DAYM2-AOD-2425-ASM-SET 4-MATH
Suggested solutions

1. (a) Volume of water $= \frac{1}{3}(4)^2\pi(12) \times \left[1 - \frac{(12-h)^3}{12^3}\right]$ 1M+1M

$= 64\pi - \frac{\pi(12-h)^3}{27}$ 1A

(b) Let V cm^3 be the volume of water.

$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\pi}{9}(12-h)^2\frac{\mathrm{d}h}{\mathrm{d}t}$ 1M

When $\frac{\mathrm{d}V}{\mathrm{d}t} = \pi$ and $h = 6$,

$\pi = \frac{\pi}{9}(12-6)^2 \times \frac{\mathrm{d}h}{\mathrm{d}t}$ 1M

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{4}$

Required rate is $\frac{1}{4}$ cm/s. 1A

2. (a) Area $= \frac{1}{2}(u)(4\ln u) = 2u\ln u$ 1A

(b) Let v be the length of PQ and A be the area of $\triangle OPQ$.

$v = 4\ln u$

$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{4}{u} \times \frac{\mathrm{d}u}{\mathrm{d}t}$

When $u = e$ and $\frac{\mathrm{d}v}{\mathrm{d}t} = -4$,

$-4 = \frac{4}{e} \times \frac{\mathrm{d}u}{\mathrm{d}t}$ 1M

$\frac{\mathrm{d}u}{\mathrm{d}t} = -e$ 1A

$\frac{\mathrm{d}A}{\mathrm{d}t} = \left(2u \times \frac{1}{u} + 2\ln u\right)\frac{\mathrm{d}u}{\mathrm{d}t}$ 1M

$= -4e$

Required rate is $-4e$/s. 1A

3. (a) $A = m(4e^{-m})$

$= 4me^{-m}$ 1A

(b) (i) $\dfrac{dA}{dm} = 4e^{-m} + 4me^{-m}(-1)$ 1M

$= 4e^{-m}(1 - m)$

When $\dfrac{dA}{dm} = 0$, $m = 1$. 1A

m	$0 < m < 1$	$m > 1$
$\dfrac{dA}{dm}$	$+$	$-$

1M

A attains its maximum when $m = 1$.

Required value $= 4(1)e^{-1}$

$= 4e^{-1}$ 1A

(ii) $\dfrac{dA}{dt} = \dfrac{dA}{dm} \cdot \dfrac{dm}{dt}$ 1M

$= 4e^{-0.5}(1 - 0.5) \cdot 0.5$

$= e^{-0.5}$ 1A

Required rate is $e^{-0.5}$ units per second.

4. (a) Radius of water surface

$= \sqrt{13^2 - (13 - h)^2}$ 1M

$= \sqrt{26h - h^2}$

$A = \pi(\sqrt{26h - h^2})^2$

$= \pi(26h - h^2)$ 1A

(b) $V = \dfrac{1}{3}\pi h^2(39 - h)$

$\dfrac{dV}{dt} = \dfrac{1}{3}\pi\left(78h\dfrac{dh}{dt} - 3h^2\dfrac{dh}{dt}\right)$ 1M

$= \pi(26h - h^2)\dfrac{dh}{dt}$

$= A\dfrac{dh}{dt}$ 1A

$\dfrac{1}{20}A = A\dfrac{dh}{dt}$

$\dfrac{dh}{dt} = \dfrac{1}{20}$ 1A

Required rate is $\dfrac{1}{20}$ cm/s.

5. (a) (i) Let C be a point on OP such that $AC \perp OP$.

$$AC = \sqrt{OA^2 - OC^2} \qquad \text{1M}$$
$$= \sqrt{4^2 - (4 - x)^2}$$
$$= \sqrt{8x - x^2} \text{ cm}$$

$$DP = AC = \sqrt{8x - x^2} \text{ cm} \qquad \text{1A}$$

(ii) $$S = \frac{(DP)(AD)}{2}$$
$$= \frac{x\sqrt{8x - x^2}}{2} \qquad \text{1A}$$

(b) $$\frac{dS}{dx} = \frac{1}{2}\left[\sqrt{8x - x^2} + \frac{x(8 - 2x)}{2\sqrt{8x - x^2}}\right] \qquad \text{1M}$$
$$= \frac{(8x - x^2) + x(4 - x)}{2\sqrt{8x - x^2}}$$
$$= \frac{x(6 - x)}{\sqrt{8x - x^2}}$$

When $\frac{dS}{dx} = 0$, $x = 6$ or 0 (rejected). 1A

x	$0 < x < 6$	$6 < x < 8$
$\frac{dS}{dx}$	+	−

1M

S attains its maximum at $x = 6$.

$$\text{Required value} = \frac{6\sqrt{8(6) - 6^2}}{2}$$
$$= 6\sqrt{3} \qquad \text{1A}$$

(c) $$\frac{dS}{dt} = \frac{dS}{dx} \cdot \frac{dx}{dt} \qquad \text{1M}$$
$$= \frac{2(6 - 2)}{\sqrt{8(2) - 2^2}}\left(-\frac{1}{\sqrt{3}}\right)$$
$$= -\frac{4}{3} \qquad \text{1A}$$

The area of $\triangle ADP$ decreases at a rate of $\frac{4}{3}$ cm^2/s.

 END OF PAPER