

M2REG-TRIG-2425-ASM-SET 5-MATH

建議題解

1. (a) $y = \cos\left(x + \frac{\pi}{8}\right) \sin\left(x - \frac{\pi}{24}\right)$
 $= \frac{1}{2} \left[\sin\left(2x + \frac{\pi}{12}\right) + \sin\frac{-\pi}{6} \right]$ 1M
 $= \frac{1}{2} \sin\left(2x + \frac{\pi}{12}\right) - \frac{1}{4}$ 1
- (b) y 的極大值 $= \frac{1}{2}(1) - \frac{1}{4}$ 1M
 $= \frac{1}{4}$ 1A
 y 的極小值 $= \frac{1}{2}(-1) - \frac{1}{4}$
 $= -\frac{3}{4}$ 1A

2. (a) 藉考慮兩根之和， $\sin \alpha + \sin \beta = \frac{7}{12}^\circ$ 1A
 藉考慮兩根之積， $\sin \alpha \sin \beta = \frac{1}{12}^\circ$ 1A
- (b) $\cos(\alpha + \beta) \cos(\alpha - \beta) = \frac{1}{2}(\cos 2\alpha + \cos 2\beta)$ 1M
 $= \frac{1}{2}(1 - 2\sin^2 \alpha + 1 - 2\sin^2 \beta)$ 1M
 $= 1 - \sin^2 \alpha - \sin^2 \beta$
 $= 1 + 2\sin \alpha \sin \beta - (\sin \alpha + \sin \beta)^2$ 1M
 $= 1 + 2\left(\frac{1}{12}\right) - \left(\frac{7}{12}\right)^2$
 $= \frac{119}{144}$ 1A

3. (a) $\sin(A + B) \sin(A - B) = \frac{1}{2}(-\cos 2A + \cos 2B)$ 1M
 $= \frac{1}{2}(-1 + 2\sin^2 A + 1 - 2\sin^2 B)$ 1M
 $= \sin^2 A - \sin^2 B$ 1
- (b) 利用 (a)，
 $\sin^2 kx - \sin^2(k - 1)x = \sin[kx + (k - 1)x] \sin[kx - (k - 1)x]$ 1M
 $= \sin[(2k - 1)x] \sin x$ 1A

$$\begin{aligned}
4. \quad & \frac{\cos 7\theta - \cos 5\theta + \cos 3\theta - \cos \theta}{\sin 7\theta - \sin 5\theta + \sin 3\theta - \sin \theta} \\
&= \frac{-2 \sin 6\theta \sin \theta - 2 \sin 2\theta \sin \theta}{2 \cos 6\theta \sin \theta + 2 \cos 2\theta \sin \theta} && 1\text{M}+2\text{A} \\
&= -\frac{\sin 6\theta + \sin 2\theta}{\cos 6\theta + \cos 2\theta} \\
&= -\frac{2 \sin 4\theta \cos 2\theta}{2 \cos 4\theta \cos 2\theta} && 1\text{M} \\
&= -\tan 4\theta && 1
\end{aligned}$$

$$\begin{aligned}
5. \quad & \cos A + \cos B - \cos C \\
&= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} - \left(1 - 2 \sin^2 \frac{C}{2}\right) && 2\text{M} \\
&= 2 \cos \frac{\pi-C}{2} \cos \frac{A-B}{2} + 2 \sin \frac{C}{2} \sin \frac{\pi-(A+B)}{2} - 1 \\
&= 2 \sin \frac{C}{2} \cos \frac{A-B}{2} + 2 \sin \frac{C}{2} \cos \frac{A+B}{2} - 1 && 1\text{M} \\
&= 2 \sin \frac{C}{2} \left[\cos \frac{(A+B)}{2} + \cos \frac{A-B}{2} \right] - 1 \\
&= 2 \sin \frac{C}{2} \left(2 \cos \frac{A}{2} \cos \frac{B}{2} \right) - 1 && 1\text{M} \\
&= 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2} - 1 && 1
\end{aligned}$$

$$\begin{aligned}
6. \quad & \sin 3A + \sin 3B + \sin 3[\pi - (A+B)] \\
&= 2 \sin \frac{3}{2}(A+B) \cos \frac{3}{2}(A-B) + \sin 3(A+B) && 2\text{M} \\
&= 2 \sin \frac{3}{2}(A+B) \cos \frac{3}{2}(A-B) + 2 \sin \frac{3}{2}(A+B) \cos \frac{3}{2}(A+B) && 1\text{M} \\
&= 2 \sin \frac{3}{2}(A+B) \left[\cos \frac{3}{2}(A-B) + \cos \frac{3}{2}(A+B) \right] \\
&= 2 \sin \frac{3}{2}(\pi - C) \left(2 \cos \frac{3A}{2} \cos \frac{3B}{2} \right) && 1\text{M} \\
&= 4 \sin \left(\frac{3\pi}{2} - \frac{3C}{2} \right) \cos \frac{3A}{2} \cos \frac{3B}{2} \\
&= -4 \cos \frac{3C}{2} \cos \frac{3A}{2} \cos \frac{3B}{2} && 1\text{M} \\
&= -4 \cos \frac{3A}{2} \cos \frac{3B}{2} \cos \frac{3C}{2} && 1
\end{aligned}$$

$$7. \quad (a) \quad \frac{\sin \alpha - \sin \beta}{\cos \alpha - \cos \beta} = \frac{2 \cos \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2}}{-2 \sin \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2}} \quad 1M$$

$$= -\cot \frac{\alpha+\beta}{2} \quad 1$$

$$(b) \quad 3 \sin \alpha - 4 \cos \beta = 3 \sin \beta - 4 \cos \alpha$$

$$\frac{3(\sin \alpha - \sin \beta)}{4(\cos \alpha - \cos \beta)} = -1$$

$$-\frac{3}{4} \cot \frac{\alpha+\beta}{2} = -1 \quad 1M$$

$$\cot \frac{\alpha+\beta}{2} = \frac{4}{3} \quad 1A$$

由於 α 及 β 為銳角， $\frac{\alpha+\beta}{2}$ 同為銳角。

$$\sin \frac{\alpha+\beta}{2} = \frac{3}{\sqrt{3^2+4^2}} \quad 1M$$

$$= \frac{3}{5} \quad 1A$$

$$8. \quad (a) \quad \sin \frac{(m+4)\pi}{8} - \sin \frac{m\pi}{8} = 2 \cos \frac{m\pi+2\pi}{8} \sin \frac{\pi}{4} \quad 1M$$

$$= \sqrt{2} \cos \frac{(m+2)\pi}{8} \quad 1A$$

$$(b) \quad \cos \frac{3\pi}{8} + \cos \frac{7\pi}{8} + \dots + \cos \frac{2015\pi}{8}$$

$$= \frac{1}{\sqrt{2}} \left(\sin \frac{5\pi}{8} - \sin \frac{\pi}{8} \right) + \frac{1}{\sqrt{2}} \left(\sin \frac{9\pi}{8} - \sin \frac{5\pi}{8} \right) + \dots + \frac{1}{\sqrt{2}} \left(\sin \frac{2017\pi}{8} - \sin \frac{2013\pi}{8} \right) \quad 1M$$

$$= \frac{1}{\sqrt{2}} \left(\sin \frac{2017\pi}{8} - \sin \frac{\pi}{8} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\sin \left(252\pi + \frac{\pi}{8} \right) - \sin \frac{\pi}{8} \right) \quad 1M$$

$$= 0 \quad 1A$$

9. 當 $n = 1$ ，

左式 = $\cos x$

$$\text{右式} = \frac{\sin 2x}{2 \sin x} = \frac{2 \sin x \cos x}{2 \sin x} = \cos x = \text{左式}$$

對 $n = 1$ ，命題為真。

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假設 $\cos x + \cos 3x + \cos 5x + \dots + \cos(2k-1)x = \frac{\sin 2kx}{2 \sin x}$ ，其中 $k \in \mathbb{Z}^+$ 。

1M

$$\cos x + \cos 3x + \cos 5x + \dots + \cos(2k-1)x + \cos(2k+1)x$$

$$= \frac{\sin 2kx}{2 \sin x} + \cos(2k+1)x \quad 1M$$

$$= \frac{1}{2 \sin x} [\sin 2kx + 2 \cos(2k+1)x \sin x]$$

$$= \frac{1}{2 \sin x} [\sin 2kx + \sin(2k+2)x + \sin(-2kx)] \quad 1M$$

$$= \frac{\sin(2k+2)x}{2 \sin x}$$

對 $n = k+1$ ，命題為真。

藉數學歸納法， $\forall n \in \mathbb{Z}^+$ ，命題為真。

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10. $2 \cos A + \cos 2B + \cos 2C = 0$ 1M
- $2 \cos A + 2 \cos(B+C) \cos(B-C) = 0$
- $\cos A + \cos(\pi - A) \cos(B-C) = 0$
- $\cos A - \cos A \cos(B-C) = 0$ 1M
- $\cos A [1 - \cos(B-C)] = 0$
- $\cos A = 0$ 或 $\cos(B-C) = 1$ 1A
- $A = \frac{\pi}{2}$ 或 $B - C = 0$
- $A = \frac{\pi}{2}$ 或 $B = C$ 1A
- 因此， $\triangle ABC$ 為直角三角形或 $\triangle ABC$ 為 $B = C$ 的等腰三角形。 1M
- $\triangle ABC$ 可以是直角三角形，其中 $A = \frac{\pi}{2}$ 但 $B \neq C$ 。
- 不同意該宣稱。 1A