

Solution	Marks
<b>ELITE-2425-MOCK-SET 2-MATH-CP 1</b>	
<b>Suggested solutions</b>	
1. $\begin{aligned} \frac{(x^9 y^5)^3}{x^{30} y^{-4}} &= \frac{x^{27} y^{15}}{x^{30} y^{-4}} \\ &= \frac{y^{15+4}}{x^{30-27}} \\ &= \frac{y^{19}}{x^3} \end{aligned}$	1M 1M 1A
2. We have $y - x = 123$ and $\frac{x}{y} = \frac{5}{8}$ .	1A
$y - \frac{5y}{8} = 123$	1M
$y = 328$	
Required product = $(328 - 123)(328)$	
$= 67240$	1A
3. $\begin{aligned} \frac{5m - n}{1 + mn} &= 2 \\ 5m - n &= 2 + 2mn \\ 5m - 2mn &= 2 + n \\ m &= \frac{2 + n}{5 - 2n} \end{aligned}$	1M 1M 1A
4. (a) Required distance = $2 \times 55$	1M
$= 110 \text{ m}$	1A
(b) $\tan \angle OPQ = \frac{110}{60}$	1M
$\angle OPQ \approx 61.4^\circ$	
Required bearing is N $61.4^\circ$ W.	1A
5. Let the marked price of the bag be $\$x$ .	
$\begin{aligned} x(1 - 10\%) - \frac{x}{1 + 25\%} &= 75 \\ 0.1x &= 75 \\ x &= 750 \end{aligned}$	2M+1A 1A
Required marked price is \$750.	

Solution	Marks
6. (a) $x - 2 \leq 0$ $x \leq 2$ $x - \frac{3x + 2}{2} < 2$ $-\frac{x}{2} < 3$ $x > -6$ <p>Thus, we have <math>-6 &lt; x \leq 2</math>.</p>	1A 1A 1M 1A
(b) 4	1A
7. (a) Let the radius of the smaller hemisphere be $r$ cm. $\frac{2}{3}\pi r^3 \left(1 + \frac{125}{64}\right) = 1008\pi$ $r^3 = 512$ $r = 8$	1M 1A
(b) Total surface area = $2\pi(8)^2 + \pi(8)^2$ $= 192\pi \text{ cm}^2$	1M 1A
8. (a) $2.6 = \frac{1(9) + 2(16) + 3(14) + 4(8) + 5(a)}{9 + 16 + 14 + 8 + a}$ $115 + 5a = 122.2 + 2.6a$ $a = 3$ <p>Median = 2.5 Mode = 2</p>	1M 1A 1A
(b) Required probability = $\frac{14 + 8 + 3}{50}$ $= \frac{1}{2}$	1M 1A
9. (a) $2.55 \text{ m} \leq \text{actual length} < 2.65 \text{ m}$ (b) Least total length of the 180 smaller pieces of ribbon $= 180 \times 1.495$ $= 269.1 \text{ cm}$ $> 2.65 \text{ m}$ <p>The claim is disagreed.</p>	1M+1A 1M 1A 1A

Solution	Marks
10. (a) $\Gamma$ is the perpendicular bisector of $AB$ .	1A
(b) (i) $\sqrt{(x - 11)^2 + (y - 17)^2} = \sqrt{(x + 7)^2 + (y + 1)^2}$	1M
$-36x - 36y + 360 = 0$	
$x + y - 10 = 0$	1A
(ii) Let $(h, k)$ be the centre of the circle.	
Since the centre lies on $\Gamma$ , we have $h + k - 10 = 0$ .	1M
Note that the centre is equidistant from $A$ and $D$ .	
$\sqrt{(h - 11)^2 + (k - 17)^2} = \sqrt{(h + 1)^2 + (k - 11)^2}$	1M
$-24h - 12k + 288 = 0$	
$2h + k - 24 = 0$	
Solving, we have $h = 14$ and $k = -4$ .	
Required coordinates are $(14, -4)$ .	1A
11. (a) Mean = $\frac{2.7 + 3.1 + 2.9 + 2.8 + 3.4 + 2.8 + 3.3}{7}$	
$= 3$ kg	1A
Range = $3.4 - 2.7$	
$= 0.7$ kg	1A
(b) (i) Let $x$ kg be the required total weight.	
$\frac{7(3) + 3.3 + 3.4 + x}{11} = 3$	1M
$x = 5.3$	1A
(ii) Let $a$ kg and $b$ kg be the weights of the two newly-added baby girls.	
Suppose the range remains unchanged, we have $2.7 \leq a \leq 3.4$ and $2.7 \leq b \leq 3.4$ .	1M
We have $a + b \geq 2.7 + 2.7 = 5.4$ , which is a contradiction.	
Thus, it is not possible.	1A

Solution	Marks
12. (a) $\angle CFE = 90^\circ$ <i>(given)</i> $\angle CFB = 90^\circ$ <i>(given)</i> $\angle DFE = \angle CFB = 90^\circ$ <i>(vert. opp. <math>\angle</math>s)</i> $\angle CED = 90^\circ$ <i>(prop. of rectangle)</i> $\angle FED = 90^\circ - \angle CEF$ $\angle CEF + \angle EFC + \angle FCE = 180^\circ$ <i>(<math>\angle</math> sum of <math>\triangle</math>)</i> $\angle CEF + 90^\circ + \angle FCE = 180^\circ$ $\angle FCE = 90^\circ - \angle CEF$ $= \angle FED$ $\triangle CEF \sim \triangle EDF$ <i>(AA)</i>	
<b>Marking Scheme</b>	
<b>Case 1</b> Any correct proof with correct reasons.	2
<b>Case 2</b> Any correct proof without reasons.	1
(b) (i) Note that $CE = AD = BC$ and $CF \perp BE$ . We have $BF = EF$ . $F$ is the mid-point of $BE$ .	1M 1
(ii) $EF = BF = 12 \text{ cm}$ $CF = \sqrt{15^2 - 12^2} = 9 \text{ cm}$ Since $\triangle CEF \sim \triangle EDF$ .	1M
$\frac{ED}{CE} = \frac{EF}{CF}$ $\frac{ED}{15} = \frac{12}{9}$ $ED = 20 \text{ cm}$	1M 1A

Solution	Marks
13. (a) Let $f(x) = a + b(x - 2)(x - 1)$ , where $a$ and $b$ are non-zero constants.	1A
$\begin{cases} 16 = a + b(10 - 2)(10 - 1) \\ -50 = a + b(4 - 2)(4 - 1) \end{cases}$	1M
Solving, we have $a = -56$ and $b = 1$ .	
$0 = -56 + (x - 2)(x - 1)$	1M
$0 = x^2 - 3x - 54$	
$x = 9 \quad \text{or} \quad -6$	
The $x$ -intercepts are 9 and $-6$ .	1A
(b) (i) $x = \frac{3}{2}$	1A
(ii) The coordinates of $A$ and $B$ are $(-6, 0)$ and $(9, 0)$ respectively.	
The coordinates of $C$ are $(0, -54)$ .	
$-54 = -56 + (x - 2)(x - 1)$	
$0 = x^2 - 3x$	
$x = 0 \quad \text{or} \quad 3$	
The coordinates of $D$ are $(3, -54)$ .	
The area of the polygon is the least when the coordinates of $Q$ are $\left(\frac{3}{2}, -\frac{1}{4}\right)$ .	1M
Minimum area of the polygon $ACDBQ$	
$= \frac{[(9 + 6) + (3 - 0)](54)}{2} - \frac{(9 + 6)\left(\frac{1}{4}\right)}{2}$	1M
$= 484.125$	
$< 485$	
The claim is disagreed.	1A

Solution	Marks
14. (a) Let $h$ cm be the required height. Note that the length of the square base of the smaller square pyramid is also $h$ cm.	1M
$27 \times \frac{1}{3} \times h^2 \times h = 4 \times \left[ 12^3 - \frac{1}{3} \times 12^2 \times 12 \right]$	1M
$h^3 = 512$	
$h = 8$	1A
(b) Required ratio = $8^2 : 12^2$ $= 4 : 9$	1M
(c) Let $k$ cm be the length of the metal cube and $H$ cm be the height of the smaller square pyramid. The height of the largest square pyramid is $k$ cm.	1A
$27 \times \frac{1}{3} \times H^2 \times H = 4 \times \left[ k^3 - \frac{1}{3} \times k^2 \times k \right]$	1M
$\left(\frac{H}{k}\right)^3 = \frac{8}{27}$	
$\frac{H}{k} = \frac{2}{3}$	
The ratio in (b) becomes $2^2 : 3^2 = 4 : 9$ , which is the same as before.	1M
The claim is disagreed.	1A
15. (a) Standard score = $\frac{74 - 64}{4}$ $= 2.5$	1M
	1A
(b) Standard score of Samuel after the adjustment $= \frac{74(1 + 10\%) - 64(1 + 10\%)}{4(1 + 10\%)}$ $= 2.5$ $< 2.75$	1M
Sophia performs better in the test.	1A
16. (a) Required probability = $\frac{C_1^4 \times C_3^5}{C_4^9}$ $= \frac{20}{63}$	1M
	1A
(b) Required probability = $1 - \frac{20}{63} - \frac{C_3^4 \times C_1^5}{C_4^9}$ $= \frac{11}{21}$	1M
	1A

Solution	Marks
17. (a) Let $G$ be the centre of the circle. The coordinates of $G$ are $(3, -2)$ . Slope of $AG = \frac{-1+2}{1-3} = -\frac{1}{2}$ Required equation is	
$y + 1 = 2(x - 1)$	1M
$2x - y - 3 = 0$	1A
(b) (i) The equation of $L$ is $y = 2x + b$ .	1M
$x^2 + (2x + b)^2 - 6x + 4(2x + b) + 8 = 0$	
$5x^2 + (4b + 2)x + (b^2 + 4b + 8) = 0$	
$L$ and $C$ intersect at two distinct points.	
$(4b + 2)^2 - 4(5)(b^2 + 4b + 8) > 0$	1M
$-4b^2 - 64b - 156 > 0$	
$-13 < b < -3$	1A
(ii) Let the equation of $L'$ be $y = 2x + k$ .	
$5x^2 + (4k + 2)x + (k^2 + 4k + 8) = 0$	
$x_1$ and $x_2$ are roots of the quadratic equation.	
$x_1x_2 = 4$	
$\frac{k^2 + 4k + 8}{5} = 4$	1M
$k^2 + 4k - 12 = 0$	
$k = -6 \quad \text{or} \quad 2 \quad (\text{rejected})$	1A
The equation of $L'$ is $y = 2x - 6$ .	

	Solution	Marks
18. (a)	$75^2 = 29^2 + 92^2 - 2(29)(92) \cos \angle ABC$ $\cos \angle ABC = \frac{20}{29}$ $BD = 29 \cos \angle ABC = 20 \text{ cm}$ $AD = \sqrt{29^2 - 20^2} = 21 \text{ cm}$	1M 1A 1A
(b) (i)	Required angle is $\angle BDC$ .	1M
	$BC^2 = 75^2 + 29^2 - 2(75)(29) \cos 60^\circ$ $BC = \sqrt{4291} \text{ cm}$	1M
	$CD = 92 - 20 = 72 \text{ cm}$ $BC^2 = 72^2 + 20^2 - 2(72)(20) \cos \angle BDC$ $\angle BDC \approx 63.3^\circ$	1A
(ii)	Required volume	
	$= \frac{1}{3} \times \left( \frac{20 \times 21}{2} \right) \times 72 \sin \angle BDC$ $\approx 4500 \text{ cm}^3$	1M 1A

Solution	Marks
19. (a) $\begin{aligned} f(x) &= x^2 - (8k + 6)x + 16k^2 + 22k + 5 \\ &= [x^2 - 2(4k + 3)x + (4k + 3)^2] - 2k - 4 \\ &= [x - (4k + 3)]^2 - 2k - 4 \end{aligned}$	1M
The coordinates of $M$ are $(4k + 3, -2k - 4)$ .	1A
(b) (i) $(4k + 9, 2k + 4)$	1A
(ii) Since $P$ is the orthocentre of $\triangle PMN$ , we have $\angle MPN = 90^\circ$ .	1M
$\frac{y - (-2k - 4)}{x - (4k + 3)} \times \frac{y - (2k + 4)}{x - (4k + 9)} = -1$	1M+1M
$y^2 - (2k + 4)^2 = -[x - (4k + 3)][x - (4k + 9)]$	
$x^2 + y^2 - (8k + 12)x + 12k^2 + 32k + 11 = 0$	1A
(iii) $UV$ is the perpendicular bisector of $MN$ .	
The coordinates of the mid-point of $MN$ are $(4k + 6, 0)$ .	
Slope of $MN = \frac{(2k + 4) - (-2k - 4)}{(4k + 9) - (4k + 3)} = \frac{2k + 4}{3}$	
Required equation is	
$y - 0 = -\frac{3}{2k + 4}[x - (4k + 6)]$	1M
$y = -\frac{3x}{2k + 4} + \frac{3(2k + 3)}{k + 2}$	1A
(iv) Note that $MN^2 = MU^2 + NU^2 = 2MU^2$ .	
Area of square $MUNV$	
$= MU^2$	
$= \frac{MN^2}{2}$	1M
$= \frac{1}{2}[((4k + 9) - (4k + 3))^2 + ((2k + 4) + (2k + 4))^2]$	
$= \frac{1}{2}[36 + (4k + 8)^2]$	1M
$= 18 + 8(k + 2)^2$	
$\geq 18$	
The claim is agreed.	1A