frac{n(n-1)(n-2)}{6}(8) = 28(2n)$ 1M

$8(n^2 - 3n + 2) = 336$

$8n^2 - 24n - 320 = 0$

$n = 8 \quad \text{or} \quad -5 \text{ (rejected)}$ 1A

When $n = 8$, $c = 2(8)(8-1) = 112$. 1A

10. $(px - 1)^n = (-1)^n + n(-1)^{n-1}px + \frac{n(n-1)}{2}(-1)^{n-2}p^2x^2 + \ldots$ 1A

Comparing like terms, n is an odd number and

$-\frac{n(n-1)}{2} \cdot p^2 = -10p^2$ 1M

$n^2 - n - 20 = 0$

$n = 5 \quad \text{or} \quad -4 \text{ (rejected)}$ 1A

$np = \frac{5}{4}$

$p = \frac{1}{4}$ 1A

11. $(1+kx)^n = 1 + C_1^n(kx) + C_2^n(kx)^2 + \ldots$ 1M

$= 1 + nkx + \dfrac{n(n-1)}{2}(k^2)x^2 + \ldots$

Comparing coefficients,

$$\begin{cases} nk = 27 \\ \dfrac{n(n-1)}{2}(k^2) = 324 \end{cases}$$ 1M

$$\frac{n(n-1)}{2}\left(\frac{27}{n}\right)^2 = 324$$ 1M

$$729n^2 - 729n = 648n^2$$

$$81n^2 - 729n = 0$$

$n = 9$ or 0 (rejected) 1A

When $n = 9$, $k = \dfrac{27}{9} = 3$. 1A

12. $(k+x)^n = x^n + C_1^n x^{n-1}(k) + C_2^n x^{n-2}(k)^2 + \ldots$ 1M

$= x^n + nkx^{n-1} + \dfrac{n(n-1)}{2}(k^2)x^{n-2} + \ldots$

Comparing coefficients,

$$\begin{cases} nk = 2n \\ \dfrac{n(n-1)}{2}(k^2) = 180 \end{cases}$$ 1M

Therefore, $k = 2$, and 1A

$$\frac{n(n-1)}{2}(2^2) = 180$$

$$n^2 - n - 90 = 0$$

$n = 10$ or -9 (rejected) 1A

13. $(1+kx^2)^n = 1 + C_1^n(kx^2) + C_2^n(kx^2)^2 + \ldots$ 1M

$= 1 + nkx^2 + \dfrac{n(n-1)}{2}(k)^2x^4 + \ldots$

Comparing coefficients,

$$\begin{cases} nk = -24 \\ \dfrac{n(n-1)}{2}(k)^2 = 28k^2 \end{cases}$$ 1M

$$\frac{n(n-1)}{2} = 28$$

$$n^2 - n - 56 = 0$$

$n = 8$ or -7 (rejected) 1A

When $n = 8$, $k = \dfrac{-24}{8} = -3$. 1A

14. $(3k+x)^n = x^n + C_1^n(3k)x^{n-1} + C_2^n(3k)^2x^{n-2} + \ldots$ 1M

$= x^n + 3nkx^{n-1} + \dfrac{n(n-1)}{2}(9k^2)x^{n-2} + \ldots$

Comparing coefficients,

$$\begin{cases} 3nk = 21 \\ \dfrac{n(n-1)}{2}(9k^2) = 189 \end{cases}$$ 1M

$$\frac{n(n-1)}{2}(9)\left(\frac{21}{3n}\right)^2 = 189$$ 1M

$3969n^2 - 3969n = 3402n^2$

$567n^2 - 3969n = 0$

$n = 7$ or 0 (rejected) 1A

When $n = 7$, $k = 1$. 1A

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