REG-GS-2425-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. A	3. D	4. D	5. D
6. A	7. C	8. B	9. B	10. A
11. C	12. D	13. B	14. D	15. C
16. D	17. C	18. A	19. B	20. A
21. D	22. C	23. D	24. A	

1. $\boxed{\text{B}}$

I. ✓. Common ratio $= 2^m$.

II. ✗. $\dfrac{2m^2}{m} = 2m$ and $\dfrac{3m^4}{2m^2} = \dfrac{3m^2}{2} \neq 2m$.

III. ✓. Sequence: $\log m, 2\log m, 4\log m, 8\log m$. Common ratio = 2.

2. $\boxed{\text{A}}$

$b^2 = ac$

$\dfrac{b}{a} = \dfrac{c}{b}$

I. ✓. We have $\dfrac{\sqrt{b}}{\sqrt{a}} = \sqrt{\dfrac{b}{a}}$ and $\dfrac{\sqrt{c}}{\sqrt{b}} = \sqrt{\dfrac{c}{b}}$.

Since $\dfrac{b}{a} = \dfrac{c}{b}$, $\dfrac{\sqrt{b}}{\sqrt{a}} = \dfrac{\sqrt{c}}{\sqrt{b}}$.

II. ✗. Take $a = 1$, $b = 2$ and $c = 4$.

$\dfrac{2^b}{2^a} = \dfrac{2^2}{2^1} = 2$ and $\dfrac{2^c}{2^b} = \dfrac{2^4}{2^2} = 4$.

$2^a, 2^b, 2^4c$ is not a geometric sequence.

III. ✗. Take $a = 10$, $b = 100$ and $c = 1000$.

$\dfrac{\log b}{\log a} = \dfrac{\log 100}{\log 10} = 2$ and $\dfrac{\log c}{\log b} = \dfrac{\log 1000}{\log 100} = \dfrac{3}{2}$.

$\log a, \log b, \log c$ is not a geometric sequence.

3. $\boxed{\text{D}}$

$$\frac{x+1}{2} = \frac{x^2-7}{x+1}$$

$$(x+1)^2 = 2(x^2-7)$$

$$0 = x^2 - 2x - 15$$

$$x = 5 \quad \text{or} \quad -3$$

4. D

$5 - h = k - 5$ and $\dfrac{3}{h} = \dfrac{k}{3}$

$h + k = 10$ $\qquad hk = 9$

$$\begin{aligned} h^2 + k^2 &= (h + k)^2 - 2hk \\ &= 10^2 - 2(9) \\ &= 82 \end{aligned}$$

5. D

$$\frac{x+7}{4} = \frac{x^2+11}{x+7}$$

$$(x+7)^2 = 4(x^2+11)$$

$$0 = 3x^2 - 14x - 5$$

$$x = -\frac{1}{3} \quad \text{or} \quad 5$$

6. A

Let the common ratio be R.

$$\begin{cases} p + r = p + pR^2 = 2 \\ q + s = pR + pR^3 = 32 \end{cases}$$

$$\frac{pR + pR^3}{p + pR^2} = \frac{32}{2}$$

$$\frac{pR(1+R^2)}{p(1+R^2)} = 16$$

$$R = 16$$

The common ratio is 16.

7. C

Let a and r be the first term and common ratio respectively.

$$\begin{cases} (a)(ar) = 48 \\ (ar)(ar^3) = 1296 \end{cases}$$

$$\frac{(ar)(ar^3)}{(a)(ar)} = \frac{1296}{48}$$

$$r^3 = 27$$

$$r = 3$$

When $r = 3$, $a^2 = \frac{48}{r} = 16$.

Required product $= (ar^2)(ar^4)$

$= a^2r^6$

$= 16(3^6)$

$= 11\,664$

8. B

Let n be the number of terms.

$$(a^4)(a^3)^{n-1} = a^{145}$$

$$a^{3n-3} = a^{141}$$

$$3n - 3 = 141$$

$$n = 48$$

There are 48 terms.

9. B

Let r be the common ratio.

$$\begin{cases} a_1 + a_1r + a_1r^2 = 1 \\ a_1r^3 + a_1r^4 + a_1r^5 = 8 \end{cases}$$

$$\frac{a_1r^3(1+r+r^2)}{a_1(1+r+r^2)} = \frac{8}{1}$$

$$r^3 = 8$$

$$r = 2$$

When $r = 2$, $a_1 = \frac{1}{1+r+r^2} = \frac{1}{7}$.

We have $a_2 = a_1r = \frac{2}{7}$.

10. A

Let r be the common ratio.

$$243r^3 = 9$$
$$r^3 = \frac{1}{27}$$
$$r = \frac{1}{3}$$

$$y - x = 243r^2 - 243r$$
$$= -54$$

11. C

Let r be the common ratio.

$$\frac{3}{r} + 3r = \frac{13}{2}$$
$$3 + 3r^2 = \frac{13r}{2}$$
$$3r^2 - \frac{13r}{2} + 3 = 0$$
$$r = \frac{2}{3} \quad \text{or} \quad \frac{3}{2}$$

12. D

Let r be the common ratio.

$$6 + 6r + 6r^2 = 186$$
$$6r^2 + 6r - 180 = 0$$
$$r = -6 \quad \text{or} \quad 5$$

13. B

Let a and r be the first term and common ratio respectively.

$$\frac{ar^6}{ar^4} = \frac{6}{24}$$
$$r^2 = \frac{1}{4}$$
$$r = \pm\frac{1}{2}$$

I. ✓. $T_1 = a = \frac{24}{r^4} > 0$

II. ✓. $T_1 - T_2 = T_1(1 - r) > 0$ for $r = \pm 1$.

III. ✗. When $r = -\frac{1}{2}$,

$$T_2 + T_3 + \ldots + T_{200} = \frac{T_2(1 - r^{199})}{1 - r}$$
$$= \frac{ar(1 - r^{199})}{1 - r}$$

Note that $a > 0$, $r < 0$, $(1 - r^{199}) > 0$ and $(1 - r) > 0$.

The sum is therefore negative.

14. D

Common ratio = $\frac{y}{x}$

Let a be the first term.

$$a\left(\frac{y}{x}\right)^2 = x$$
$$a = \frac{x^3}{y^2}$$

Sum to infinity $= \frac{\left(\frac{x^3}{y^2}\right)}{1 - \frac{y}{x}}$

$$= \frac{x^3}{y^2} \div \frac{x - y}{x}$$
$$= \frac{x^4}{y^2(x - y)}$$

15. C

Sum to infinity $= \frac{-a}{1 - \left(-\frac{1}{a}\right)}$

$$= -a \div \frac{a + 1}{a}$$
$$= -\frac{a^2}{1 + a}$$

16. D

Common ratio $= \dfrac{n}{m}$

Let a be the first term.

$$a\left(\frac{n}{m}\right)^3 = m$$

$$a = \frac{m^4}{n^3}$$

$$\text{Sum to infinity} = \frac{\left(\frac{m^4}{n^3}\right)}{1-\frac{n}{m}}$$

$$= \frac{m^4}{n^3} \div \frac{m-n}{m}$$

$$= \frac{m^5}{n^3(m-n)}$$

17. C

$$\text{Sum to infinity} = \frac{18}{1-\frac{1}{3}}$$

$$= 27$$

18. A

Let r be the common ratio.

First term $= \dfrac{-9}{r}$

$$\frac{\left(\frac{-9}{r}\right)}{1-r} = \frac{81}{4}$$

$$\frac{-9}{r(1-r)} = \frac{81}{4}$$

$$-36 = 81r - 81r^2$$

$$81r^2 - 81r - 36 = 0$$

$$r = -\frac{1}{3} \quad \text{or} \quad \frac{4}{3} \text{ (rejected)}$$

Note that $-1 < r < 1$ for the existence of sum to infinity.

19. B

$$\frac{x}{1-x} = \frac{1}{2}$$

$$2x = 1 - x$$

$$x = \frac{1}{3}$$

20. A

Let the first term and common ratio be a and r respectively.

$$\begin{cases} \dfrac{a}{1-r} = \dfrac{81}{4} \\ ar = -9 \end{cases}$$

$$ar \div \frac{a}{1-r} = -9 \div \frac{81}{4}$$

$$r(1-r) = -\frac{4}{9}$$

$$-r^2 + r + \frac{4}{9} = 0$$

$$r = -\frac{1}{3} \quad \text{or} \quad \frac{4}{3} \text{ (rejected)}$$

21. D

First term = $3^{6-2(1)} = 81$ and second term = $3^{6-2(2)} = 9$

Common ratio = $\dfrac{9}{81} = \dfrac{1}{9}$

22. C

Let a and r be the first term and common ratio respectively.

$$\frac{a}{1-r} = 3a$$

$$a = 3a(1-r)$$

$$0 = a[3(1-r) - 1]$$

$$0 = a(2-3r)$$

$$r = \frac{2}{3} \quad \text{or} \quad a = 0 \text{ (rejected)}$$

23. D

Required sum = $\dfrac{-81}{1 - \left(\frac{-9}{-81}\right)}$

$= -\dfrac{729}{8}$

24. A

Required sum = $\dfrac{12}{1 - \frac{3}{12}}$

$= 16$

Conventional Questions

25. (a) $30 + 30r + 30r^2 = 52.5$ 1M

$30r^2 + 30r - 22.5 = 0$

$r = \frac{1}{2}$ or $-\frac{3}{2}$ (rejected) 1A

(b) Height of the balloon $\leq \frac{30}{1-\frac{1}{2}}$ 1M

$= 60\text{ m} < 75\text{ m}$

The balloon cannot reach a height of 75 m. 1A

(c) Let the time required be n minutes.

$\frac{30(1-0.5^n)}{1-0.5} = 58.125$ 1M

$0.5^n = 0.03125$

$n \log 0.5 = \log 0.03125$ 1M

$n = 5$

The time required is 5 minutes. 1A

26. (a) Required probability $= \frac{3}{7} \times \frac{3}{8} + \frac{4}{7} \times \frac{6}{8}$ 1M

$= \frac{33}{56}$ 1A

(b) Required probability $= \frac{33}{56} + \left(\frac{23}{56}\right)^2 \frac{33}{56} + \left(\frac{23}{56}\right)^4 \frac{33}{56} + \ldots$ 1M

$= \frac{\frac{33}{56}}{1-\frac{529}{3136}}$ 1M

$= \frac{56}{79}$ 1A

(c) Required probability $= \frac{\left(\frac{23}{56}\right)\frac{33}{56} + \left(\frac{23}{56}\right)^3 \frac{33}{56}}{1-\frac{56}{79}}$ 1M+1A

≈ 0.972 1A

27. (a) (i) Required number

$= 120\,000(1+10\%)^n - 3000(1+10\%)^{n-1} - 3000(1+10\%)^{n-2} - \ldots - 3000$ 1M

$= 120\,000(1.1)^n - \frac{3000(1.1^n - 1)}{1.1-1}$ 1M

$= 90\,000(1.1)^n + 30\,000$ 1A

(ii) $90\,000(1.1)^n + 30\,000 > 200\,000$ 1M

$1.1^n > \frac{17}{9}$

$n \log 1.1 > \log \frac{17}{9}$ 1M

$n > 6.67$

The total number of applicants will exceed 20 000 in year 2020. 1A

(b) We have

$$\begin{cases} 10\,000p + q = 37\,100 \\ 10\,000p^2 + q = 39\,641 \end{cases}$$ 1M

$$10\,000p^2 + (37\,100 - 10\,000p) = 39\,641$$ 1M

$$10\,000p^2 - 10\,000p - 2541 = 0$$

$$p = 1.21 \quad \text{or} \quad -0.21 \text{ (rejected)}$$

When $p = 1.21$, $q = 25\,000$. 1A

Let m be the number of years after 2013 such that the number of flats does not fulfil the demand.

$$90\,000(1.1)^m + 30\,000 > 10\,000(1.21)^m + 25\,000$$ 1M

$$0 > 1.21^m - 9(1.1)^m - 0.5$$

$$-0.055\,216\,789\,57 < 1.1^m < 9.055\,216\,790$$

$$m \log 1.1 < \log 9.055\,216\,790$$

$$m < 23.1$$ 1A

Therefore, the number of flats does not fulfil the demand from 2014 to 2036.

The claim is agreed. 1A

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