Solution	Marks
ELITE-2425-MOCK-SET 5-MATH-CP 1	
Suggested solutions	
1. $\dfrac{(x^{-1}y)^8}{x^{16}y^{-3}} = \dfrac{x^{-8}y^8}{x^{16}y^{-3}}$	1M
$= \dfrac{y^{8+3}}{x^{16+8}}$	1M
$= \dfrac{y^{11}}{x^{24}}$	1A
2. $\dfrac{3-2ab}{a} = 3-b$	
$3-2ab = 3a-ab$	1M
$-ab-3a = -3$	1M
$a = \dfrac{3}{b+3}$	1A
3. (a) $2x^2-3x-2 = (2x+1)(x-2)$	1A
(b) $6x^2y+3xy-2x^2+3x+2 = 3xy(2x+1)-(2x+1)(x-2)$	1M
$= (2x+1)(3xy-x+2)$	1A
4. (a) $\dfrac{28+3x}{4} \geq 3x-2$	
$28+3x \geq 12x-8$	
$-9x \geq -36$	
$x \leq 4$	1A
(b) $x \leq 4$ or $x < 3$.	1A
Thus, $x \leq 4$.	1M
Required positive integers are 1, 2, 3 and 4.	1A
5. (a) Required number $= \dfrac{81}{1+35\%} = 60$	1A
(b) Suppose Peter gives Simon n stamps.	
$81-n = 60+n$	1M
$n = 10.5$ (rejected)	1A
Since n must be a non-negative integer, it is impossible for them to have equal number of stamps.	1A
6. Let the original number of girls in the summer camp be $7x$. Then the original number of boys is $8x$.	1M
$8x-16 = 7x-11$	1M
$x = 5$	1A
Required number $= 7 \times 5 = 35$	1A

Solution	Marks
7. (a) $55 - 49 > 64 - k$	1M
$k > 58$	
Required value is 59.	1A
(b) Median score of Test B (63) is greater than the highest score of Test A (k).	1A
The claim is agreed.	1A
8. Since $AF = BF$, $\angle FAB = \angle ABF = 35°$.	1A
$\angle BFC = 35° + 35° = 70°$	
Since $BE // CD$, $\angle GCD = \angle BFC = 70°$.	1M
$\angle BDC = \angle BAC = 35°$	1M
Consider $\triangle CDG$, $\angle BGC = 35° + 70° = 105°$.	1A
9. (a) $DE = DE$ *(common side)*	
$\angle DFE = 90°$ *(given)*	
$\angle DCE = 90°$ *(property of rectangle)*	
$= \angle DFE$	
$AD = AE$ *(given)*	
$\angle AED = \angle ADE$ *(base $\angle$s, isos. $\triangle$)*	
$AD // BC$ *(property of rectangle)*	
$\angle CED = \angle ADE$ *(alt. $\angle$s, $AD // BC$)*	
$= \angle AED$	
$\triangle CDE \cong \triangle FDE$ *(AAS)*	

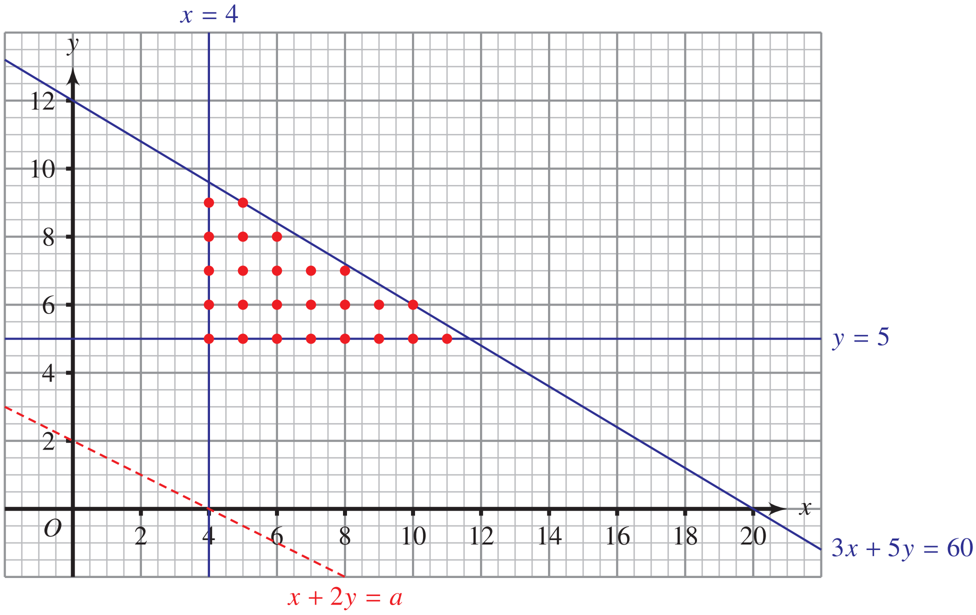
Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) $AF = AE - FE = AE - CE = 5 - 1 = 4\text{ cm}$	1M
$DF = \sqrt{AD^2 - AF^2} = \sqrt{5^2 - 4^2} = 3\text{ cm}$	1M
Area of $\triangle ADF = \frac{1}{2}(4)(3) = 6\text{ cm}^2$	1A

Solution	Marks
10. (a) Let $C = a + br^2$, where a and b are non-zero constants.	1A
$\begin{cases} 67 = a + b \\ 112 = a + 4^2 b \end{cases}$	1M
Solving, we have $a = 64$ and $b = 3$.	1A
Required cost $= 64 + 3(3)^2 = \$91$.	1A
(b) Let the radius of the larger sphere be r cm.	
$\left(\frac{r}{3}\right)^3 = \frac{8}{1}$	
$r = 6$	1A
Required cost $= 64 + 3(6)^2 = \$172$	1A
11. (a) $f(x) = (x-2)(x^2 - 3x + 4) + r$	1A
So, $(x-1)g(x) \equiv (x-2)(x^2 - 3x + 4) + r$.	
Put $x = 1$,	
$0 = (-1)(1 - 3 + 4) + r$	1M
$r = 2$	1A
(b) $f(x) = (x-2)(x^2 - 3x + 4) + 2$	
$= x^3 - 5x^2 + 10x - 6$	
$= (x-1)(x^2 - 4x + 6)$	1M
Therefore, $g(x) = x^2 - 4x + 6$. When $g(x) = 0$,	1A
$\Delta = 4^2 - 4(1)(6) = -8 < 0$	1M
The roots of $g(x) = 0$ are not real, i.e., they are not rational numbers.	
The claim is disagreed.	1A

Solution	Marks
12. (a) The coordinates of V are $(-3,\ -4)$.	1A
When $x = 0$, $y = 3^2 - 4 = 5$.	
The coordinates of A are $(0,\ 5)$.	1A
(b) Slope of $VA = \dfrac{5+4}{0+3} = 3$	
Slope of $L = -\dfrac{1}{3}$	1M
Equation of L is	
$y + 4 = -\dfrac{1}{3}(x+3)$	1M
$x + 3y + 15 = 0$	1A
(c) The coordinates of C and D are $(-15,\ 0)$ and $(0,\ -5)$ respectively.	
The coordinates of mid-point of CD are $\left(-\dfrac{15}{2},\ -\dfrac{5}{2}\right)$.	1M
Required equation of locus is	
$\left(x + \dfrac{15}{2}\right)^2 + \left(y + \dfrac{5}{2}\right)^2 = 4^2$	1M
$\left(x + \dfrac{15}{2}\right)^2 + \left(y + \dfrac{5}{2}\right)^2 = 16$	1A
13. (a) Suppose the frustum is formed by cutting off a small cone of height h cm from a larger cone of height $(h+3)$ cm.	
$\dfrac{h+3}{h} = \dfrac{4}{2}$	1M
$h = 3$	
Volume of the cream $= 4^2\pi(7-3) + \dfrac{1}{3}\pi(4)^2(3+3) - \dfrac{1}{3}\pi(2)^2(3)$	1M+1A
$= 92\pi \text{ cm}^3$	1A
(b) New volume of cream $= 4^2\pi(7-4) + \dfrac{1}{2} \times \dfrac{4}{3}\pi(4)^3$	1M+1A
$= \dfrac{272\pi}{3} \text{ cm}^3$	
Percentage change	
$= \dfrac{\frac{272\pi}{3} - 92\pi}{92\pi} \times 100\%$	1M
$\approx -1.45\%$	
$> -5\%$	
The claim is incorrect.	1A

Solution	Marks
14. (a) (i) $\dfrac{(20+b)+30}{2}=29$	
$b=8$	1A
$43-(10+a)=27$	
$a=6$	1A
(ii) Mean $=\dfrac{16+17+18+\ldots+43}{20}$	
$=28.8$	1A
(b) (i) Sum of ages of the new member $=16+43=59$	1M
If the ages of them are 29 and 30, the new median is 29.5.	
It is possible that the median of the distribution is changed.	1A
(ii) As the mode has two values in the new distribution, there are only three cases of the ages of the new members: $(18,\ 41)$, $(21,\ 38)$ and $(25,\ 34)$	
In any of the case, new range $=42-17=25\neq 27$	1M
It is impossible to keep the range unchanged.	1A
15. (a) $\dfrac{2}{1+i}\times\dfrac{2}{1-i}=\dfrac{2}{a}$	1M
$\dfrac{4}{1+1}=\dfrac{2}{a}$	
$a=1$	1A
$\dfrac{2}{1+i}+\dfrac{2}{1-i}=-\dfrac{b}{1}$	
$\dfrac{(2-2i)+(2+2i)}{1+1}=-b$	
$b=-2$	1A
(b) Let $g(x)=f(x)+k=x^2-2x+2+k$, where $k\neq 0$.	
If $y=g(x)$ has two x-intercepts, then $g(x)=0$ has two distinct real roots and	
$\Delta=2^2-4(1)(2+k)>0$	1M
$-4-4k>0$	
$k<-1$	
The graph of $y=g(x)$ is obtained by translating the graph of $y=f(x)$ downwards by more than 1 unit.	1A
16. (a) Required probability $=\dfrac{C_5^7+C_5^5}{C_5^{15}}$	1M
$=\dfrac{2}{273}$	1A
(b) Required probability $=1-\dfrac{2}{273}-\dfrac{C_4^7C_1^8+C_4^5C_1^{10}}{C_5^{15}}$	1M
$=\dfrac{241}{273}$	1A

Solution	Marks
17. (a) The constraints are	
$\begin{cases} x \geq 4 \\ y \geq 5 \\ 9000x + 15\,000y \leq 180\,000 \text{ or } 3x + 5y \leq 60 \\ x \text{ and } y \text{ are non-negative integers} \end{cases}$	1A+1A
(b) The dots represent the feasible solutions.	
	
(for any one line)	1A
(for the region)	1A
(c) Let \$$P$ be the monthly profit from recruiting x assistant account managers and y designers.	
$P = 4x(3000) + 5y(4800)$	
$= 12\,000(x + 2y)$	1A
Draw the line $x + 2y = a$, where a is a constant.	
P attains its maximum value at $(5,\ 9)$.	1M
The total number of recruited staff is 14, not 15.	
The claim is disagreed.	1A

Solution	Marks
18. (a) $AM = 10\sin 60° \approx 8.66\text{ cm}$	1A
$\angle MAP = \dfrac{60°}{4} = 15°$	
$PQ = 2PM$	
$= 2 \times AM \tan 15°$	1M
$\approx 4.64\text{ cm}$	1A
(b) As the planes APB and AQC are identical and vertical, vertical distance from M to the ground is equal to the vertical distance from P to the ground.	
With reference of the second figure, the vertical distance = PM.	1M
Vertical distance $= 10\sin 60° \tan 15°$	
$\approx 2.32\text{ cm}$	1A
(c) Let P' and Q' be the projections of P and Q on the horizontal ground respectively.	
$AP' = AQ' = AM \approx 8.66\text{ cm}$	1A
$P'Q' = PQ \approx 4.64\text{ cm}$	
In $\triangle AP'Q'$,	
$P'Q'^2 = AP'^2 + AQ'^2 - 2(AP')(AQ')\cos\angle P'AQ'$	1M
$\angle P'AQ' \approx 31.1°$	
Thus, $\angle BAC \approx 31.1°$	1A

Solution	Marks

19. (a) $\angle CAB = \angle BAD$ *(common ∠)*

$\angle ABC = \angle ADB$ *(∠ in alt. segment)*

$\triangle ABC \sim \triangle ADB$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $\dfrac{AC}{AB} = \dfrac{AB}{AD}$ — 1M

$$\frac{\sqrt{9^2+12^2}}{36+9} = \frac{36+9}{\sqrt{9^2+12^2}+CD}$$

$CD = 120$

Radius of Γ is 60.

Note that $\angle EBA = 90°$.

The coordinates of E are $(60,\ 36)$. — 1M

Required equation is

$$(x-60)^2 + (y-36)^2 = 60^2$$

$$(x-60)^2 + (y-36)^2 = 3600$$ — 1A

(ii) E is the mid-point of CD.

The coordinates of D are $(108,\ 72)$. — 1A

Let $x^2 + y^2 + dx + ey + f = 0$ be the equation of the circumcircle of $\triangle BED$.

$$\begin{cases} 0^2 + 36^2 + 0 + 36e + f = 0 \\ 60^2 + 36^2 + 60d + 36e + f = 0 \\ 108^2 + 72^2 + 108d + 72e + f = 0 \end{cases}$$ — 1M

Solving, we have $d = -60$, $e = -252$ and $f = 7776$.

Required equation is $x^2 + y^2 - 60x - 252y + 7776 = 0$. — 1A

> E is the mid-point of CD.
>
> The coordinates of D are $(108,\ 72)$. — 1A
>
> Note that the centre of the circumcircle lies on the perpendicular bisector of BE, which is $x = 30$.
>
> Let the coordinates of the centre of the circumcentre of $\triangle BED$ be $(30,\ k)$.
>
> $$\sqrt{(30-0)^2 + (k-36)^2} = \sqrt{(30-108)^2 + (k-72)^2}$$ — 1M
>
> $$k^2 - 72k + 2196 = k^2 - 144k + 11\,268$$
>
> $$k = 126$$
>
> Required equation is
>
> $$(x-30)^2 + (y-126)^2 = (0-30)^2 + (36-126)^2$$
>
> $$(x-30)^2 + (y-126)^2 = 9000$$ — 1A

Solution	Marks
(iii) Area of the circumcircle of $\triangle BED$	
$= (\sqrt{30^2 + 126^2 - 7776})^2 \pi$	1M
$\approx 28\,300$	
Let r be the radius of the inscribed circle of $\triangle AED$.	
Consider the area of $\triangle AED$.	
$\frac{(36+9)(108)}{2} = \frac{(AB)(r)}{2} + \frac{(BD)(r)}{2} + \frac{(AD)(r)}{2}$	1M
$r \approx 16.5$	
(Area of the inscribed circle of $\triangle BED$) $\times 30$	
$= \pi r^2 \times 30$	
$\approx 25\,800$	
$<$ area of the circumcircle of $\triangle BED$	
The claim is agreed.	1

END OF PAPER