REG-GS-2425-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. A	3. C	4. A	5. A
6. D	7. A	8. B	9. D	10. A
11. A	12. C	13. C	14. B	15. A
16. A	17. C	18. B	19. B	20. C
21. B	22. A	23. B	24. A	25. D
26. A	27. D	28. A	29. A	30. D

1. $\boxed{\text{A}}$

$$\text{Required sum} = \frac{\frac{3}{4}(4^n - 1)}{4 - 1}$$
$$= \frac{1}{4}(4^n - 1)$$

2. $\boxed{\text{A}}$

$$\text{Required sum} = \frac{2\left[1 - \left(\frac{\sqrt{2}}{2}\right)^{10}\right]}{1 - \frac{\sqrt{2}}{2}}$$
$$= \frac{2\left(1 - \frac{1}{32}\right)}{\left(\frac{2-\sqrt{2}}{2}\right)}$$
$$= \frac{31}{8(2 - \sqrt{2})}$$

3. $\boxed{\text{C}}$

$$\text{Required sum} = \frac{-1[1 - (-4)^7]}{1 - (-4)}$$
$$= -3277$$

4. $\boxed{\text{A}}$

$$\text{5th term} = \left(32 - 2^{5-5}\right) - \left(32 - 2^{5-4}\right)$$
$$= 31 - 30$$
$$= 1$$

5. A

$$640\left(-\frac{1}{2}\right)^{n-1} = -5$$
$$\left(-\frac{1}{2}\right)^{n-1} = -\frac{1}{128}$$
$$= \left(-\frac{1}{2}\right)^{7}$$
$$n = 8$$

There are 8 terms.

$$\text{Required sum} = \frac{640\left[1-\left(-\frac{1}{2}\right)^{8}\right]}{1-\left(-\frac{1}{2}\right)}$$
$$= 425$$

6. D

$$\text{Required sum} = \frac{7(5^7-1)}{5-1}$$
$$= 136\,717$$

7. A

$$\frac{-1(3^k-1)}{3-1} < -200$$
$$3^k > 401$$
$$k\log 3 > \log 401$$
$$k > 5.46$$

Minimum value of k is 6.

8. B

$$\frac{3(3^k-1)}{3-1} > 780$$
$$3^k > 521$$
$$k\log 3 > \log 521$$
$$k > 5.69$$

Minimum value of k is 6.

9. D

$$\text{Required sum} = \frac{18}{1-\frac{1}{3}}$$
$$= 27$$

10. A

Let the common ratio be r.

$$\frac{30}{1-r} = 25$$

$$r = \frac{1}{6}$$

11. A

Let the first term be a.

$$\frac{a}{1-\left(-\frac{1}{2}\right)} = \frac{1}{6}$$

$$a = \frac{1}{4}$$

$$\text{Required sum} = \frac{\frac{1}{4}\left[1-\left(-\frac{1}{2}\right)^5\right]}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{11}{64}$$

12. C

Sum of first four terms $= -10 - 5 - 2.5 - 1.25$

$= -18.75$

$$\text{Sum to infinity} = \frac{-10}{1-\frac{1}{2}}$$

$= -20$

Error $= -18.75 - (-20)$

$$= \frac{5}{4}$$

13. C

Let the first term be a.

$$\frac{a}{1-\frac{1}{2}} = 16$$

$$a = 8$$

14. B

$$\frac{2x-1}{5x+2} = \frac{x-2}{2x-1}$$

$$(2x-1)^2 = (x-2)(5x+2)$$

$$4x^2 - 4x + 1 = 5x^2 - 8x - 4$$

$$0 = x^2 - 4x - 5$$

$$x = 5 \quad \text{or} \quad -1 \text{ (rejected)}$$

$$\text{Required sum} = \frac{27}{1 - \frac{9}{27}}$$

$$= \frac{81}{2}$$

15. A

$$\text{Common ratio} = \frac{q}{p}$$

$$\text{First term} = \frac{p}{\left(\frac{q}{p}\right)^2} = \frac{p^3}{q^2}$$

$$\text{Required sum} = \frac{\frac{p^3}{q^2}}{1 - \frac{q}{p}}$$

$$= \frac{p^3}{q^2} \div \frac{p-q}{p}$$

$$= \frac{p^4}{q^2(p-q)}$$

16. A

$$\frac{36}{2k} = \frac{72k}{36}$$

$$36 = 4k^2$$

$$k = 3 \quad \text{or} \quad -3 \text{ (rejected)}$$

The geometric sequence is 216, 36, 6, . . .

$$\text{Required sum} = \frac{216}{1 - \frac{6}{36}}$$

$$= \frac{1296}{5}$$

17. C

$$\frac{a}{1 - \left(-\frac{1}{3}\right)} = \frac{45}{8}$$

$$a = \frac{15}{2}$$

18. B

Ratio $= \dfrac{-24}{72} = -\dfrac{1}{3}$

Sum to infinity $= \dfrac{72}{1-\left(-\frac{1}{3}\right)} = 54$

19. B

Let the first term and common ratio be a and r respectively.

$$\begin{cases} \dfrac{a(1-r^3)}{1-r} = -\dfrac{3}{8} \\ \dfrac{a}{1-r} = -\dfrac{1}{3} \end{cases}$$

$$\begin{aligned} \left(-\frac{1}{3}\right)(1-r^3) &= -\frac{3}{8} \\ r^3 &= -\frac{1}{8} \\ r &= -\frac{1}{2} \end{aligned}$$

20. C

All positive terms: $36, 9, \dfrac{9}{4}, \ldots$.

Sum $= \dfrac{36}{1-\frac{1}{4}} = 48$

21. B

$$\begin{aligned} \text{Required sum} &= \left(\frac{1}{3} + \frac{1}{3^5} + \ldots\right) + \left(\frac{2}{3^3} + \frac{2}{3^7} + \ldots\right) \quad = \frac{\frac{1}{3}}{1-\frac{1}{3^4}} + \frac{\frac{2}{27}}{1-\frac{1}{3^4}} \\ &= \frac{33}{80} \end{aligned}$$

22. A

Let the first term and common ratio be a and r respectively.

$$\begin{aligned} \frac{ar^7}{ar^5} &= \frac{3}{48} \\ r^2 &= \frac{1}{16} \\ r &= \pm\frac{1}{4} \end{aligned}$$

I. ✓. $x_4 = \dfrac{48}{r^2} = 768$.

II. ✗. $\dfrac{x_9}{x_{12}} = \dfrac{1}{r^3} < 0 < 1$ when $r = -\dfrac{1}{4}$.

III. ✗. There are n terms.

$$x_4 + x_6 + \ldots + x_{2n+2} = \frac{768\left[1-\left(\frac{1}{16}\right)^n\right]}{1-\frac{1}{16}} = 819.2\left[1-\left(\frac{1}{16}\right)^n\right] < 819.2 < 1000$$

23. $\boxed{\text{B}}$

I. ✓. General term $= (1 - 2^{-n}) - (1 - 2^{-(n-1)})$

$= 2^{-n}(-1 + 2)$

$= 2^{-n}$

We have $2^{-n} < 1$ for all positive integers n.

II. ✗. The nth term $= \dfrac{1}{2^n}$ is a rational number for all positive integers n.

III. ✓. $\log T_{n+1} - \log T_n = \log 2^{-n-1} - \log 2^{-n}$

$= (-n - 1)\log 2 + n \log 2$

$= -\log 2 = \text{constant}$

Thus, it is an arithmetic sequence.

24. $\boxed{\text{A}}$

General term $= (2^{n+1} - 2) - (2^n - 2)$

$= 2^n(2 - 1)$

$= 2^n$

I. ✓. $\dfrac{T(n+1)}{T(n)} = \dfrac{2^{n+1}}{2^n} = 2 = \text{constant}.$

II. ✗. Second term $= 2^2 = 4 \neq 6$.

III. ✗.

25. D

Let the first term and common ratio be a and r respectively.

$$\frac{ar^7}{ar^2} = \frac{6}{192}$$
$$r^5 = \frac{1}{32}$$
$$r = \frac{1}{2}$$

I. ✗.

II. ✓. $a = \dfrac{192}{r^2} = 768$.

$$768\left(\frac{1}{2}\right)^{n-1} > 10^{-2}$$
$$(n-1)\log\frac{1}{2} > \log\frac{1}{76\,800}$$
$$n < 17.2$$

17 terms are greater than 10^{-2}.

III. ✓. Sum $= \dfrac{768\left(1-\left(\frac{1}{2}\right)^{13}\right)}{1-\frac{1}{2}} \approx 1535.8$

> 1535

26. A

I. ✓. 4th term $= 2 \times 3^4 = 162$.

II. ✗. Sum of first n terms $= \dfrac{2(3)(3^n - 1)}{3-1} = 3(3^n - 1)$.

III. ✗. Sum to infinity does not exists as common ratio > 1.

27. D

Let first term and common ratio be a and r respectively.

$$\frac{ar^7}{ar^3} = \frac{4}{243} \div \frac{1}{12}$$
$$r^4 = \frac{16}{81}$$
$$r = \pm\frac{2}{3}$$

When $r = \frac{2}{3}$, $a = \frac{1}{12} \div r^3 = \frac{9}{32}$ and $S(\infty) = \frac{a}{1-r} = \frac{27}{32} > \frac{1}{2}$ (rejected).
When $r = -\frac{2}{3}$, $a = \frac{1}{12} \div r^3 = -\frac{9}{32}$.

I. ✗.

II. ✓. $a_1 + a_2 + \ldots + a_{10} = \frac{a(r^9 - 1)}{r - 1} \approx -0.173 < -\frac{1}{10}$.

III. ✓. $a_2 + a_4 + \ldots = \frac{ar}{1 - r^2} = \frac{27}{80}$.

28. A

Let the first term and common ratio be a and r respectively.

$$\frac{ar^6}{ar^4} = \frac{15}{375}$$
$$r^2 = \frac{1}{25}$$
$$r = \pm\frac{1}{5}$$

I. ✓. $a_1 = \frac{375}{r^4} = 234\,375$.

II. ✗. When $r = -\frac{1}{5}$, $a_2 = \frac{ar^4}{r^3} < 0$ and $a_2 \neq 46\,875$.

III. ✗. When $r = -\frac{1}{5}$,

$$\begin{aligned} a_1 + a_2 + a_3 + \ldots &= \frac{234\,375}{1 - \left(-\frac{1}{5}\right)} \\ &= 195\,312.5 < 290\,000 \end{aligned}$$

29. A

Let the first term and common ratio be a and r respectively.

$$\frac{ar^6}{ar^2} = \frac{48}{3}$$
$$r^4 = 16$$
$$r = \pm 2$$

I. ✓. 5th term = $3 \times r^2 = 12$.

II. ✗. The sequence has negative terms when $r = -2$.

III. ✗. Sum to infinity does not exist.

30. D

Let the common ratio be r.

$$r^{6-2} = \frac{1}{18} \div \frac{9}{32}$$
$$r^4 = \frac{16}{81}$$
$$r = \pm\frac{2}{3}$$

When $r = \frac{2}{3}$, $a_1 = \frac{a_2}{r} = \frac{27}{64}$.
Sum to infinity $= \frac{a_1}{1-r} = \frac{81}{64} > \frac{1}{2}$.
When $r = -\frac{2}{3}$, $a_1 = \frac{a_2}{r} = -\frac{27}{64}$.
Sum to infinity $= \frac{a_1}{1-r} = -\frac{81}{320} < \frac{1}{2}$.
Thus, $r = -\frac{2}{3}$.

I. ✓. $a_4 = a_2 \times r^2 = \frac{1}{8}$

II. ✓.

$$\begin{aligned} a_1 - a_2 + a_3 - a_4 + \ldots + a_9 - a_{10} &= \frac{a_1[1-(-r)^{10}]}{1-(-r)} \\ &= \frac{\frac{a_2}{r}(1-r^{10})}{1+r} \\ &\approx -1.24 < -1 \end{aligned}$$

III. ✓.

$$\begin{aligned} a_2 + a_4 + \ldots + &= \frac{a_2}{1-r^2} \\ &= \frac{81}{160} \\ &= \frac{9}{320\left(\frac{1}{18}\right)} \end{aligned}$$

Conventional Questions

31. (a) $A(1)+A(2)+A(3)+\ldots+A(n) = 1000(2+2^2+2^3+\ldots+2^n)$

$= 1000\times\dfrac{2(2^n-1)}{2-1}$ 1M

$= 2000(2^n-1)$ 1

(b) $B(1)+B(2)+B(3)+\ldots+B(n) > A(1)+A(2)+A(3)+\ldots+A(n)$

$\dfrac{4(4^n-1)}{4-1} > 2000(2^n-1)$

$\dfrac{4}{3}(2^n)^2 - 2000(2^n) + \dfrac{5996}{3} > 0$ 1A

$2^n < 1$ (rejected) or $2^n > 1499$

$n\log 2 > \log 1499$ 1M

$n > 10.5$

The least value of n is 11. 1A

32. (a) $S(n) = \dfrac{16\left[1-\left(\frac{3}{4}\right)^n\right]}{1-\frac{3}{4}}$ 1M

$= 64\left[1-\left(\dfrac{3}{4}\right)^n\right]$ 1A

(b) Required sum $= 64\left[1-\left(\dfrac{3}{4}\right)^7\right]$

$= \dfrac{14\,197}{256}$ 1A

(c) $64\left[1-\left(\dfrac{3}{4}\right)^k\right] > 60$ 1M

$\left(\dfrac{3}{4}\right)^k < \dfrac{1}{16}$

$k\log\dfrac{3}{4} < \log\dfrac{1}{16}$ 1M

$k > 9.64$

The least value of k is 10. 1A

33. (a) Let the first term and common ratio be a and r respectively.

$\begin{cases} ar^4+ar^5+ar^6 = 6318 \\ a+ar+ar^2 = 78 \end{cases}$ 1M

$\dfrac{ar^4(1+r+r^2)}{a(1+r+r^2)} = \dfrac{6318}{78}$ 1M

$r^4 = 81$

$r = \pm 3$ 1A

When $r=3$, $a = \dfrac{78}{1+r+r^2} = 6$ and $T(n) = 6(3)^{n-1}$.

When $r = -3$, $a = \dfrac{78}{1+r+r^2} = \dfrac{78}{7}$ and $T(n) = \dfrac{78}{7}(-3)^{n-1}$. 1A

(b) (i) $T(n) = 6(3)^{n-1}$

$$\frac{6(3^m - 1)}{3-1} > 5000 \quad \text{1M}$$

$$3^m > \frac{5003}{3}$$

$$m \log 3 > \log \frac{5003}{3} \quad \text{1M}$$

$$m > 6.75$$

Least value of m is 7. 1A

(ii) Required sum $= \dfrac{6[(3^3)^8 - 1]}{3^3 - 1}$ 1M+1A

$\approx 6.52 \times 10^{10}$ 1A

34. (a) Let the common ratio be r.

$$\begin{cases} G_1 r + G_1 r^2 = 1944 \\ G_1 r^4 + G_1 r^5 = 72 \end{cases}$$

$$\frac{G_1 r^4(1+r)}{G_1 r(1+r)} = \frac{72}{1944} \quad \text{1M}$$

$$r^3 = \frac{1}{27}$$

$$r = \frac{1}{3} \quad \text{1A}$$

So, $G_1 = \dfrac{1944}{r+r^2} = 4374$. 1A

(b) $$\frac{4374}{1-\frac{1}{3}} - \frac{4374\left[1-\left(\frac{1}{3}\right)^n\right]}{1-\frac{1}{3}} < 5 \times 10^{-19} \quad \text{1M+1M}$$

$$6561\left(\frac{1}{3}\right)^n < 5 \times 10^{-19}$$

$$\log 6561 + n \log \frac{1}{3} < \log 5 \times 10^{-19} \quad \text{1M}$$

$$n > 46.4$$

The least value of n is 47. 1A

35. Let first term and common ratio be a and r respectively, and 243 be the nth term.

Consider the sum from the term 243 to infinity,

$$\frac{243}{1-r} = 4096 - 3367 + 243 \quad \text{1M+1A}$$

$$243 = 972 - 972r$$

$$r = \frac{3}{4} \quad \text{1A}$$

Consider the sum to infinity,

$$\frac{a}{1-r} = 4096 \qquad \text{1M}$$

$$a = 1024 \qquad \text{1A}$$

The first term is 1024.

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