

# REG-POC-2425-ASM-SET 3-MATH

## Suggested solutions

### Multiple Choice Questions

- |       |       |       |       |       |
|-------|-------|-------|-------|-------|
| 1. B  | 2. C  | 3. A  | 4. B  | 5. D  |
| 6. C  | 7. D  | 8. A  | 9. C  | 10. C |
| 11. C | 12. C | 13. D | 14. B | 15. C |
| 16. D | 17. C | 18. C | 19. C | 20. D |
| 21. B | 22. A | 23. B | 24. D | 25. C |
| 26. C | 27. B | 28. C | 29. B | 30. C |

1. B

Join  $OC$  (to use the property from equal arcs)

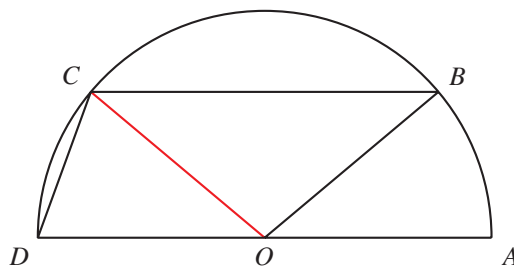
$$\text{Reflex } \angle BOD = 2 \times 110^\circ = 220^\circ$$

$$\angle AOB = 220^\circ - 180^\circ = 40^\circ$$

$$\angle COD = \angle AOB = 40^\circ$$

$$\angle COB = 180^\circ - 2 \times 40^\circ = 100^\circ$$

$$\angle OBC = \angle OCB = \frac{180^\circ - 100^\circ}{2} = 40^\circ$$



2. C

$$\angle POS = \angle ROS = \frac{136^\circ}{2} = 68^\circ$$

$$\angle SPO = \angle PSO = \frac{180^\circ - 68^\circ}{2} = 56^\circ$$

3. A

$$\angle COD = \angle BOC = \angle AOB = 70^\circ$$

$$\angle AOD = 360^\circ - 70^\circ \times 3 = 150^\circ$$

$$\angle ACD = \frac{\angle AOD}{2} = \frac{150^\circ}{2} = 75^\circ$$

4. B

$$\angle AOC = \angle BOD$$

$$\angle AOD = \angle BOC = \frac{180^\circ - 48^\circ}{2} = 66^\circ$$

$$\angle ABD = \frac{180^\circ - 66^\circ - 48^\circ}{2} = 33^\circ$$

5. D

$$\angle OCB = 68^\circ \text{ and } \angle BOC = 180^\circ - 68^\circ - 68^\circ = 44^\circ.$$

$$\angle COD = \angle AOB = x$$

$$x + 44^\circ + x = 180^\circ$$

$$x = 68^\circ$$

6. C

$$\angle AOB = \angle BOC = \frac{100^\circ}{2} = 50^\circ$$

$$\angle OAB = \angle OBA$$

$$\angle OAB = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

7. D

I. ✓. Note that  $\angle AOB = \angle BOC = \angle COD = \dots = \angle FOA$ .

$$\angle EOF = \frac{360^\circ}{6} = 60^\circ$$

II. ✓. Since  $\widehat{AB} = \widehat{DE}$ ,  $AB = DE$ .

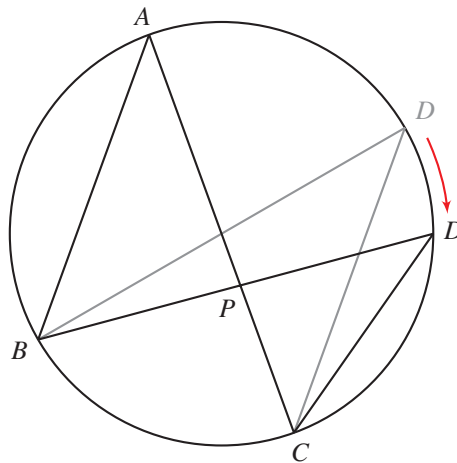
III. ✓. Since all arcs are equal, all chords are also equal.

It is a regular hexagon.

8. A

I. ✓.  $\angle APB = \angle CPD$  (vert. opp.  $\angle$ s)

II. ✗. Consider the following figure. Point  $D$  is shifted clockwise. Chords  $AC$  and chords  $BD$  intersect at  $P$ .



Note that  $\widehat{AB}$  remains unchanged, while  $\widehat{CD}$  becomes shorter.

Thus, the equation  $\widehat{AB} = \widehat{CD}$  cannot be true.

III. ✗. Consider the above figure.

$AB$  remains unchanged, while  $CD$  becomes shorter.

Thus, the equation  $AB = CD$  cannot be true.

9. C

$$\begin{aligned}\angle PQR &= 90^\circ \\ \angle PRQ &= \frac{180^\circ - 90^\circ}{2} = 45^\circ \\ \angle PQS &= 90^\circ - 18^\circ = 72^\circ \\ \angle SRP &= \angle SQR = 72^\circ \\ \angle QRS &= 72^\circ + 45^\circ = 117^\circ\end{aligned}$$

10. C

$$\begin{aligned}\angle AOP &= \angle POQ = \angle QOC = \frac{180^\circ - 90^\circ}{3} = 30^\circ \\ \angle CPQ &= \frac{1}{2} \angle COP = 15^\circ \\ \angle AQP &= \frac{1}{2} \angle AOP = 15^\circ \\ \angle ARC &= \angle PRQ = 180^\circ - 15^\circ - 15^\circ = 150^\circ\end{aligned}$$

11. C

$$\begin{aligned}\angle AOC &= \angle COE = \frac{180^\circ}{2} = 90^\circ \\ \text{Reflex } \angle AOC &= 360^\circ - 90^\circ = 270^\circ \\ \angle ABC &= \frac{270^\circ}{2} = 135^\circ\end{aligned}$$

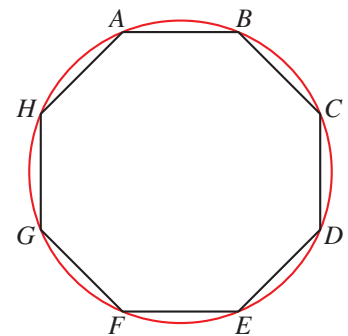
12. C

$$\begin{aligned}\angle FDE &= 145^\circ - 15^\circ = 130^\circ \\ \angle ADC &= 180^\circ - 130^\circ = 50^\circ \\ \angle AOC &= 2\angle ADC = 100^\circ \\ \angle BOC &= \angle AOB = \frac{100^\circ}{2} = 50^\circ \\ \angle OCE &= \angle BOC - \angle CEO = 50^\circ - 15^\circ = 35^\circ\end{aligned}$$

13. D

Refer to the figure. Note that every regular polygon is cyclic.

- I. ✓. The graph is symmetric about  $DH$ .  
So,  $AG \perp DH$  and  $BF \perp DH$ .  
Thus,  $AG \parallel BF$ .
- II. ✓. Since  $\widehat{BD} = \widehat{EG}$ ,  $BD = EG$ .
- III. ✓.  $\angle CAG : \angle BDH = \widehat{CEG} : \widehat{BAH} = 2 : 1$   
Thus,  $\angle CAG = 2\angle BDH$ .



14. B

Note that regular hexagon is cyclic.

Since  $BC = CD = DE = EF$ , we have  $\angle BAC = \angle CAD = \angle DAE = \angle EAF$ .

$$\angle BAF = \frac{(6-2)180^\circ}{6} = 120^\circ$$

$$x + y = 120^\circ \times \frac{2}{4} = 60^\circ$$

15. C

Let  $\angle ADB = x$ . Then  $\angle CAD = 2x$ .

$$\angle CED = \angle ADB + \angle CAD$$

$$87^\circ = x + 2x$$

$$x = 29^\circ$$

16. D

$$\angle AOB = 24^\circ \times 2 = 48^\circ$$

$$\angle BOC = 48^\circ \times \frac{3}{2} = 72^\circ$$

$$\angle AOC = 48^\circ + 72^\circ = 120^\circ$$

$$\angle OAC = \frac{180^\circ - 120^\circ}{2} = 30^\circ$$

17. C

$$\angle PRQ : \angle QPR : \angle PQR = \widehat{PQ} : \widehat{QR} : \widehat{PR} = 2 : 3 : 4$$

$$\angle PQR = 180^\circ \times \frac{4}{2+3+4} = 80^\circ$$

18. C

Let  $O$  be the centre.

$$\angle OBA = \angle BAC = 36^\circ$$

$$\angle AOB = 180^\circ - 36^\circ - 36^\circ = 108^\circ$$

$$\frac{\widehat{AC}}{\widehat{AB}} = \frac{\angle AOC}{\angle AOB}$$

$$\frac{\widehat{AC}}{9} = \frac{180^\circ}{108^\circ}$$

$$\widehat{AC} = 15 \text{ cm}$$

19. C

$$\angle BDA : \angle BDC = \widehat{AB} : \widehat{BC} = 1 : 2$$

$$\angle BDC = 84^\circ \times \frac{2}{1+2} = 56^\circ$$

20. D

$$\frac{\angle ABD}{\angle BAC} = \frac{\widehat{AD}}{\widehat{BC}}$$

$$\frac{\angle ABD}{28^\circ} = \frac{1}{2}$$

$$\angle ABD = 14^\circ$$

$$\angle ADB = 90^\circ$$

$$\angle CAD = 180^\circ - 90^\circ - 28^\circ - 14^\circ = 48^\circ$$

21. B

Let  $\angle ABD = 3\theta$ . Then  $\angle ACB = 3\theta$ ,  $\angle BAC = 2\theta$  and  $\angle CBD = 4\theta$ .

$$2\theta + (3\theta + 4\theta) + 3\theta = 180^\circ$$

$$\theta = 15^\circ$$

$$\angle BFC = 2\theta + 3\theta = 75^\circ$$

$$\angle ACE = 75^\circ - 40^\circ = 35^\circ$$

22. A

$$\angle POQ = 360^\circ - 100^\circ - 90^\circ - 50^\circ = 120^\circ$$

$$\frac{\widehat{SR}}{\widehat{PQ}} = \frac{\angle SOR}{\angle POQ}$$

$$= \frac{90^\circ}{120^\circ}$$

$$\widehat{SR} : \widehat{PQ} = 3 : 4$$

23. B

$$\angle AOB : \angle BOC : \angle COD = \widehat{AB} : \widehat{BC} : \widehat{CD}$$

$$= 2 : 1 : 1$$

$$\angle OAD = \angle ODA = 34^\circ$$

$$\angle AOD = 180^\circ - 34^\circ - 34^\circ = 112^\circ$$

$$\text{Reflex } \angle AOD = 360^\circ - 112^\circ = 248^\circ$$

$$\angle COD = 248^\circ \times \frac{1}{2+1+1} = 62^\circ$$

$$\angle AOC = 112^\circ + 62^\circ = 174^\circ$$

$$\angle ABC = \frac{1}{2}\angle AOC = 87^\circ$$

24. D

$$\angle AOB : \angle BOC : \angle COA = 5 : 4 : 3$$

$$\angle AOB = 360^\circ \times \frac{5}{5+4+3} = 150^\circ$$

$$\angle OBA = \frac{180^\circ - 150^\circ}{2} = 15^\circ$$

25. C

Let  $\angle PRQ = \theta$ .

Since  $PQ = QR = RS$ , we have  $\angle QSR = \angle SQR = \theta$ .

Since  $\widehat{PS} : \widehat{RS} = 2 : 1$ , we have  $\angle PRS = 2\theta$ .

In  $\triangle QRS$ ,

$$\theta + (\theta + 2\theta) + \theta = 180^\circ$$

$$\theta = 36^\circ$$

$$\angle PES = 2\theta + \theta = 108^\circ$$

26. C

Let  $O$  be the centre of the semi-circle  $APQRSB$ .

$$\angle AOQ = 180^\circ \times \frac{2}{5} = 72^\circ$$

$$\angle ABQ = \frac{\angle AOQ}{2} = 36^\circ$$

$$\angle BOS = 180^\circ \times \frac{1}{5} = 36^\circ$$

$$\angle BAS = \frac{\angle BOS}{2} = 18^\circ$$

$$\angle AKQ = 18^\circ + 36^\circ = 54^\circ$$

27. B

I.  $\checkmark$ . Note that  $12^2 + 12.6^2 = 302.76 = 17.4^2$ .

We have  $\angle BAC = 90^\circ$  and  $BC$  is a diameter of the circle.

$$\text{Radius} = \frac{BC}{2} = 8.7 \text{ cm}$$

II.  $\times$ . Note that  $\frac{\widehat{AC}}{\widehat{AB}} \neq \frac{AC}{AB} = \frac{21}{20}$  in general.

Let  $O$  be the centre of the circle.

$$12.6^2 = 8.7^2 + 8.7^2 - 2(8.7)(8.7) \cos \angle AOC$$

$$\angle AOC \approx 92.8^\circ$$

$$12^2 = 8.7^2 + 8.7^2 - 2(8.7)(8.7) \cos \angle AOB$$

$$\angle AOB \approx 87.2^\circ$$

$$\frac{\widehat{AC}}{\widehat{AB}} = \frac{\angle AOC}{\angle AOB} \approx 1.06 \neq \frac{21}{20}$$

III.  $\checkmark$ .  $BC$  is a diameter and  $\angle BAC = 90^\circ$ .

$$\cos \angle ACB = \frac{AC}{BC} = \frac{21}{29}$$

28. C

$$\angle DCA = 180^\circ - 120^\circ = 60^\circ$$

$$\angle BDC = \angle ABD \times \frac{3+4}{4+6} = 84^\circ$$

$$\angle EDB = 180^\circ - 84^\circ = 96^\circ$$

$$\angle AEC = 120^\circ - 96^\circ = 24^\circ$$

29. B

$$\angle ABD = 15^\circ + 25^\circ = 40^\circ$$

$$\angle ADB = 90^\circ$$

$$\angle BAD = 180^\circ - 90^\circ - 40^\circ = 50^\circ$$

$$\frac{\widehat{AD}}{\widehat{BQD}} = \frac{\angle ABD}{\angle BAD} = \frac{40^\circ}{50^\circ} = \frac{4}{5}$$

30. C

Let  $\angle ADB = \theta$ .

Then  $\angle BDC = 2\theta$ ,  $\angle CAD = 2\theta$  and  $\angle ACD = 3\theta$ .

$$\angle CAD + \angle ACD + \angle ADC = 180^\circ$$

$$2\theta + 3\theta + (\theta + 2\theta) = 180^\circ$$

$$\theta = 22.5^\circ$$

$$x = 180^\circ - 2\theta - \theta = 112.5^\circ$$