

M2REG-BT-2425-ASM-SET 1-MATH

建議題解

$$1. (x+8)^4 = x^4 + 4(x)^3(8) + 6(x)^2(8)^2 + 4(x)(8)^3 + 8^4$$

$$= x^4 + 32x^3 + 384x^2 + 2048x + 4096 \quad 1A$$

$$2. (5-x)^5 = (5)^5 + 5(5)^4(-x) + 10(5)^3(-x)^2 + 10(5)^2(-x)^3 + 5(5)(-x)^4 + (-x)^5$$

$$= 3125 - 3125x + 1250x^2 - 250x^3 + 25x^4 - x^5 \quad 1A$$

$$3. (a) \left(y - \frac{1}{y}\right)^3 = y^3 + 3(y)^2\left(-\frac{1}{y}\right) + 3(y)\left(-\frac{1}{y}\right)^2 + \left(-\frac{1}{y}\right)^3 \quad 1M$$

$$27 = y^3 - \frac{1}{y^3} - 3\left(y - \frac{1}{y}\right)$$

$$y^3 - \frac{1}{y^3} = 36 \quad 1A$$

$$(b) \left(y - \frac{1}{y}\right)^5 = (y)^5 + 5(y)^4\left(-\frac{1}{y}\right) + 10(y)^3\left(-\frac{1}{y}\right)^2 + 10(y)^2\left(-\frac{1}{y}\right)^3 + 5(y)\left(-\frac{1}{y}\right)^4 + \left(-\frac{1}{y}\right)^5 \quad 1M$$

$$243 = y^5 - \frac{1}{y^5} - 5\left(y^3 - \frac{1}{y^3}\right) + 10\left(y - \frac{1}{y}\right)$$

$$y^5 - \frac{1}{y^5} = 393 \quad 1A$$

$$4. \text{ 包含 } x^4 \text{ 的項}$$

$$= C_4^{10}(3)^6(2x)^4 \quad 1M$$

$$= 2449440x^4 \quad 1A$$

$$5. (2x+1)^{13} = 1^{13} + C_1^{13}(1)^{12}(2x) + C_2^{13}(1)^{11}(2x)^2 + C_3^{13}(1)^{10}(2x)^3 + \dots \quad 1M$$

$$= 1 + 26x + 312x^2 + 2288x^3 + \dots \quad 1A$$

$$6. \text{ 所求係數} = C_{10}^{17}(2)^{10}\left(-\frac{1}{4}\right)^{17-10} \quad 1M$$

$$= -\frac{2431}{2} \quad 1A$$

$$7. \text{ 第 5 項} = C_4^n (ax^2)^{n-4} \left(\frac{1}{3x}\right)^4 \quad 1M$$

$$= C_4^n a^{n-4} \left(\frac{1}{81}\right) x^{2n-12}$$

故此，

$$2n - 12 = 0$$

$$n = 6$$

1A

及

$$C_4^6 a^{6-4} \left(\frac{1}{81}\right) = \frac{5}{27}$$

$$a^2 = 1$$

$$a = 1 \quad \text{或} \quad -1$$

1A

$$8. \text{ 所求項} = C_3^{10} (3)^{10-3} (-x)^3 \quad 1M$$

$$= -262\,440x^3$$

1A

$$9. (2x^3 + 1)^n = 1 + C_1^n (2x^3) + C_2^n (2x^3)^2 + C_3^n (2x^3)^3 + \dots \quad 1M$$

$$= 1 + 2nx^3 + 2n(n-1)x^6 + \frac{n(n-1)(n-2)}{6} (8)x^9 + \dots$$

若 $d = 28b$ ，

$$\frac{n(n-1)(n-2)}{6} (8) = 28(2n)$$

1M

$$8(n^2 - 3n + 2) = 336$$

$$8n^2 - 24n - 320 = 0$$

$$n = 8 \quad \text{或} \quad -5 \quad (\text{捨去})$$

1A

當 $n = 8$ 時， $c = 2(8)(8-1) = 112$ 。

1A

$$10. (px - 1)^n = (-1)^n + n(-1)^{n-1} px + \frac{n(n-1)}{2} (-1)^{n-2} p^2 x^2 + \dots \quad 1A$$

比較同類項， n 為奇數及

$$-\frac{n(n-1)}{2} \cdot p^2 = -10p^2$$

1M

$$n^2 - n - 20 = 0$$

$$n = 5 \quad \text{或} \quad -4 \quad (\text{捨去})$$

1A

$$np = \frac{5}{4}$$

$$p = \frac{1}{4}$$

1A

$$11. (1+kx)^n = 1 + C_1^n(kx) + C_2^n(kx)^2 + \dots \quad 1M$$

$$= 1 + nkx + \frac{n(n-1)}{2}(k^2)x^2 + \dots$$

比較係數，

$$\begin{cases} nk = 27 \\ \frac{n(n-1)}{2}(k^2) = 324 \end{cases} \quad 1M$$

$$\frac{n(n-1)}{2} \left(\frac{27}{n} \right)^2 = 324 \quad 1M$$

$$729n^2 - 729n = 648n^2$$

$$81n^2 - 729n = 0$$

$$n = 9 \quad \text{或} \quad 0 \quad (\text{捨去}) \quad 1A$$

當 $n = 9$ 時， $k = \frac{27}{9} = 3$ 。 1A

$$12. (k+x)^n = x^n + C_1^n x^{n-1}(k) + C_2^n x^{n-2}(k)^2 + \dots \quad 1M$$

$$= x^n + nkx^{n-1} + \frac{n(n-1)}{2}(k^2)x^{n-2} + \dots$$

比較係數，

$$\begin{cases} nk = 2n \\ \frac{n(n-1)}{2}(k^2) = 180 \end{cases} \quad 1M$$

故此， $k = 2$ 及 1A

$$\frac{n(n-1)}{2}(2^2) = 180$$

$$n^2 - n - 90 = 0$$

$$n = 10 \quad \text{或} \quad -9 \quad (\text{捨去}) \quad 1A$$

$$13. (1+kx^2)^n = 1 + C_1^n(kx^2) + C_2^n(kx^2)^2 + \dots \quad 1M$$

$$= 1 + nkx^2 + \frac{n(n-1)}{2}(k)^2x^4 + \dots$$

比較係數，

$$\begin{cases} nk = -24 \\ \frac{n(n-1)}{2}(k)^2 = 28k^2 \end{cases} \quad 1M$$

$$\frac{n(n-1)}{2} = 28$$

$$n^2 - n - 56 = 0$$

$$n = 8 \quad \text{或} \quad -7 \quad (\text{捨去}) \quad 1A$$

當 $n = 8$ 時， $k = \frac{-24}{8} = -3$ 。 1A

$$14. (3k+x)^n = x^n + C_1^n(3k)x^{n-1} + C_2^n(3k)^2x^{n-2} + \dots \quad 1M$$

$$= x^n + 3nkx^{n-1} + \frac{n(n-1)}{2}(9k^2)x^{n-2} + \dots$$

比較係數，

$$\begin{cases} 3nk = 21 \\ \frac{n(n-1)}{2}(9k^2) = 189 \end{cases} \quad 1M$$

$$\frac{n(n-1)}{2}(9)\left(\frac{21}{3n}\right)^2 = 189 \quad 1M$$

$$3969n^2 - 3969n = 3402n^2$$

$$567n^2 - 3969n = 0$$

$$n = 7 \quad \text{或} \quad 0 \quad (\text{捨去}) \quad 1A$$

當 $n = 7$ 時， $k = 1$ 。 1A