

M2REG-TRIG-2425-ASM-SET 1-MATH

建議題解

1. 由於 $\sin \alpha > 0$ 及 $\sec \beta < 0$ ， α 及 β 在第二象限中。

當 $\sin \alpha = \frac{3}{5}$ 時，

$$\cos \alpha = \frac{-\sqrt{5^2 - 3^2}}{5} = -\frac{4}{5} \quad 1M$$

$$\tan \alpha = \frac{3}{-\sqrt{5^2 - 3^2}} = -\frac{3}{4} \quad 1A$$

當 $\sec \beta = -\frac{13}{12}$ 時，

$$\csc \beta = \frac{13}{\sqrt{13^2 - 12^2}} = \frac{13}{5}$$

$$\tan \beta = \frac{-\sqrt{13^2 - 12^2}}{12} = -\frac{5}{12} \quad 1A$$

$$\begin{aligned} \frac{\cos \alpha + \csc \beta}{\tan \alpha - \tan \beta} &= \frac{-\frac{4}{5} + \frac{13}{5}}{-\frac{3}{4} + \frac{5}{12}} \\ &= -\frac{27}{5} \end{aligned} \quad 1A$$

$$2. \tan^2 \theta (\csc^2 \theta + \sec^2 \theta) = \frac{\sin^2 \theta}{\cos^2 \theta} \left(\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \right) \quad 1M$$

$$\begin{aligned} &= \frac{1}{\cos^2 \theta} \left(\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} \right) \\ &= \sec^4 \theta \end{aligned} \quad 1M+1A$$

$$3. \left(\cot \theta + \frac{1}{\cot \theta} \right) \cos \theta = \left(\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right) \cos \theta \quad 1M$$

$$\begin{aligned} &= \frac{\cos^2 \theta}{\sin \theta} + \sin \theta \\ &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta} \\ &= \csc \theta \end{aligned} \quad 1M+1A$$

$$4. \csc^2 \alpha + \csc^2 \beta = (1 + \cot^2 \alpha) + (1 + \cot^2 \beta) \quad 1M$$

$$= 2 + (\cot^2 \alpha + \cot^2 \beta)$$

$$= 3 \quad 1$$

$$\begin{aligned}
 5. \quad \frac{\cos \theta + \cot \theta}{\tan \theta + \sec \theta} &= \frac{\cos \theta + \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}} & 1M \\
 &= \frac{\cos \theta + \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}} \times \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} & 1M \\
 &= \frac{\cos^2 \theta (\sin \theta + 1)}{\sin \theta (\sin \theta + 1)} \\
 &= \cos \theta \times \cot \theta & 1
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{\sin \theta}{\csc \theta - \cot \theta} - \frac{\tan \theta}{\csc \theta + \cot \theta} &= \frac{\sin \theta (\csc \theta + \cot \theta) - \tan \theta (\csc \theta - \cot \theta)}{\csc^2 \theta - \cot^2 \theta} & 1M \\
 &= \frac{1 + \cos \theta - \frac{1}{\cos \theta} + 1}{1} & 2M \\
 &= 2 + \cos \theta - \frac{1}{\cos \theta} \\
 &= 2 - \frac{1 - \cos^2 \theta}{\cos \theta} \\
 &= 2 - \sec \theta \sin^2 \theta & 1
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \sin x \cos x (\sin x + \cos x) - \frac{1 + 2 \sin x \cos x}{\sec x + \csc x} \\
 &= \frac{(\sin^2 x \cos x + \sin x \cos^2 x) \left(\frac{1}{\cos x} + \frac{1}{\sin x} \right) - 1 - 2 \sin x \cos x}{\sec x + \csc x} & 1M \\
 &= \frac{\sin^2 x + \sin x \cos x + \sin x \cos x + \cos^2 x - 1 - 2 \sin x \cos x}{\sec x + \csc x} & 1M \\
 &= 0 \\
 \text{因此, } \sin x \cos x (\sin x + \cos x) &= \frac{1 + 2 \sin x \cos x}{\sec x + \csc x} . & 1
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \frac{1 + \sin x + \cos x}{1 + \sin x - \cos x} - \frac{1 + \cos x + \cot x + \csc x}{1 + \sin x} \\
 &= \frac{(1 + \sin x + \cos x)(1 + \sin x) - (1 + \cos x + \cot x + \csc x)(1 + \sin x - \cos x)}{(1 - \sin x - \cos x)(1 + \sin x)} \\
 &= \frac{(1 + 2 \sin x + \sin^2 x + \cos x + \sin x \cos x) - \left(1 + \sin x - \cos x + \cos x + \sin x \cos x - \cos^2 x + \cot x + \cos x - \frac{\cos^2 x}{\sin x} + \csc x + 1 - \cot x \right)}{(1 + \sin x - \cos x)(1 + \sin x)} & 1M \\
 &= \frac{-1 + \sin x + \sin^2 x + \cos^2 x + \frac{1 - \cos^2 x}{\sin x}}{(1 + \sin x - \cos x)(1 + \sin x)} \\
 &= \frac{\sin x + \frac{-\sin^2 x}{\sin x}}{(1 + \sin x - \cos x)(1 + \sin x)} & 1M \\
 &= 0 \\
 \text{因此, } \frac{1 + \sin x + \cos x}{1 + \sin x - \cos x} &= \frac{1 + \cos x + \cot x + \csc x}{1 + \sin x} . & 1
 \end{aligned}$$

$$\begin{aligned}
9. \quad & \frac{1 + \cos^2 A - \cos^2(A + C)}{\cos A \sin(A + C)} \\
&= \frac{1 + \cos^2 A - \cos^2(\pi - B)}{\cos A \sin(\pi - B)} \\
&= \frac{1 + \cos^2 A - \cos^2 B}{\cos A \sin B} & 1M \\
&= \frac{\cos^2 A + \sin^2 B}{\cos A \sin B} & 1M \\
&= \frac{\cos A}{\sin B} + \frac{\sin B}{\cos A} \\
&= \cos A \csc B + \sec A \sin B & 1
\end{aligned}$$

$$\begin{aligned}
10. \quad & (2 \sin x + \sec x)(2 \cos x + \csc x) \\
&= 4 \sin x \cos x + 2 + 2 + \frac{1}{\sin x \cos x} & 1M \\
&= \frac{(2 \sin x \cos x)^2 + 4 \sin x \cos x + 1}{\sin x \cos x} & 1M \\
&= \frac{(2 \sin x \cos x + 1)^2}{\sin x \cos x} & 1
\end{aligned}$$

11. 對任意實數 θ ，

$$\begin{aligned}
& \csc^2 \theta \geq 1 & 1M \\
& 2 \csc^2 \theta + 1 \geq 3 \\
& \frac{1}{2 \csc^2 \theta + 1} \leq \frac{1}{3} \\
& \frac{6}{2 \csc^2 \theta + 1} \leq 2 & 1M
\end{aligned}$$

所求數值為 2。 & 1A

12. 對任意實數 θ ，

$$\begin{aligned}
& \sec^2 \theta \geq 1 & 1M \\
& 2 \sec^2 \theta - 1 \geq 1 \\
& \frac{1}{2 \sec^2 \theta - 1} \leq 1 \\
& -\frac{5}{2 \sec^2 \theta - 1} \geq -5 \\
& 3 - \frac{5}{2 \sec^2 \theta - 1} \geq -2 & 1M
\end{aligned}$$

所求數值為 -2。 & 1A