

Solution	Marks
<p>ELITE-2425-MOCK-SET 2-MATH-CP 1</p> <p>Suggested solutions</p> <p>1. $\frac{(x^9y^5)^3}{x^{30}y^{-4}} = \frac{x^{27}y^{15}}{x^{30}y^{-4}}$</p> <p>$= \frac{y^{15+4}}{x^{30-27}}$</p> <p>$= \frac{y^{19}}{x^3}$</p> <p>2. We have $y - x = 123$ and $\frac{x}{y} = \frac{5}{8}$.</p> <p>$y - \frac{5y}{8} = 123$</p> <p>$y = 328$</p> <p>Required product = $(328 - 123)(328)$</p> <p>$= 67\,240$</p> <p>3. $\frac{5m - n}{1 + mn} = 2$</p> <p>$5m - n = 2 + 2mn$</p> <p>$5m - 2mn = 2 + n$</p> <p>$m = \frac{2 + n}{5 - 2n}$</p> <p>4. (a) Required distance = 2×55</p> <p>$= 110 \text{ m}$</p> <p>(b) $\tan \angle OPQ = \frac{110}{60}$</p> <p>$\angle OPQ \approx 61.4^\circ$</p> <p>Required bearing is N61.4°W.</p> <p>5. Let the marked price of the bag be \$x.</p> <p>$x(1 - 10\%) - \frac{x}{1 + 25\%} = 75$</p> <p>$0.1x = 75$</p> <p>$x = 750$</p> <p>Required marked price is \$750.</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>2M+1A</p> <p>1A</p>

Solution	Marks
<p>6. (a) $x - 2 \leq 0$</p> $x \leq 2$ $x - \frac{3x+2}{2} < 2$ $-\frac{x}{2} < 3$ $x > -6$ <p>Thus, we have $-6 < x \leq 2$.</p> <p>(b) 4</p>	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>
<p>7. (a) Let the radius of the smaller hemisphere be r cm.</p> $\frac{2}{3}\pi r^3 \left(1 + \frac{125}{64}\right) = 1008\pi$ $r^3 = 512$ $r = 8$ <p>(b) Total surface area $= 2\pi(8)^2 + \pi(8)^2$</p> $= 192\pi \text{ cm}^2$	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>
<p>8. (a) $2.6 = \frac{1(9) + 2(16) + 3(14) + 4(8) + 5(a)}{9 + 16 + 14 + 8 + a}$</p> $115 + 5a = 122.2 + 2.6a$ $a = 3$ <p>Median = 2.5</p> <p>Mode = 2</p> <p>(b) Required probability $= \frac{14 + 8 + 3}{50}$</p> $= \frac{1}{2}$	<p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>
<p>9. (a) $2.55 \text{ m} \leq \text{actual length} < 2.65 \text{ m}$</p> <p>(b) Least total length of the 180 smaller pieces of ribbon</p> $= 180 \times 1.495$ $= 269.1 \text{ cm}$ $> 2.65 \text{ m}$ <p>The claim is disagreed.</p>	<p>1M+1A</p> <p>1M</p> <p>1A</p> <p>1A</p>

Solution	Marks
<p>10. (a) Γ is the perpendicular bisector of AB. 1A</p> <p>(b) (i) $\sqrt{(x-11)^2 + (y-17)^2} = \sqrt{(x+7)^2 + (y+1)^2}$ 1M</p> $-36x - 36y + 360 = 0$ $x + y - 10 = 0$ 1A <p>(ii) Let (h, k) be the centre of the circle.</p> <p>Since the centre lies on Γ, we have $h + k - 10 = 0$. 1M</p> <p>Note that the centre is equidistant from A and D.</p> $\sqrt{(h-11)^2 + (k-17)^2} = \sqrt{(h+1)^2 + (k-11)^2}$ 1M $-24h - 12k + 288 = 0$ $2h + k - 24 = 0$ <p>Solving, we have $h = 14$ and $k = -4$.</p> <p>Required coordinates are $(14, -4)$. 1A</p>	
<p>11. (a) Mean = $\frac{2.7 + 3.1 + 2.9 + 2.8 + 3.4 + 2.8 + 3.3}{7}$</p> $= 3 \text{ kg}$ 1A <p>Range = $3.4 - 2.7$</p> $= 0.7 \text{ kg}$ 1A <p>(b) (i) Let x kg be the required total weight.</p> $\frac{7(3) + 3.3 + 3.4 + x}{11} = 3$ 1M $x = 5.3$ 1A <p>(ii) Let a kg and b kg be the weights of the two newly-added baby girls.</p> <p>Suppose the range remains unchanged, we have $2.7 \leq a \leq 3.4$ and $2.7 \leq b \leq 3.4$. 1M</p> <p>We have $a + b \geq 2.7 + 2.7 = 5.4$, which is a contradiction.</p> <p>Thus, it is not possible. 1A</p>	

Solution			Marks									
12. (a)	$\angle CFE = 90^\circ$ (given) $\angle CFB = 90^\circ$ (given) $\angle DFE = \angle CFB = 90^\circ$ (vert. opp. \angle s) $\angle CED = 90^\circ$ (prop. of rectangle) $\angle FED = 90^\circ - \angle CEF$ $\angle CEF + \angle EFC + \angle FCE = 180^\circ$ (\angle sum of \triangle) $\angle CEF + 90^\circ + \angle FCE = 180^\circ$ $\angle FCE = 90^\circ - \angle CEF$ $= \angle FED$ $\triangle CEF \sim \triangle EDF$ (AA)											
<table border="1"><tr><th colspan="3">Marking Scheme</th></tr><tr><td>Case 1</td><td>Any correct proof with correct reasons.</td><td>2</td></tr><tr><td>Case 2</td><td>Any correct proof without reasons.</td><td>1</td></tr></table>				Marking Scheme			Case 1	Any correct proof with correct reasons.	2	Case 2	Any correct proof without reasons.	1
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(b) (i)	Note that $CE = AD = BC$ and $CF \perp BE$. We have $BF = EF$. F is the mid-point of BE .		1M 1									
(ii)	$EF = BF = 12$ cm $CF = \sqrt{15^2 - 12^2} = 9$ cm Since $\triangle CEF \sim \triangle EDF$. $\frac{ED}{CE} = \frac{EF}{CF}$ $\frac{ED}{15} = \frac{12}{9}$ $ED = 20$ cm		1M 1A									

Solution	Marks
<p>13. (a) Let $f(x) = a + b(x - 2)(x - 1)$, where a and b are non-zero constants.</p> $\begin{cases} 16 = a + b(10 - 2)(10 - 1) \\ -50 = a + b(4 - 2)(4 - 1) \end{cases}$ <p>Solving, we have $a = -56$ and $b = 1$.</p> $0 = -56 + (x - 2)(x - 1)$ $0 = x^2 - 3x - 54$ $x = 9 \quad \text{or} \quad -6$ <p>The x-intercepts are 9 and -6.</p> <p>(b) (i) $x = \frac{3}{2}$</p> <p>(ii) The coordinates of A and B are $(-6, 0)$ and $(9, 0)$ respectively. The coordinates of C are $(0, -54)$.</p> $-54 = -56 + (x - 2)(x - 1)$ $0 = x^2 - 3x$ $x = 0 \quad \text{or} \quad 3$ <p>The coordinates of D are $(3, -54)$.</p> <p>The area of the polygon is the least when the coordinates of Q are $\left(\frac{3}{2}, -\frac{1}{4}\right)$.</p> <p>Minimum area of the polygon $ACDBQ$</p> $= \frac{[(9 + 6) + (3 - 0)](54)}{2} - \frac{(9 + 6)\left(\frac{1}{4}\right)}{2}$ $= 484.125$ < 485 <p>The claim is disagreed.</p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p>

Solution		Marks
14. (a)	Let h cm be the required height. Note that the length of the square base of the smaller square pyramid is also h cm. $27 \times \frac{1}{3} \times h^2 \times h = 4 \times \left[12^3 - \frac{1}{3} \times 12^2 \times 12 \right]$ $h^3 = 512$ $h = 8$	1M 1M 1A
(b)	Required ratio = $8^2 : 12^2$ $= 4 : 9$	1M 1A
(c)	Let k cm be the length of the metal cube and H cm be the height of the smaller square pyramid. The height of the largest square pyramid is k cm. $27 \times \frac{1}{3} \times H^2 \times H = 4 \times \left[k^3 - \frac{1}{3} \times k^2 \times k \right]$ $\left(\frac{H}{k} \right)^3 = \frac{8}{27}$ $\frac{H}{k} = \frac{2}{3}$ The ratio in (b) becomes $2^2 : 3^2 = 4 : 9$, which is the same as before. The claim is disagreed.	1M 1A
15. (a)	Standard score = $\frac{74 - 64}{4}$ $= 2.5$	1M 1A
(b)	Standard score of Samuel after the adjustment $= \frac{74(1 + 10\%) - 64(1 + 10\%)}{4(1 + 10\%)}$ $= 2.5$ < 2.75 Sophia performs better in the test.	1M 1A
16. (a)	Required probability = $\frac{C_1^4 \times C_3^5}{C_4^9}$ $= \frac{20}{63}$	1M 1A
(b)	Required probability = $1 - \frac{20}{63} - \frac{C_3^4 \times C_1^5}{C_4^9}$ $= \frac{11}{21}$	1M 1A

Solution	Marks
<p>17. (a) Let G be the centre of the circle. The coordinates of G are $(3, -2)$. Slope of $AG = \frac{-1+2}{1-3} = -\frac{1}{2}$ Required equation is</p> $y + 1 = 2(x - 1)$ $2x - y - 3 = 0$ <p>(b) (i) The equation of L is $y = 2x + b$.</p> $x^2 + (2x + b)^2 - 6x + 4(2x + b) + 8 = 0$ $5x^2 + (4b + 2)x + (b^2 + 4b + 8) = 0$ <p>L and C intersect at two distinct points.</p> $(4b + 2)^2 - 4(5)(b^2 + 4b + 8) > 0$ $-4b^2 - 64b - 156 > 0$ $-13 < b < -3$ <p>(ii) Let the equation of L' be $y = 2x + k$.</p> $5x^2 + (4k + 2)x + (k^2 + 4k + 8) = 0$ <p>x_1 and x_2 are roots of the quadratic equation.</p> $x_1x_2 = 4$ $\frac{k^2 + 4k + 8}{5} = 4$ $k^2 + 4k - 12 = 0$ $k = -6 \quad \text{or} \quad 2 \text{ (rejected)}$ <p>The equation of L' is $y = 2x - 6$.</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>

Solution		Marks
18. (a)	$75^2 = 29^2 + 92^2 - 2(29)(92) \cos \angle ABC$ $\cos \angle ABC = \frac{20}{29}$ $BD = 29 \cos \angle ABC = 20 \text{ cm}$ $AD = \sqrt{29^2 - 20^2} = 21 \text{ cm}$	1M
(b) (i)	Required angle is $\angle BDC$.	
	$BC^2 = 75^2 + 29^2 - 2(75)(29) \cos 60^\circ$ $BC = \sqrt{4291} \text{ cm}$	1A
	$CD = 92 - 20 = 72 \text{ cm}$ $BC^2 = 72^2 + 20^2 - 2(72)(20) \cos \angle BDC$ $\angle BDC \approx 63.3^\circ$	1A
	(ii) Required volume	
	$= \frac{1}{3} \times \left(\frac{20 \times 21}{2} \right) \times 72 \sin \angle BDC$ $\approx 4500 \text{ cm}^3$	1M
		1A

Solution	Marks
<p>19. (a) $f(x) = x^2 - (8k + 6)x + 16k^2 + 22k + 5$ $= [x^2 - 2(4k + 3)x + (4k + 3)^2] - 2k - 4$ $= [x - (4k + 3)]^2 - 2k - 4$ The coordinates of M are $(4k + 3, -2k - 4)$.</p> <p>(b) (i) $(4k + 9, 2k + 4)$</p> <p>(ii) Since P is the orthocentre of $\triangle PMN$, we have $\angle MPN = 90^\circ$. $\frac{y - (-2k - 4)}{x - (4k + 3)} \times \frac{y - (2k + 4)}{x - (4k + 9)} = -1$ $y^2 - (2k + 4)^2 = -[x - (4k + 3)][x - (4k + 9)]$ $x^2 + y^2 - (8k + 12)x + 12k^2 + 32k + 11 = 0$</p> <p>(iii) UV is the perpendicular bisector of MN. The coordinates of the mid-point of MN are $(4k + 6, 0)$. Slope of $MN = \frac{(2k + 4) - (-2k - 4)}{(4k + 9) - (4k + 3)} = \frac{2k + 4}{3}$ Required equation is $y - 0 = -\frac{3}{2k + 4}[x - (4k + 6)]$ $y = -\frac{3x}{2k + 4} + \frac{3(2k + 3)}{k + 2}$</p> <p>(iv) Note that $MN^2 = MU^2 + NU^2 = 2MU^2$. Area of square $MUNV$ $= MU^2$ $= \frac{MN^2}{2}$ $= \frac{1}{2} [((4k + 9) - (4k + 3))^2 + ((2k + 4) + (2k + 4))^2]$ $= \frac{1}{2} [36 + (4k + 8)^2]$ $= 18 + 8(k + 2)^2$ ≥ 18 The claim is agreed.</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1M+1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p>