REG-GS-2425-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. A	3. B	4. D	5. C
6. A	7. D	8. C	9. D	10. B
11. A	12. C	13. B	14. A	15. D
16. C	17. B	18. D		

1. $\boxed{\text{B}}$

$$1 + 2^2 + 2^4 + \ldots + 2^{2n} = \frac{1[(2^2)^{n+1} - 1]}{2^2 - 1}$$
$$= \frac{4^{n+1} - 1}{3}$$

2. $\boxed{\text{A}}$

$$\text{Common ratio} = \frac{9}{27} = \frac{1}{3}$$
$$\text{Required sum} = \frac{27\left[1 - \left(\frac{1}{3}\right)^6\right]}{1 - \frac{1}{3}}$$
$$= \frac{364}{9}$$

3. $\boxed{\text{B}}$

$$\text{Required sum} = \frac{10\,368\left[1 - \left(\frac{1}{2}\right)^{10}\right]}{1 - \frac{1}{2}}$$
$$= 20\,715.75$$

4. $\boxed{\text{D}}$

$$\frac{7(2^n - 1)}{2 - 1} = 114\,681$$
$$2^n = 16\,384$$
$$n \log 2 = \log 16\,384$$
$$n = 14$$

5. C

Note that $\log 2^2 = 2\log 2$ and $\log 2^4 = 4\log 2$.
Common ratio of the sequence is 2.

$$\begin{aligned} \log 2 + 2\log 2 + 4\log 2 + 2^{n-1}\log 2 &> \log 2^{4095} \\ \frac{\log 2[2^n - 1]}{2-1} &> 4095\log 2 \\ 2^n &> 4096 \\ n\log 2 &> \log 4096 \\ n &> 12 \end{aligned}$$

Least possible value of n is 13.

6. A

Let the first term and common ratio be a and r respectively.

$$\begin{aligned} \frac{ar^8}{ar^6} &= \frac{8}{32} \\ r^2 &= \frac{1}{4} \\ r &= \pm\frac{1}{2} \end{aligned}$$

I. ✓. $a_1 = \dfrac{a_7}{r^6} > 0$.

II. ✓. $a_1 - a_2 = a_1(1-r) > 0$ (since $a_1 > 0$ and $1 - r > 0$).

III. ✗. If $r = \dfrac{1}{2}$, then all the terms are positive and you don't need to evaluate the sum.
If $r = -\dfrac{1}{2}$, $a_2 + a_3 + \ldots + a_{100} = \dfrac{a_1 r(1-r^{99})}{1-r} = \dfrac{\oplus \ominus (\oplus)}{\oplus} < 0$.
(since $a_1 > 0$, $r < 0$, $1 - r > 0$ and $1 - r^{99} > 0$)

7. D

Let the first term and common ratio be a and r respectively.

$$\begin{aligned} \frac{ar^7}{ar^5} &= \frac{96}{216} \\ r^2 &= \frac{4}{9} \\ r &= \pm\frac{2}{3} \end{aligned}$$

I. ✗. When $r = -\dfrac{2}{3}$, $x_3 = \dfrac{x_6}{r^3} < 0$. x_3 may not be 729.

II. ✓. $\dfrac{x_5}{x_7} = \dfrac{1}{r^2} = \dfrac{9}{4} > 1$.

III. ✓. $x_2 = \dfrac{x_6}{r^4} = \dfrac{2187}{2}$, common ratio $= \dfrac{x_4}{x_2} = r^2$.
$x_2 + x_4 + x_6 + \ldots + x_{2n} = \dfrac{2187}{2} \times \dfrac{1-(r^2)^n}{1-r^2} = 1968.3\left[1 - \left(\dfrac{4}{9}\right)^n\right] < 1968.3 < 2015$.

8. C

$$\begin{aligned}\frac{5}{6} &= 1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \ldots \\ &= \frac{1}{1 - \left(-\frac{1}{x}\right)} \\ &= \frac{x}{x+1}\end{aligned}$$

$$\begin{aligned}5x + 5 &= 6x \\ x &= 5\end{aligned}$$

9. D

Sum to infinity $= \dfrac{-1}{1 - \left(-\frac{1}{x}\right)} = \dfrac{-x}{x+1}$.

10. B

Sum to infinity exists when $-1 < r < 1$.

The answer is B.

11. A

$$\begin{aligned}\text{Required sum} &= \frac{\left(\frac{1}{x^3}\right)}{1 - (-x^2)} \\ &= \frac{1}{x^3(1+x^2)} \\ &= \frac{1}{x^3 + x^5}\end{aligned}$$

12. C

Let the common ratio be r.

$$\begin{aligned}\frac{\frac{3}{2}}{1-r} &= 2 \\ r &= \frac{1}{4}\end{aligned}$$

13. B

We have

$$\begin{cases} \dfrac{a}{1-r} = \dfrac{64}{5} \\ ar = -4 \end{cases}$$

$$\frac{\left(\frac{-4}{r}\right)}{1-r} = \frac{64}{5}$$

$$-20 = 64r(1-r)$$

$$0 = -64r^2 + 64r + 20$$

$$r = -\frac{1}{4} \quad \text{or} \quad \frac{5}{4} \text{ (rejected)}$$

$$\text{First term} = \frac{-4}{r}$$

$$= 16$$

14. A

Let the common ratio be r.

$$\frac{a}{1-r} = \frac{3}{4}a$$

$$r = -\frac{1}{3}$$

15. D

We have

$$\begin{cases} a + ar = 24 \\ \dfrac{a}{1-r} = 27 \end{cases}$$

$$(a+ar) \div \frac{a}{1-r} = \frac{24}{27}$$

$$(1+r)(1-r) = \frac{8}{9}$$

$$1 - r^2 = \frac{8}{9}$$

$$r^2 = \frac{1}{9}$$

$$r = \pm\frac{1}{3}$$

16. C

$$\text{Required sum} = (-9) + (-9)\left(\frac{1}{9}\right) + (-9)\left(\frac{1}{9}\right)^2 + \ldots$$

$$= \frac{-9}{1 - \frac{1}{9}}$$

$$= -\frac{81}{8}$$

17. B

$$\text{Required sum} = 9 + 1 + \frac{1}{9} + \ldots$$
$$= \frac{9}{1 - \frac{1}{9}}$$
$$= \frac{81}{8}$$

18. D

$$\text{Required sum} = 8 + 2 + \ldots$$
$$= \frac{8}{1 - \frac{1}{4}}$$
$$= \frac{32}{3}$$

Conventional Questions

19. (a) Let a and r be the first term and common ratio respectively.

$$\frac{ar^5}{ar^2} = \frac{486}{144} \qquad \text{1M}$$
$$r^3 = \frac{27}{8}$$
$$r = \frac{3}{2}$$

First term = $a = 144 \div \left(\frac{3}{2}\right)^2 = 64$. 1A

(b) $64 + 96 + \ldots + 64(1.5)^{n-1} > 8 \times 10^{18}$

$$\frac{64(1.5^n - 1)}{1.5 - 1} > 8 \times 10^{18} \qquad \text{1M}$$
$$1.5^n > 6.25 \times 10^{16} + 1$$
$$n \log 1.5 > \log\left(6.25 \times 10^{16} + 1\right) \qquad \text{1M}$$
$$n > 95.4$$

The least value of n is 96. 1A

20. (a) (i) Required amount $= P\left(1 + \frac{3\%}{12}\right)^2 - x\left(1 + \frac{3\%}{12}\right) - x$ 1M

$= \$(1.0025^2 P - 1.0025x - x)$ 1A

(ii) Required amount $= 1.0025^n P - 1.0025^{n-1}x - 1.0025^{n-2}x - \ldots - x$ 1M

$= 1.0025^n P - \frac{x(1.0025^n - 1)}{1.0025 - 1}$ 1M

$= \$[1.0025^n P - 400x(1.0025^n - 1)]$ 1A

(b) (i) $P = 3\,000\,000 \times 70\% = 2\,100\,000$.

Put $P = 2\,100\,000$ and $x = 10\,000$,

$$1.0025^n(2\,100\,000) - 400(10\,000)(1.0025^n - 1) \le 0 \qquad \text{1M}$$
$$1.0025^n \ge \frac{40}{19}$$
$$n \log 1.0025 \ge \log\frac{40}{19} \qquad \text{1M}$$
$$n \ge 298.1 \qquad \text{1A}$$

It takes Jason 299 months to repay the loan. 1A

(ii) First month interest $= 2\,100\,000 \times \frac{3\%}{12}$

$= \$5250 > \$5000$ 1M

Therefore, Jason will never repay the loan, and hence the request will not be approved. 1A

21. (a) (i) $P(1+r\%)^2 = 1.0201P$ 1M

$1+r\% = 1.01$

$r = 1$ 1A

(ii) Required amount

$= P\left(1+\frac{6\%}{12}\right)^n + P(1.01)\left(1+\frac{6\%}{12}\right)^{n-1} + P(1.01)^2\left(1+\frac{6\%}{12}\right)^{n-2} + \ldots$

$+ P(1.01)^{n-1}\left(1+\frac{6\%}{12}\right)$ 1M

$= P[1.005^n + (1.01)(1.005)^{n-1} + (1.01)^2(1.005)^{n-2} + \ldots + (1.01)^{n-1}(1.005)]$

$= P \times \dfrac{1.005^n\left[\left(\frac{1.01}{1.005}\right)^n - 1\right]}{\frac{1.01}{1.005} - 1}$ 1M

$= P \times \dfrac{1.01^n - 1.005^n}{\frac{0.005}{1.005}}$

$= \$201P[(1.01)^n - (1.005)^n]$ 1

(b) (i) Required selling price $= 3\,000\,000(1+0.5\%)^{n+24}$ 1M

$= \$3\,000\,000(1.005)^{n+24}$ 1A

(ii) Amount in Ms Wong's account

$= 201(20\,000)[(1.01)^n - (1.005)^n]$

$= \$4\,020\,000[(1.01)^n - (1.005)^n]$

Required down payment

$= 3\,000\,000(1.005)^{n+24} \times 30\%$

$= \$900\,000(1.005)^{n+24}$

$$4\,020\,000[(1.01)^n - (1.005)^n] \geq 900\,000(1.005)^{n+24}$$ 1M+1A

$$1.01^n - 1.005^n \geq \frac{15}{67}(1.005)^{n+24}$$

$$\left(\frac{1.01}{1.005}\right)^n \geq 1 + \frac{15}{67} \times 1.005^{24}$$

$$n\log\frac{1.01}{1.005} \geq \log\left(1 + \frac{15}{67} \times 1.005^{24}\right)$$ 1M

$$n \geq 45.3$$

The amount will be enough at the end of October 2015. 1A

22. (a) (i) Required number

$= 7\times10^6(1+2\%)^n - 5\times10^3(1+2\%)^{n-1} - 5\times10^3(1+2\%)^{n-2} - \ldots - 5\times10^3$ 1M

$= 7\times10^6(1.02)^n - \dfrac{5\times10^3(1.02^n - 1)}{1.02-1}$ 1M

$= 7\times10^6(1.02)^n - 2.5\times10^5(1.02^n - 1)$

$= 6.75\times10^6(1.02)^n + 2.5\times10^5$ 1

(ii) $6.75 \times 10^6(1.02)^n + 2.5 \times 10^5 > 7 \times 10^6 \times 2$

$$1.02^n > \frac{55}{27}$$

$$n \log 1.02 > \log \frac{55}{27} \qquad \text{1M}$$

$$n > 35.9$$

Thus, $n = 36$. 1A

(b) $$4 \times 10^6(1.0404)^n + 2 \times 10^5 > 6.75 \times 10^6(1.02)^n + 2.5 \times 10^5$$

$$40(1.02)^{2n} - 67.5(1.02)^n - 0.5 > 0 \qquad \text{1M}$$

$1.02^n < 0.0711$ (rejected) or $1.02^n > 1.69$

$$n \log 1.02 > \log 1.69 \qquad \text{1M}$$

$$n > 26.6$$

Thus, the required time is 27th second after the start of observation. 1A

(c) Number of particles after n seconds

$= 7 \times 10^6(1 + 1\%)^n - 1 \times 10^5(1 + 1\%)^{n-1} - 1 \times 10^5(1 - 2\%)(1 + 1\%)^{n-2}$

$\quad - 1 \times 10^5(1 - 2\%)^2(1 + 1\%)^{n-3} - \ldots - 1 \times 10^5(1 - 2\%)^{n-1}$ 1M

$$= 7 \times 10^6(1.01)^n - \frac{1 \times 10^5(1.01)^{n-1}\left[1 - \left(\frac{0.98}{1.01}\right)^n\right]}{1 - \frac{0.98}{1.01}}$$

$$= 7 \times 10^6(1.01)^n - \frac{100}{3} \times 10^5(1.01^n - 0.98^n)$$

$$= \left[\frac{110}{3}(1.01)^n + \frac{100}{3}(0.98)^n\right] 10^5$$

If the number of particles after n seconds starts increasing,

$$\frac{110}{3}(1.01)^n + \frac{100}{3}(0.98)^n > \frac{110}{3}(1.01)^{n-1} + \frac{100}{3}(0.98)^{n-1}$$

$$\frac{11}{30}(1.01)^{n-1} > \frac{2}{3}(0.98)^{n-1}$$

$$\left(\frac{1.01}{0.98}\right)^{n-1} > \frac{20}{11}$$

$$(n-1) \log \frac{1.01}{0.98} > \log \frac{20}{11}$$

$$n > 20.8 \qquad \text{1A}$$

The number of particles starts increasing after 21 seconds since the start of observation.

The claim is agreed. 1A

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