1. (a) $\dfrac{AB}{\sin 65^\circ} = \dfrac{15}{\sin 58^\circ}$ 1M

$AB \approx 16.0\text{ cm}$ 1A

$AC^2 = AB^2 + 17^2 - 2(AB)(17)\cos 116^\circ$ 1M

$AC \approx 28.0\text{ cm}$ 1A

(b) $\angle BAD = 180^\circ - 58^\circ - 65^\circ = 57^\circ$

$AK = AB\cos 57^\circ \approx 8.73\text{ cm}$ 1M

$27^2 = AC^2 + 15^2 - 2(AC)(15)\cos\angle CAD$

$\angle CAD \approx 70.5^\circ$

$AC\cos\angle CAD \approx 9.36\text{ cm} \neq AK$

So, CK is not perpendicular to AD. 1M

The claim is disagreed. 1A

2. (a) Required area $= \dfrac{1}{2}(20)(24)\sin 60^\circ$ 1M

$\approx 208\text{ cm}^2$ 1A

(b) Let E be a point on CD such that $AE \perp CD$.

Required angle is $\angle AEB$. 1M

$CD^2 = 20^2 + 24^2 - 2(20)(24)\cos 60^\circ$ 1M

$CD \approx 22.3\text{ cm}$

Consider the area of $\triangle ACD$.

$\dfrac{1}{2}(CD)(AE) = \dfrac{1}{2}(20)(24)\sin 60^\circ$ 1M

$AE \approx 18.7\text{ cm}$

Consider $\triangle ABE$.

$\sin\angle AEB = \dfrac{18}{AE}$

$\angle AEB \approx 74.7^\circ$ 1A

(c) The shortest distance from A to the plane BCD is equal to AB, which is 18 cm.

Let F be the projection of B on the plane ACD.

Note that F lies on AE and BF is the shortest distance between B and the plane ACD.

$BE = \sqrt{AE^2 - 18^2} \approx 4.94\text{ cm}$

Consider $\triangle BEF$.

$\sin\angle AEB = \dfrac{BF}{BE}$ 1M

$BF \approx 4.76\text{ cm} < 18\text{ cm}$

The claim is agreed. 1A

Let h cm be the shortest distance from B to the plane ACD.

Note that $\cos\angle AEB = \dfrac{\text{area of }\triangle BCD}{\text{area of }\triangle ACD}$.

Consider the volume of tetrahedron $ABCD$.

$$\frac{1}{3}(\text{area of } \triangle ACD)(h) = \frac{1}{3}(\text{area of } \triangle BCD)(AB) \quad \text{1M}$$

$$h = AB \times \frac{\text{area of } \triangle BCD}{\text{area of } \triangle ACD}$$

$$= AB \cos \angle AEB$$

$$< AB$$

The claim is agreed. 1A

3. (a) (i) $CD = 20\cos 30° = 10\sqrt{3}$ cm 1A

$BD = AD = 20 \sin 30° = 10$ cm

$AB = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ cm 1M

$BC^2 = AB^2 + 20^2 - 2(AB)(20)\cos \angle BAC$ 1M

$BC \approx 17.8$ cm 1A

(ii) Let $s = \dfrac{10 + CD + BC}{2} \approx 22.6$ cm.

Area of $\triangle BDC$

$= \sqrt{s(s-10)(s-CD)(s-BC)}$ 1M

$\approx 84.1\text{ cm}^2$ 1A

(iii) Let h cm be the required height.

$$\frac{1}{3}(\text{area of } \triangle ABC)(h) = \frac{1}{3}(\text{area of } \triangle BDC)(AD) \quad \text{1M}$$

$$\frac{1}{3}\left(\frac{1}{2}(AB)(20)\sin 60°\right) h = \frac{1}{3}(\text{area of } \triangle BDC)(10) \quad \text{1M}$$

$$h \approx 6.87 \quad \text{1A}$$

(b) Volume of the tetrahedron

$= \dfrac{1}{3}\left(\dfrac{1}{2}(BD)(CD)\sin \angle BDC\right)(AD)$ 1M

$= \dfrac{500\sqrt{3}}{3} \sin \angle BDC$

The volume of the tetrahedron is maximum when $\angle BDC$ is 90°.

When $\angle BDC$ increases from 45° to 90°, the volume of the tetrahedron increases.

When $\angle BDC$ increases from 90° to 120°, the volume of the tetrahedron decreases. 1A

4. (a) $\dfrac{18}{\sin\angle ABD} = \dfrac{22}{\sin 80°}$ 1M

$\angle ABD \approx 53.7°$ or $126°$ (rejected) 1A

(b) (i) $25^2 = 22^2 + 18^2 - 2(22)(18)\cos\angle ADC$ 1M

$\angle ADC \approx 76.6°$ 1A

(ii) Let E be a point on BD such that $AE \perp BD$.

Let F be a point on CD such that $EF \perp BD$.

Required angle is $\angle AEF$. 1M

$AE = 18\sin 80° \approx 17.7\text{ cm}$ 1M

$DE = 18\cos 80° \approx 3.13\text{ cm}$

$EF = DE\tan\angle CDB = DE\tan\angle ABD \approx 4.25\text{ cm}$

$DF = \sqrt{DE^2 + EF^2} \approx 5.28\text{ cm}$

Consider $\triangle ACD$.

$$AF^2 = 18^2 + DF^2 - 2(18)(DF)\cos\angle ADC$$

$$AF \approx 17.5\text{ cm}$$

Consider $\triangle AEF$.

$$AF^2 = AE^2 + EF^2 - 2(AE)(EF)\cos\angle AEF$$

$$\angle AEF \approx 80.7°$$

$$> 80°$$

The claim is agreed. 1A

5. (a) (i) $AD^2 = 24^2 + 20^2 - 2(24)(20)\cos 80°$ 1M

$AD \approx 28.4\text{ cm}$ 1A

(ii) $AE = AB = CD = 20\text{ cm}$

$AD^2 = 20^2 + 18^2 - 2(20)(18)\cos\angle AED$ 1M

$\angle AED \approx 96.8°$ 1A

(b) Area of $\triangle ADE = \dfrac{1}{2}(20)(18)\sin\angle AED$ 1M

$= 180\sin\angle AED\text{ cm}^2$

The area of $\triangle ADE$ varies directly as $\sin\angle AED$. 1M

The area of $\triangle ADE$ is maximum when $\angle AED = 90°$.

Suppose $\angle AED = 90°$.

$AD = \sqrt{20^2 + 18^2} = \sqrt{724}\text{ cm}$

$AD^2 = 24^2 + 20^2 - 2(24)(20)\cos\angle ACD$

$\angle ACD \approx 74.8°$

When $\angle ACD$ decreases from $80°$ to $74.8°$, the area of $\triangle ADE$ increases.

when $\angle ACD$ decreases from $74.8°$ to $20°$, the area of $\triangle ADE$ decreases. 1A

(c) (i) $AD^2 = 24^2 + 20^2 - 2(24)(20)\cos 60°$

$AD = \sqrt{496}\text{ cm}$

$BC = AD = \sqrt{496}\text{ cm}$

$BC^2 = 20^2 + 18^2 - 2(20)(18)\cos\angle BDC$ 1M

$\cos\angle BDC = \dfrac{19}{60}$ 1A

(ii) Required angle is $\angle BAP$.

$\angle APC = 180° - 25° - 60° = 95°$

$\dfrac{AP}{\sin 60°} = \dfrac{24}{\sin 95°}$ 1M

$AP \approx 20.9\text{ cm}$

$\dfrac{CP}{\sin 25°} = \dfrac{24}{\sin 95°}$

$CP \approx 10.2\text{ cm}$

$PD = CD - CP \approx 9.82\text{ cm}$

$BP^2 = PB^2 + 18^2 - 2(PD)(18)\cos\angle BDC$ 1M

$BP \approx 17.6\text{ cm}$

$BP^2 = 20^2 + AP^2 - 2(20)(AP)\cos\angle BAP$ 1M

$\angle BAP \approx 50.9°$ 1A

6. (a) Consider $\triangle ABD$.

$$AD^2 = 75^2 + 60^2 - 2(75)(60)\cos 105° \qquad \text{1M}$$

$$AD \approx 107\text{ cm} \qquad \text{1A}$$

$$60^2 = 75^2 + AD^2 - 2(75)(AD)\cos(2\angle CAD) \qquad \text{1M}$$

$$\angle CAD \approx 16.3°$$

Consider $\triangle ACD$.

$\angle ACD = \angle ABC + \angle BAC \approx 121°$ 1M

$$\frac{CD}{\sin\angle CAD} = \frac{AD}{\sin\angle ACD} \qquad \text{1M}$$

$$CD \approx 35.3\text{ cm} \qquad \text{1A}$$

(b) (i) Let E be a point on AC produced such that $DE \perp AC$.

Required distance $= DE$

$= AD\sin\angle CAD$ 1M

$\approx 30.2\text{ cm}$ 1A

(ii) $CE = \sqrt{CD^2 - DE^2} \approx 18.4\text{ cm}$

$\angle BCE = 180° - \angle ACB = \angle ACD \approx 121°$

$BC = 60 - CD \approx 24.7\text{ cm}$

Consider $\triangle BCE$.

$$BE^2 = BC^2 + CE^2 - 2(BC)(CE)\cos\angle BCE \qquad \text{1M}$$

$$BE \approx 37.6\text{ cm}$$

Consider $\triangle BDE$.

$$BD = \sqrt{BE^2 + DE^2} \qquad \text{1M}$$

$$\approx 48.2\text{ cm}$$

Consider $\triangle BCD$.

$$BD^2 = BC^2 + CD^2 - 2(BC)(CD)\cos\angle BCD \qquad \text{1M}$$

$$\angle BCD \approx 105.7°$$

$$> 105°$$

The claim is agreed. 1A

7. (a) $\angle BAC = 180° - 104° - 18° = 58°$

$$\frac{AB}{\sin 18°} = \frac{56}{\sin 58°}$$ 1M

$AB \approx 20.4\text{ cm}$ 1A

$$\frac{AC}{\sin 104°} = \frac{56}{\sin 58°}$$

$AC \approx 64.1\text{ cm}$ 1A

(b) $AP = \dfrac{AC}{4} \approx 16.0\text{ cm}$

$BP^2 = AP^2 + AB^2 - 2(AP)(AB)\cos 58°$ 1M

$BP \approx 18.1\text{ cm}$

Let Q and R be the projections of P and C on the horizontal ground respectively.

Required angle is $\angle PBQ$. 1M

$CR = 56\sin 37° \approx 33.7\text{ cm}$

$PQ = \dfrac{CR}{4} \approx 8.43\text{ cm}$ 1M

$\sin\angle PBQ = \dfrac{PQ}{BP}$

$\angle PBQ \approx 27.8° < 28°$

The claim is correct. 1A

8. (a) Let M be the mid-point of AC.

$BM = 20\sin 60° = 10\sqrt{3}\text{ cm}$

$BE = BM\sin 60° = 15\text{ cm}$ 1A

$\angle BEC = 90°$. So, BC is a diameter of the circumcircle of $\triangle BCE$. 1M

Thus, $DE = DB = DC = \dfrac{20}{2} = 10\text{ cm}$ 1A

(b) $AE = \sqrt{AB^2 - BE^2}$ 1M

$= \sqrt{175}\text{ cm}$

$AD = BM = 10\sqrt{3}\text{ cm}$

$AE^2 = AD^2 + DE^2 - 2(AD)(DE)\cos\angle ADE$ 1M

$\angle ADE \approx 49.5° \neq 90°$

The claim is disagreed. 1A

9. (a) Let X be the mid-point of BC.

Required angle is $\angle DAX$. 1M

Note that $\angle AXD = 70°$.

$AX = \sqrt{16^2 - \left(\frac{18}{2}\right)^2} = \sqrt{175}$ cm

$\frac{\sqrt{175}}{\sin \angle ADX} = \frac{15}{\sin 70°}$ 1M

$\angle ADX \approx 56.0°$

$\angle DAX + \angle ADX + 70° = 180°$

$\angle DAX \approx 54.0°$ 1A

(b) $DX^2 = 15^2 + AX^2 - 2(15)(AX) \cos \angle DAX$ 1M

$DX \approx 12.9$ cm

$BD = \sqrt{DX^2 + XB^2} \approx 15.7$ cm

$BD^2 = 16^2 + 15^2 - 2(16)(15) \cos \angle BAD$

$\angle BAD \approx 60.9°$

Let P be a point on AD such that $BP \perp AD$ and $CP \perp AD$.

$\sin \angle BAP = \frac{BP}{16}$

$BP \approx 14.0$ cm

$CP = BP \approx 14.0$ cm

Required distance

$= BP + CP$ 1M

≈ 28.0 cm

> 27 cm

The total distance cannot be less than 27 cm. 1A

10. (a) (i) $AC = 24$ cm

$\frac{\sin \angle BAC}{23} = \frac{\sin 70°}{24}$ 1M

$\angle BAC \approx 64.2°$ 1A

(ii) Required distance

$= BC \sin \angle BCA$

$= 23 \sin(180° - 70° - \angle BAC)$ 1M

≈ 16.5 cm 1A

(b) $\angle ACB = 180° - 70° - \angle BAC \approx 45.8°$

$\frac{AB}{\sin \angle ACB} = \frac{24}{\sin 70°}$

$AB \approx 18.3$ cm

Let E be a point on AC such that $BE \perp AC$.

$AE = \sqrt{AB^2 - BE^2} \approx 7.96$ cm 1A

Let F be a point on AD such that $FE \perp AC$.

Required angle is $\angle BEF$. 1M

$EF = AE \tan 60° \approx 13.8\,\text{cm}$

$AF = \dfrac{AE}{\cos \angle EAF} \approx 15.9\,\text{cm}$

$BF^2 = AB^2 + AF^2 - 2(AB)(AF)\cos 45°$ 1M

$BF \approx 13.3\,\text{cm}$

$BF^2 = BE^2 + EF^2 - 2(BE)(EF)\cos \angle BEF$

$\angle BEF \approx 51.1°$ 1A

(c) Required area

$= \dfrac{1}{2}(AB)(24)\sin 45°$ 1M

$\approx 155\,\text{cm}^2$ 1A

(d) Let h cm be the required distance.

Consider the volume of the tetrahedron.

$\dfrac{h}{3}(\text{area of } \triangle ABD) = \dfrac{1}{3}(BE \sin \angle BEF)\left(\dfrac{1}{2}(24)(24)\sin 60°\right)$ 1M

$h \approx 20.6$ 1A

11. (a) $2 \times \dfrac{1}{2}(8)(7)\sin \angle ABC = 28\sqrt{3}$ 1M

$\angle ABC = 120°$

$AC^2 = 8^2 + 7^2 - 2(8)(7)\cos 120°$ 1M

$AC = 13\,\text{cm}$ 1A

(b) (i) $\dfrac{8}{\sin \angle ACB} = \dfrac{13}{\sin 120°}$ 1M

$\angle ACB \approx 32.2°$ or $148°$ (rejected)

Let E be a point on AC such that $BE \perp AC$.

$BE = 7 \sin \angle ACB \approx 3.73\,\text{cm}$ 1M

Note that $\angle BED = 80°$.

$\sin \dfrac{\angle BED}{2} = \dfrac{\frac{1}{2}BD}{BE}$

$BD \approx 4.80\,\text{cm}$

Required distance is 4.80 cm. 1A

(ii) Let X be the projection of B on the plane ACD.

We have $\alpha = \angle BCX$ and $\beta = \angle BDX$. 1M

Note that $BC > BD$.

$\sin \alpha = \dfrac{BX}{BC} < \dfrac{BX}{BD} = \sin \angle BDX = \sin \beta$

Since α and β are acute angles, α is smaller. 1A

12. (a) In $\triangle ABC$,

$$BC^2 = 5^2 + 10^2 - 2(5)(10)\cos 80^\circ \quad \text{1M}$$

$$BC \approx 10.4\text{ cm}$$

$$5^2 = 10^2 + BC^2 - 2(10)(BC)\cos\angle ABD$$

$$\angle ABD \approx 28.3^\circ$$

In $\triangle ABD$,

$$BD = 10\cos\angle ABD \quad \text{1M}$$

$$\approx 8.80\text{ cm} \quad \text{1A}$$

$DC = BC - BD \approx 1.57\text{ cm}$ 1A

(b) (i) In $\triangle ABC$, when $\theta = 45^\circ$,

$$BC^2 = 10^2 + 5^2 - 2(10)(5)\cos 45^\circ \quad \text{1M}$$

$$BC \approx 7.37\text{ cm} \quad \text{1A}$$

The angle between the faces ABD and ADC is $\angle BDC$. 1A

In $\triangle BDC$,

$$BC^2 = BD^2 + CD^2 - 2(BD)(CD)\cos\angle BDC$$

$$\angle BDC \approx 22.1^\circ \quad \text{1A}$$

$$< 25^\circ$$

The claim is agreed. 1A

(ii) In $\triangle ABC$, when $\theta = 40^\circ$,

$$BC^2 = 10^2 + 5^2 - 2(10)(5)\cos 40^\circ$$

$$BC \approx 6.96\text{ cm}$$

So, $BC + CD \approx 8.53\text{ cm} < BD$. 1M

This violates the triangle inequality, implying that this situation is impossible. 1

13. (a) Let K be a point on EF such that $CK \perp EF$.

Consider $\triangle CFK$.

$$\cos\angle CFK = \frac{\left(\frac{60-30}{2}\right)}{50}$$

$$\angle CFK \approx 72.5^\circ$$

$\angle DCF = 180^\circ - \angle CFK \approx 107^\circ$ 1A

Consider $\triangle DCF$.

$$DF^2 = 30^2 + 50^2 - 2(30)(50)\cos\angle DCF$$

$$DF \approx 65.6\,\text{cm}$$ 1A

(b) Let J be a point on FH such that $CJ \perp FH$.

$AC = \sqrt{30^2 + 30^2} = 30\sqrt{2}\,\text{cm}$

$HF = \sqrt{60^2 + 60^2} = 60\sqrt{2}\,\text{cm}$

Consider $\triangle CFJ$.

$$CJ = \sqrt{50^2 - \left(\frac{HF - AC}{2}\right)^2}$$ 1M

$$= 5\sqrt{82}\,\text{cm}$$

Height of the frustum is $5\sqrt{82}\,\text{cm}$. 1A

(c) $BD = AC = 30\sqrt{2}\,\text{cm}$

$BF = DF \approx 65.6\,\text{cm}$

Consider $\triangle BDF$.

Let $s = \dfrac{BD + BF + DF}{2}$.

Area of $\triangle BDF = \sqrt{s(s - BD)(s - DF)(s - BF)}$ 1M

$\approx 1320\,\text{cm}^2$

Let h cm be the required distance.

Consider the volume of the tetrahedron $CBDF$.

$$\frac{1}{3}(\text{area of } \triangle BDF)(h) = \frac{1}{3}\left[\frac{(30)(30)}{2}\right](5\sqrt{82})$$ 1M

$$h \approx 15.5$$ 1A

Required distance is 15.5 cm.

14. (a) $\angle QPS = 180° - 35° - 45° = 100°$

$\frac{PS}{\sin 35°} = \frac{58}{\sin 100°}$ 1M

$PS \approx 33.8$ cm 1A

(b) (i) $\frac{PQ}{\sin 45°} = \frac{58}{\sin 100°}$

$PQ \approx 41.6$ cm

$RS = PQ \approx 41.6$ cm

$30^2 = PS^2 + RS^2 - 2(PS)(RS)\cos\angle PSR$ 1M

$\angle PSR \approx 45.4°$ 1A

(ii) Let T be the projection of P on the plane QRS.

Required distance is PT. 1M

Let X be a point on QS such that $PX \perp QS$.

Let Y be a point on RS such that XTY is a straight line.

$SX = PS\cos 45° \approx 23.9$ cm 1M

$PX = PS\sin 45° \approx 23.9$ cm

$XY = SX\tan 35° \approx 16.7$ cm

$SY = \frac{SX}{\cos 35°} \approx 29.2$ cm

$PY^2 = PS^2 + SY^2 - 2(PS)(SY)\cos\angle PSY$

$PY \approx 24.7$ cm

$PY^2 = PX^2 + XY^2 - 2(PX)(XY)\cos\angle PXY$

$\angle PXY \approx 72.4°$

$PT = PX\sin\angle PXT$ 1M

≈ 22.8 cm

< 23 cm

The claim is agreed. 1A

15. (a) $MS = 2\sin 60° = \sqrt{3}$ cm

$PS = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ cm

$\sin\angle MPS = \frac{\sqrt{3}}{2\sqrt{2}}$ 1M

$\angle MPS \approx 37.8°$ 1A

(b) Let A be a point on PS such that $MA \perp PS$.

Let B be a point on ST such that $AB \perp PS$.

Required angle is $\angle MAB$. 1A

$MA = MP\sin\angle MPS \approx 1.37$ cm 1M

$PA = MP\cos\angle MPS \approx 1.77$ cm

$AS = PS - PA \approx 1.06$ cm

$AB = AS\tan 45° \approx 1.06$ cm

$BS = \frac{AS}{\cos 45°} = 1.5$ cm

$BM^2 = BS^2 + MS^2 - 2(BS)(MS)\cos 30°$ 1M

$BM \approx 0.866$ cm

$BM^2 = AB^2 + MA^2 - 2(AB)(MA)\cos\angle MAB$

$\angle MAB \approx 39.2°$ 1A

Required angle is 39.2°.

(c) Let E be a point on WS such that $DE \perp WS$.

Let F be a point on RQ such that $DF \perp RQ$.

Area of $\triangle DSW = \dfrac{(WS)(DE)}{2} = \dfrac{WS\sqrt{DF^2 + EF^2}}{2}$ 1M

Since WS and EF are constant, area of $\triangle DSW$ is minimum when DF is minimum, i.e., $DF = 0$.

When D is at P, $DF \neq 0$. The area is therefore not minimum when D is at P. 1A

16. (a) $\angle ACB = 180° - 40° - 56° = 84°$

$\dfrac{BC}{\sin 40°} = \dfrac{72}{\sin 84°}$ 1M

$BC \approx 46.5\text{ cm}$

Required area

$= \dfrac{1}{2}(72)(BC)\sin 56°$ 1M

$\approx 1390\text{ cm}^2$ 1A

(b) (i) $BD = \sqrt{72^2 + 40^2} = \sqrt{6784}\text{ cm}$ 1M

$\dfrac{AC}{\sin 56°} = \dfrac{72}{\sin 84°}$

$AC \approx 60.0\text{ cm}$

$CD = \sqrt{AC^2 + AD^2} \approx 72.1\text{ cm}$

Consider $\triangle BCD$.

Let $s = \dfrac{BD + BC + CD}{2} \approx 101\text{ cm}$.

Required area

$= \sqrt{s(s - BD)(s - BC)(s - CD)}$ 1M

$\approx 1670\text{ cm}^2$ 1A

(ii) Let h cm be the required distance.

$\dfrac{1}{3}(\text{area of }\triangle BCD)(h) = \dfrac{1}{3}(\text{area of }\triangle ABC)(AD)$ 1M

$h \approx 33.2$ 1A

17. (a) $\dfrac{\sin \angle BAD}{85} = \dfrac{\sin 60^\circ}{102}$ 1M

$\angle BAD \approx 46.2^\circ$ or 134° (rejected)

$\angle BDA = 180^\circ - \angle BAD - 60^\circ \approx 73.8^\circ$

$AB^2 = 102^2 + 85^2 - 2(102)(85)\cos \angle BDA$

$AB \approx 113\text{ cm}$ 1A

$\angle BDC = 140^\circ - \angle BDA \approx 66.2^\circ$

$BC = 85 \tan \angle BDC \approx 193\text{ cm}$ 1A

$CD = \dfrac{85}{\cos \angle BDC} \approx 211\text{ cm}$ 1A

(b) (i) Let G be a point on BC such that $AG \perp BC$.

Let H be a point on CD such that $GH \perp BC$.

Required angle is $\angle AGH$. 1M

$AC = \sqrt{CD^2 - AD^2} \approx 184\text{ cm}$

$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos \angle ACB$

$\angle ACB \approx 34.8^\circ$

$AG = AC \sin \angle ACB \approx 105\text{ cm}$ 1M

$CG = AC \cos \angle ACB \approx 151\text{ cm}$

$\dfrac{GH}{85} = \dfrac{CG}{BC}$

$GH \approx 66.7\text{ cm}$

$CH = \sqrt{GH^2 + CG^2} \approx 165\text{ cm}$

$\angle ACD = \tan^{-1} \dfrac{102}{AC} \approx 29.0^\circ$

$AH^2 = AC^2 + CH^2 - 2(AC)(CH)\cos \angle ACD$

$AH \approx 89.3\text{ cm}$

$AH^2 = AG^2 + GH^2 - 2(AG)(GH)\cos \angle AGH$ 1M

$\angle AGH \approx 57.5^\circ$ 1A

(ii) Note that B is at a direction of N15°E from F.

Consider the area of the shadow.

$$\frac{BD(BF \sin 15^\circ)}{2} + \frac{(BC)(BF \cos 15^\circ)}{2} = 2 \times 100^2$$ 1M

$$BF \approx 192\text{ cm}$$

Let K be the projection of A on the horizontal ground.

Note that F, B and K are collinear.

$AK = AG \sin \angle AGH \approx 88.8\text{ cm}$ 1M

$BK = \sqrt{AB^2 - AK^2} \approx 70.1\text{ cm}$

$\tan \phi = \dfrac{AK}{BF + BK}$ 1M

$\phi \approx 18.7^\circ < 20^\circ$

The claim is not correct. 1A

18. (a) $AB = 52 \times \frac{3}{4}$ 1M

$= 39\,\text{cm}$ 1A

$BC = 52 - 39 = 13\,\text{cm}$ 1A

(b) (i) $\frac{\sin \angle BAC}{13} = \frac{\sin 77^\circ}{39}$ 1M

$\angle BAC \approx 19.0^\circ$

$\angle ABC = 180^\circ - 77^\circ - \angle BAC \approx 84.0^\circ$ 1M

$AC^2 = 39^2 + 13^2 - 2(39)(13)\cos \angle ABC$ 1M

$AC \approx 39.8\,\text{cm}$ 1A

(ii) Let Q be a point on CF such that $BQ \perp CF$.

Area of $\triangle CDF = \frac{1}{2}(CF)(DQ)$

Area of $\triangle BCF = \frac{1}{2}(CF)(BQ)$

Note that $\angle BDQ = 90^\circ$. We have $BQ > DQ$, and

area of $\triangle BCF >$ area of $\triangle CDF$.

Required sum

$< 2(\text{area of } \triangle BCF)$

$= 2\left(\frac{1}{2}(13)(13)\sin(180^\circ - \angle ABC)\right)$ 1M

$\approx 168\,\text{cm}^2$

$< 168.5\,\text{cm}^2$

The sum of the areas is less than $168.5\,\text{cm}^2$. 1A

(iii) $FC^2 = 52^2 + AC^2 - 2(52)(AC)\cos \angle BAC$

$FC \approx 19.3\,\text{cm}$

$52^2 = AC^2 + FC^2 - 2(AC)(FC)\angle ACF$ 1M

$\angle ACF \approx 119^\circ$

$> 90^\circ$

Note that when P lies on the line segment CF, we have $\angle APF \geq ACF > 90^\circ$.

Thus, P does not lie on CF.

The claim is disagreed. 1A

19. (a) (i) $\dfrac{PR}{\sin 38°} = \dfrac{10}{\sin(180° - 96° - 38°)}$ 1M

$PR \approx 8.56\text{ cm}$ 1A

$TR^2 = 10^2 + PR^2 - 2(10)(PR)\cos 96°$ 1M

$TR \approx 13.8\text{ cm}$

$QR = TR - 8 \approx 5.83\text{ cm}$ 1A

(ii) $PQ^2 = 10^2 + 8^2 - 2(10)(8)\cos 38°$

$PQ \approx 6.16\text{ cm}$

$\dfrac{\sin \angle QPR}{QR} = \dfrac{\sin 46°}{PQ}$

$\angle QPR \approx 42.9°$ 1A

(b) (i) $\angle VPQ = 96° - \angle QPR \approx 53.1°$

$CP = VP\cos \angle VPQ \approx 6.00\text{ cm}$

$CR^2 = CP^2 + PR^2 - 2(CP)(PR)\cos \angle CPR$

$CR \approx 5.83\text{ cm}$ 1A

(ii) $CR^2 + CP^2 \approx 70.0\text{ cm}^2$

$PR^2 \approx 73.3\text{ cm}^2 \neq CR^2 + CP^2$

Thus, $\angle PCR \neq 90°$. 1M

$\angle VCR$ is not the angle between the face VPQ and the face PQR.

The claim is incorrect. 1A

20. (a) $\angle CTA = 180° - 42° - 30° = 108°$

$\dfrac{CA}{\sin 108°} = \dfrac{145}{\sin 42°}$ 1M

$CA \approx 206\text{ m}$ 1A

$AB^2 = 240^2 + AC^2 - 2(240)(AC)\cos 25°$ 1M

$AB \approx 102\text{ m}$ 1A

(b) Let T' be a point on AC such that $TT' \perp AC$.

The angle of elevation of T from P is $\angle TPT'$.

$\tan \angle TPT' = \dfrac{TT'}{T'P}$

The angle of elevation is greater when PT' is shorter. 1M

$240^2 = AC^2 + AB^2 - 2(AC)(AB)\cos \angle CAB$ 1M

$\angle CAB \approx 96.4° > 90°$

The length of $T'P$ is the shortest when P is at A.

The angle of elevation of T from P is the greatest when P is at A.

The claim is agreed. 1A

21. Let F be the mid-point of AD such that $EF \perp AD$.

$AF = \dfrac{2}{2} = 1\,\text{cm}$

$BF = \sqrt{2^2 + 1^2} = \sqrt{5}\,\text{cm}$

$EF = \sqrt{5}\tan 60^\circ = \sqrt{15}\,\text{cm}$

$$\begin{aligned}\text{Required volume} &= \frac{1}{3}(2^2)(\sqrt{15}) \\ &= \frac{4\sqrt{15}}{3}\,\text{cm}^3\end{aligned}$$

22. Let M be the mid-point of FG.

Then $\theta = \angle QPM$.

$$\begin{aligned}\tan\theta &= \frac{QM}{MP} \\ &= \frac{AF}{\frac{1}{2}(EF)} \\ &= 2\end{aligned}$$

23. Since BV is perpendicular to the plane VAC, $\angle BVA = \angle BVC = 90^\circ$.

$AB = BC = \sqrt{6^2 + 8^2} = 10\,\text{cm}$ and $BM = BN = 5\,\text{cm}$

$\triangle BMN$ is equilateral. So, $MN = 5\,\text{cm}$.

$\angle VBM = \tan^{-1}\dfrac{8}{6}$

$$VM^2 = 6^2 + 5^2 - (2)(6)(5)\cos\angle VBM$$

$$VM = 5$$

So, $VM = VN = MN = 5\,\text{cm}$, and the required area is $\dfrac{1}{2}(5)^2\sin 60^\circ = \dfrac{25\sqrt{3}}{4}\,\text{cm}^2$.

24. Let K be the mid-point of EF. The required angle is $\angle MNK$.

In $\triangle DEF$,

$$\frac{7}{\sin 50^\circ} = \frac{6}{\sin\angle DFE}$$

$$\angle DFE \approx 41.0^\circ \quad \text{or} \quad 139^\circ \text{ (rejected)}$$

$\angle DEF = 180^\circ - \angle DFE - 50^\circ \approx 89.0^\circ$

$$NK^2 = \left(\frac{6}{2}\right)^2 + \left(\frac{7}{2}\right)^2 - 2\left(\frac{6}{2}\right)\left(\frac{7}{2}\right)\cos\angle DEF$$

$$NK \approx 4.57\,\text{cm}$$

In $\triangle MNK$,

$$\tan\angle MNK = \frac{5}{NK}$$

$$\angle MNK \approx 48^\circ$$

25. $AP = \sqrt{36^2 + 48^2 + 63^2} = 87\,\text{cm}$

Let $s = \dfrac{AE + EP + AP}{2} = (k + 12)\,\text{cm}$.

$$\begin{aligned}\text{Required area} &= \sqrt{s(s - 87)(s - k)(s - (k - 63))} \\ &= \sqrt{(k + 12)(k - 75)(12)(75)} \\ &= \sqrt{900(k^2 - 63k - 900)}\,\text{cm}^2\end{aligned}$$

26. Let K be the mid-point of EF.

Note that $M_5K \perp M_4M_5$.

We have $M_1M_5 \perp M_4M_5$ by theorem of three perpendiculars.

Thus, $\theta = \angle KM_5M_1$.

Let $AB = 2x\,\text{cm}$.

$$\begin{aligned}\tan\theta &= \frac{KM_1}{M_1M_5} \\ &= \frac{2x}{\sqrt{x^2 + x^2}} \\ &= \sqrt{2}\end{aligned}$$

27. Let K be a point on AC such that $DK \perp AC$.

We have $x = \angle DKE$.

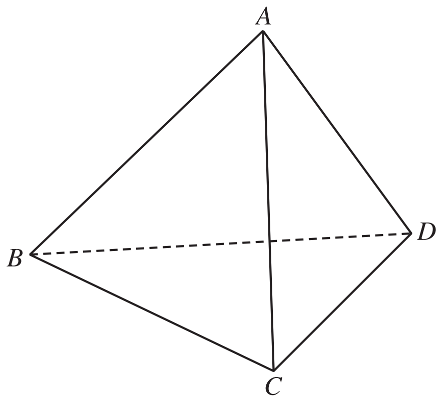
Consider the area of $\triangle ACD$.

$$\begin{aligned}\frac{(AD)(CD)}{2} &= \frac{(AC)(DK)}{2} \\ \frac{(3)(4)}{2} &= \frac{\sqrt{3^2 + 4^2}(DK)}{2} \\ DK &= 2.4\,\text{cm}\end{aligned}$$

Consider $\triangle DKE$.

$$\begin{aligned}\tan x &= \frac{DE}{DK} \\ &= \frac{2}{2.4} \\ &= \frac{5}{6}\end{aligned}$$

28. Refer to the figure. Let E be the mid-point of BC and length of each side be x cm.



In $\triangle AED$, $AE = DE = x\sin 60^\circ = \dfrac{\sqrt{3}x}{2}$ cm.

$$x^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$

$$\angle AED = \cos^{-1}\frac{1}{3}$$

Consider the volume of the tetrahedron.

$$\frac{1}{3}\left(\frac{x^2}{2}\sin 60^\circ\right)(AE\sin\angle AED) = 576$$

$$x \approx 17.0$$

Let X be the projection of A on the plane BCD.

Note that D, X, E are collinear.

Consider $\triangle AXE$.

Required height $= AE\sin\angle AED$

≈ 13.9 cm

29. Required angle is $\angle AMF$.

$FM = \sqrt{6^2 + 4^2} = \sqrt{52}$ cm

$\tan\angle AMF = \dfrac{14}{\sqrt{52}}$

$\angle AMF \approx 63^\circ$

30. Note that $\theta = \angle BKA$.

$AK = \sqrt{12^2 + 9^2} = 15\,\text{cm}$

$BK = \sqrt{15^2 + 8^2} = 17\,\text{cm}$

$$\cos\theta = \frac{AK}{BK} = \frac{15}{17}$$

31. $AY = \sqrt{4^2 + 3^2} = 5\,\text{cm}$

$XY = \dfrac{BV}{2} = \dfrac{6}{2} = 3\,\text{cm}$

Consider $\triangle AXY$.

Let $s = \dfrac{5 + 3 + 4k}{2} = 4 + 2k$.

$$\begin{aligned}\text{Required area} &= \sqrt{s(s-5)(s-3)(s-4k)} \\ &= \sqrt{(2k+4)(2k-1)(2k+1)(-2k+4)} \\ &= \sqrt{(4k^2-1)(4k^2-16)} \\ &= 2\sqrt{(4k^2-1)(k^2-4)}\end{aligned}$$

Note that when $k = 1$, the lengths of three sides of $\triangle AXY$ are 3 cm, 4 cm and 5 cm.

$\triangle AXY$ is a right-angled triangle of area $6\,\text{cm}^2$ when $k = 1$.

Check the value of each option when $k = 1$.

A. 0

B. 0

C. 6

D. ERROR

The answer is C.

32. Let K be a point on ME such that $FK \perp ME$. [In fact, K is at the position of point M.]

Since $AF \perp EM$, we also have $AK \perp ME$. The angle required is therefore $\angle AKF$.

Since $MH = EH = 12\,\text{cm}$, $\angle EMH = 45°$ and so $\angle FEM = 45°$

$FK = FE \sin\angle FEM = 12\sqrt{2}\,\text{cm}$

Required angle $= \tan^{-1}\dfrac{AF}{FK} = \dfrac{10}{12\sqrt{2}} \approx 31°$

33. Let N be a point on AC such that $BN \perp AC$.
Required angle is $\angle BND$.
Let $AB = AC = BC = CD = x$ cm.
$BD = \sqrt{x^2 + x^2} = \sqrt{2}x$ cm
$\sin 60° = \dfrac{BN}{x}$
$BN = \dfrac{\sqrt{3}x}{2}$ cm
Similarly, $DN = BN = \dfrac{\sqrt{3}x}{2}$ cm.
$BD^2 = BN^2 + DN^2 - 2(BN)(DN)\cos\angle BND$
$\angle BND \approx 109°$

34. A. Let $AB = BC = CD = DA = ED = FG = GH = HE = x$.
$BX = \sqrt{x^2 + \left(\frac{x}{2}\right)^2} = \dfrac{\sqrt{5}x}{2}$
$\tan\angle BXF = \dfrac{BF}{BX}$
$\tan\angle BXF = \dfrac{2x}{\left(\frac{\sqrt{5}x}{2}\right)}$
$\angle BXF \approx 60.8°$

B. Required angle is $\angle BHF$.
$FH = \sqrt{x^2 + x^2} = \sqrt{2}x$
$\tan\angle BHF = \dfrac{BF}{FH}$
$\tan\angle BHF = \dfrac{2x}{\sqrt{2}x}$
$\angle BHF \approx 54.7°$

C. Required angle is $\angle FXH$.
$HX = \sqrt{(2x)^2 + \left(\frac{x}{2}\right)^2} = \dfrac{\sqrt{17}x}{2}$
$AF = \sqrt{x^2 + (2x)^2} = \sqrt{5}x$
$FX = \sqrt{\left(\frac{x}{2}\right)^2 + (\sqrt{5}x)^2} = \dfrac{\sqrt{21}x}{2}$
$FH^2 = FX^2 + HX^2 - 2(FX)(HX)\cos\angle FXH$
$\angle FXH \approx 37.5°$

D. Let Y be a point on EH such that $XY \perp EH$.
Required angle is $\angle XHY$.
$\tan\angle XHY = \dfrac{XY}{HY}$
$\tan\angle XHY = \dfrac{2x}{\left(\frac{x}{2}\right)}$
$\angle XHY \approx 76.0°$

35. Let $s = \dfrac{6 + x + (x + 2)}{2} = (x + 4)$ cm.

Area of $\triangle ABC = \sqrt{s(s - AB)(s - BC)(s - AC)}$

$= \sqrt{(x + 4)(x - 2)(4)(2)}$

$= \sqrt{8(x + 4)(x - 2)}$ cm^2

$\sqrt{8(x + 4)(x - 2)} = \sqrt{216}$

$8x^2 + 16x - 280 = 0$

$x = 5 \quad \text{or} \quad -7$ (rejected)

$BD = \sqrt{5^2 + 5^2} = \sqrt{50}$ cm

$AD = \sqrt{(5 + 2)^2 + 5^2} = \sqrt{74}$ cm

$6^2 = AD^2 + BD^2 - 2(AD)(BD)\cos\angle ADB$

$\angle ADB \approx 44°$

36. I. ✓.

II. ✓.

III. ✗.

Note that θ, α and β lie between 0° and 90°.

Since $AB > AX > AC$, we have $\dfrac{1}{AB} < \dfrac{1}{AX} < \dfrac{1}{AC}$.

We have $\tan\theta = \dfrac{VA}{AB}$, $\tan\beta = \dfrac{VA}{AX}$ and $\tan\alpha = \dfrac{VA}{AC}$.

Thus, $\tan\theta < \tan\beta < \tan\alpha$ and $\theta < \beta < \alpha$.

37. Let E be the mid-point of AC.

We have $\theta = \angle BED$.

Let $BC = 2x$.

$BE = \sqrt{(2a)^2 - a^2} = \sqrt{3}a$

$DE = BE = \sqrt{3}a$

$(2a)^2 = BE^2 + DE^2 - 2(BE)(DE)\cos\theta$

$\cos\theta = \dfrac{1}{3}$

38. Let $PQ = 2$. Then $RS = 3$.

$RQ = \dfrac{PQ}{\tan 45^\circ} = 2$

$QS = \dfrac{PQ}{\tan 30^\circ} = 2\sqrt{3}$

Consider $\triangle QRS$.

$$RS^2 = RQ^2 + QS^2 - 2(RQ)(QS)\cos\angle RQS$$

$$\angle RQS \approx 60^\circ$$

39. Let E be a point on the plane BCD such that AE is perpendicular to the plane BCD.

Required angle is $\angle ABE$.

$\dfrac{1}{3}(335)(AE) = 2010$

$$AE = 18\,\text{cm}$$

$\sin\angle ABE = \dfrac{18}{26}$

$$\angle ABE \approx 43.8^\circ$$

40. Let M be a point on BC such that $AM \perp BC$.

Note that AMD is a plane of reflection of the tetrahedron and it is perpendicular to the plane BCD.

We have $\angle AMD = 80^\circ$ and the required angle is $\angle ADM$.

Consider $\triangle ABC$.

$AM = 56\sin 60^\circ = 28\sqrt{3}\,\text{cm}$

Consider $\triangle BCD$.

$DM = \sqrt{60^2 - 28^2} = \sqrt{2816}\,\text{cm}$

Consider $\triangle AMD$.

$AD^2 = AM^2 + DM^2 - 2(AM)(DM)\cos 80^\circ$

$AD \approx 65.4\,\text{cm}$

$AM^2 = AD^2 + DM^2 - 2(AD)(DM)\cos\angle ADM$

$\angle ADM \approx 47^\circ$

END OF PAPER