M2REG-TRIG-2425-ASM-SET 4-MATH
建議題解

1. 當 $\sin\theta = -\frac{2}{\sqrt{5}}$ 及 $\pi < \theta < \frac{3\pi}{2}$，

$$\cos\theta = -\frac{\sqrt{(\sqrt{5})^2 - 2^2}}{\sqrt{5}} \qquad \text{1M}$$
$$= -\frac{1}{\sqrt{5}}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \text{1M}$$
$$= 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right)$$
$$= \frac{4}{5} \qquad \text{1A}$$

2. $$\frac{\sin 5\theta}{\cos\theta} + \frac{\cos 5\theta}{\sin\theta} = \frac{\sin 5\theta\sin\theta + \cos 5\theta\cos\theta}{\sin\theta\cos\theta}$$
$$= \frac{\cos(5\theta - \theta)}{\frac{1}{2}\sin 2\theta} \qquad \text{2M}$$
$$= \frac{2\cos 4\theta}{\sin 2\theta}$$
$$= \frac{2(1 - 2\sin^2 2\theta)}{\sin 2\theta} \qquad \text{1M}$$
$$= 2\csc 2\theta - 4\sin 2\theta \qquad \text{1}$$

3. (a) $$\cot A - \cot 2A = \frac{\cos A}{\sin A} - \frac{\cos 2A}{\sin 2A} \qquad \text{1M}$$
$$= \frac{\cos A\sin 2A - \sin A\cos 2A}{\sin A\sin 2A}$$
$$= \frac{\sin(2A - A)}{\sin A\sin 2A} \qquad \text{1M}$$
$$= \frac{\sin A}{\sin A\sin 2A}$$
$$= \csc 2A \qquad \text{1}$$

(b) $\csc 2\theta + \csc 4\theta + \csc 8\theta$

$= (\cot\theta - \cot 2\theta) + (\cot 2\theta - \cot 4\theta) + (\cot 4\theta - \cot 8\theta)$ [From (a)] 1M

$= \cot\theta - \cot 8\theta$ 1

4. $$\frac{1-2\sin^2 x}{1+\sin 2x} = \frac{1-2\sin^2 x}{1+2\sin x\cos x} \qquad \text{1M}$$

$$= \frac{\cos^2 x-\sin^2 x}{\sin^2 x+\cos^2 x+2\sin x\cos x} \qquad \text{1M}$$

$$= \frac{(\cos x-\sin x)(\cos x+\sin x)}{(\sin x+\cos x)^2} \qquad \text{1M}$$

$$= \frac{\cos x-\sin x}{\sin x+\cos x}$$

$$= \frac{1-\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x}+1}$$

$$= \frac{1-\tan x}{1+\tan x} \qquad 1$$

5. (a) $\dfrac{1-\cos 2\theta}{1+\cos 2\theta}=\dfrac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}$ 1M+1A

$=\dfrac{2\sin^2\theta}{2\cos^2\theta}$

$=\tan^2\theta$ 1

(b) 代 $\theta=\dfrac{\pi}{8}$，

$\tan^2\dfrac{\pi}{8}=\dfrac{1-\cos 2(\frac{\pi}{8})}{1+\cos 2(\frac{\pi}{8})}$ 1M

$=\dfrac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$

$=\dfrac{\sqrt{2}-1}{\sqrt{2}+1}\times\dfrac{\sqrt{2}-1}{\sqrt{2}-1}$ 1M

$=\dfrac{2-2\sqrt{2}+1}{2-1}$

$=3-2\sqrt{2}$ 1A

代 $\theta=\dfrac{3\pi}{8}$，

$\tan^2\dfrac{3\pi}{8}=\dfrac{1-\cos 2(\frac{3\pi}{8})}{1+\cos 2(\frac{3\pi}{8})}$ 1M

$=\dfrac{1-(-\frac{1}{\sqrt{2}})}{1+(-\frac{1}{\sqrt{2}})}$

$=\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$ 1M

$=\dfrac{2+2\sqrt{2}+1}{2-1}$

$=3+2\sqrt{2}$ 1A

所求方程為

$x^2-\left(\tan^2\dfrac{\pi}{8}+\tan\dfrac{3\pi}{8}\right)x+\left(\tan^2\dfrac{\pi}{8}\tan^2\dfrac{3\pi}{8}\right)=0$ 1M

$x^2-6x+1=0$ 1A

6. (a) $\sin 2\beta = \dfrac{3\sin 2\alpha}{2}$

$= \dfrac{3(2\sin\alpha\cos\alpha)}{2}$ 1M

$= 3\sin\alpha\cos\alpha$ 1A

$\cos 2\beta = 1 - 2\sin^2\beta$

$= 1 - (1 - 3\sin^2\alpha)$ 1M

$= 3\sin^2\alpha$ 1A

(b) $\cos(\alpha + 2\beta) = \cos\alpha\cos 2\beta - \sin\alpha\sin 2\beta$ 1M

$= 3\sin^2\alpha\cos\alpha - 3\sin^2\alpha\cos\alpha$

$= 0$ 1A

7. $\dfrac{\sin A}{1 - \cos A} = \dfrac{\sin(180^\circ - 2B)}{1 - \cos(180^\circ - 2B)}$ 1M

$= \dfrac{\sin 2B}{1 + \cos 2B}$ 1M

$= \dfrac{2\sin B\cos B}{1 + 2\cos^2 B - 1}$ 1M

$= \tan B$ 1

8. $\dfrac{\cos^2 A - \cos^2 B}{\cos^2 A + \cos^2 B} = \dfrac{\cos^2 A - \cos^2\left(\frac{\pi}{2} - A\right)}{\cos^2 A + \cos^2\left(\frac{\pi}{2} - B\right)}$

$= \dfrac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$ 1M

$= \dfrac{\cos 2A}{1}$ 1M

$= \cos 2A$ 1

9. (a) $\cos 4\theta = \cos 2(2\theta)$

$= 2\cos^2 2\theta - 1$ 1M

$= 2(2\cos^2\theta - 1)^2 - 1$ 1M

$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 1$ 1

(b) 代 $x = \cos 15°$，

$16\cos^4 15° - 16\cos^2 15° + 1 = 2(8\cos^4 15° - 8\cos^2 15° + 1) - 1$

$= 2\cos 4(15°) - 1$ 1M

$= 2\cos 60° - 1$

$= 1 - 1$ 1M

$= 0$

因此，$\cos 15°$ 為 $16x^4 - 16x^2 + 1 = 0$ 的根。 1

(c) $16\cos^4 15° - 16\cos^2 15° + 1 = 0$

$\cos^2 15° = \dfrac{-(-16) \pm \sqrt{(-16)^2 - 4(16)(1)}}{2(16)}$ 1M

$= \dfrac{16 \pm 8\sqrt{3}}{32}$

$= \dfrac{2+\sqrt{3}}{4}$ 或 $\dfrac{2-\sqrt{3}}{4}$ 1A

由於 $\cos 15° > \cos 60° > 0$，可得 $\cos^2 15° > \cos^2 60° = \dfrac{1}{4}$ 及 $\cos^2 15° = \dfrac{2+\sqrt{3}}{4}$。 1A

因此，$\cos 15° = \dfrac{\sqrt{2+\sqrt{3}}}{2}$。 1A

10. (a) $\cos 3\theta = \sin 2\theta$

$$\cos 3\theta = \cos(90^\circ - 2\theta)$$
$$3\theta = 90^\circ - 2\theta$$
$$\theta = 18^\circ \qquad \text{1A}$$

(b)
$$\begin{aligned}
\cos 3\theta &= \sin 2\theta \\
\cos 2\theta \cos\theta - \sin 2\theta \sin\theta &= \sin 2\theta && \text{1M} \\
(2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta\cos\theta &= 2\sin\theta\cos\theta \\
(2\cos^2\theta - 1) - 2(1 - \cos^2\theta) &= 2\sin\theta \\
4\cos^2\theta - 3 &= 2\sin\theta \\
16\cos^4\theta - 24\cos^2\theta + 9 &= 4\sin^2\theta && \text{1M} \\
16\cos^4\theta - 24\cos^2\theta + 9 &= 4 - 4\cos^2\theta \\
16\cos^4\theta - 20\cos^2\theta + 5 &= 0 && 1
\end{aligned}$$

(c) 當 $\theta = 18^\circ$，$\cos 3\theta = \sin 2\theta$ 及

$$16\cos^4 18^\circ - 20\cos^2 18^\circ + 5 = 0$$
$$\cos^2 18^\circ = \frac{20 \pm \sqrt{20^2 - 4(16)(5)}}{2(16)}$$
$$= \frac{5+\sqrt{5}}{8} \quad \text{或} \quad \frac{5-\sqrt{5}}{8}\text{（捨去）} \qquad \text{1M}$$

$$\begin{aligned}
\sin 54^\circ &= \cos 36^\circ \\
&= 2\cos^2 18^\circ - 1 && \text{1M} \\
&= \frac{1+\sqrt{5}}{4} && \text{1A}
\end{aligned}$$