DAYCP-EOC-2425-ASM-SET 3-MATH
Suggested solutions

Conventional Questions

1. (a) Let the coordinates of G be $(h, 26)$.
Note that G lies on the perpendicular bisector of AB.

$h = \frac{5 + 13}{2}$ 1M

$= 9$

The equation of C is

$(x - 9)^2 + (y - 26)^2 = (5 - 9)^2 + (23 - 26)^2$ 1M

$(x - 9)^2 + (y - 26)^2 = 25$ 1A

(b) $\sqrt{(k - 9)^2 + (38 - 26)^2} = 15$ 1M

$k^2 - 18k = 0$

$k = 18$ or 0 (rejected) 1A

(c) (i) T, P and G are collinear. 1A

(ii) Radius of C is 5.
Required ratio $= GP : PT$ 1M

$= 5 : (15 - 5)$

$= 1 : 2$ 1A

2. (a) Let the equation of the circle be $x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants.

$$\begin{cases} 0^2 + 2^2 + 0 + 2y + F = 0 \\ 4^2 + 0^2 + 4D + 0 + F = 0 \\ 9^2 + 5^2 + 9D + 5E + F = 0 \end{cases}$$ 1M

Subtract one equation from another.

$$\begin{cases} 102 + 9D + 3E = 0 \\ 90 + 5D + 5E = 0 \end{cases}$$ 1M

Solving, we have $D = -8$, $E = -10$ and $F = 16$.

Required equation is $x^2 + y^2 - 8x - 10y + 16 = 0$. 1A

(b) The coordinates of the centre of C are $(4,\ 5)$.

Radius $= \sqrt{4^2 + t^2 - 16} = 5$

Distance from M to the centre of C

$= \sqrt{(5-4)^2 + (8-5)^2}$ 1M

$= \sqrt{10}$

< 5

Thus, M lies inside C. 1

(c) (i) G, M and N are collinear. 1A

(ii) Slope $= \dfrac{8-5}{5-4} = 3$ 1M

Required equation is

$y - 5 = 3(x - 4)$

$3x - y - 7 = 0$ 1A

3. (a) $(x-6)^2 + (y+5)^2 = 6^2 + 5^2$ 1M

$(x-6)^2 + (y+5)^2 = 61$ 1A

(b) (i) $H = (12,\ 0)$ and $K = (0,\ -10)$ 1A+1A

(ii) O, P and Q are collinear. 1A

(iii) Required area $= 12 \times 10$ 1M

$= 120$ 1A

4. (a) Slope of $L = \dfrac{5-0}{15-12} = \dfrac{5}{3}$ 1M

Slope of $L' = -\dfrac{3}{5}$

Required equation is

$$\frac{y-0}{x-12} = -\frac{3}{5} \qquad \text{1M}$$

$$3x + 5y - 36 = 0 \qquad \text{1A}$$

(b) (i) $3x + 5k - 36 = 0$

$$x = \frac{36-5k}{3}$$

The coordinates of G are $\left(\dfrac{36-5k}{3},\ k\right)$. 1M

The equation of C is

$$x^2 + y^2 - 2\left(\frac{36-5k}{3}\right)x - 2ky + F = 0$$

$$3x^2 + 3y^2 - 2(36-5k)x - 6ky + 3F = 0$$

where F is a constant.

C passes through $Q\ (12,\ 0)$.

$$3(12)^2 + 0 - 2(36-5k)(12) - 0 + 3F = 0$$

$$3F = 432 - 120k$$

The equation of C is $3x^2 + 3y^2 - 2(36-5k)x - 6ky + 432 - 120k = 0$. 1

(ii) $3(4)^2 + 3(8)^2 - 2(36-5k)(4) - 6k(8) + 432 - 120k = 0$ 1M

$$k = 3$$

The coordinates of G are $(7,\ 3)$.

PG is a diameter of the required circle.

$PG = \sqrt{(15-7)^2 + (5-3)^2} = \sqrt{68}$ 1M

Required area $= \pi\left(\dfrac{\sqrt{68}}{2}\right)^2$

$= 17\pi$ 1A

5. (a) Let $f(x) = ax + bx^2$, where a and b are non-zero constants. 1A

$$\begin{cases} -160 = -5a + 25b \\ -16 = 4a + 16b \end{cases}$$ 1M

Solving, we have $a = 12$ and $b = -4$.

$f(3) = 12(3) - 4(3)^2 = 0$ 1A

(b) $p = f(3) = 0$

$12q - 4q^2 = 8$ 1M

$-4q^2 + 12q - 8 = 0$

$q = 1$ or 2

Note that $\angle QRP = 90°$.

PQ is a diameter of C. 1M

When $q = 1$, area of $C = \pi \left(\frac{\sqrt{2^2 + 8^2}}{2} \right)^2 = 17\pi$. 1M

When $q = 2$, area of $C = \pi \left(\frac{\sqrt{1^2 + 8^2}}{2} \right)^2 = \frac{65\pi}{4} < 17\pi$.

Thus, it is not possible. 1A

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