REG-GS-2425-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. A	3. C	4. C	5. A
6. A	7. C	8. D	9. D	10. C
11. A	12. A	13. C	14. B	15. B
16. D	17. D	18. B	19. C	20. A
21. A	22. B	23. B	24. B	25. C
26. D	27. B	28. B	29. B	30. B

1. $\boxed{\text{A}}$

We have $b - a = c - b$.

I. ✓. $\frac{2^b}{2^a} = 2^{b-a}$ and $\frac{2^c}{2^b} = 2^{c-b} = 2^{b-a}$.

II. ✗. Consider when $a = 1$, $b = 2$ and $c = 3$.

III. ✗. The terms are all undefined when $k = 0$.

2. $\boxed{\text{A}}$

I. ✓. We have $\frac{y_2}{y_1} = \frac{y_3}{y_2} = \frac{y_4}{y_3} = \ldots = \frac{y_n}{y_{n-1}}$.
Thus, we have $\frac{x_1y_2}{x_1y_1} = \frac{x_1y_3}{x_1y_2} = \frac{x_1y_4}{x_1y_3} = \ldots = \frac{x_1y_n}{x_1y_{n-1}}$.
It is a geometric sequence.

II. ✓. We have $\frac{x_2}{x_1} = \frac{x_3}{x_2} = \ldots = \frac{x_n}{x_{n-1}}$ and $\frac{y_2}{y_1} = \frac{y_3}{y_2} = \ldots = \frac{y_n}{y_{n-1}}$.
Thus, we have $\frac{x_2y_2}{x_1y_1} = \frac{x_3y_3}{x_2y_2} = \ldots = \frac{x_ny_n}{x_{n-1}y_{n-1}}$.
It is a geometric sequence.

III. ✗. Consider the geometric sequences 1, 3, 9, 27 and 1, 2, 4, 8.
$x_1 - y_1 = 0, x_2 - y_2 = 1, x_3 - y_3 = 5, x_4 - y_4 = 19$ is not a geometric sequence.

3. $\boxed{\text{C}}$

We have $\frac{b}{a} = \frac{c}{b}$.

I. ✓. $\frac{b^2}{a^2} = \left(\frac{b}{a}\right)^2 = \left(\frac{c}{b}\right)^2 = \frac{c^2}{b^2}$

II. ✗. Take $a = 1$, $b = 2$, $c = 4$ and $k = 2$.
The sequence 3, 4, 6 is not a geometric sequence.

III. ✓. $\frac{kb}{ka} = \frac{b}{a} = \frac{c}{b} = \frac{kc}{kb}$

4. $\boxed{\text{C}}$

I. ✓. Common ratio $= 3^3$

II. ✗. $\dfrac{33}{3} = 11$ and $\dfrac{333}{33} \neq 11$.

III. ✓. Common ratio $= -10$

5. $\boxed{\text{A}}$

I. ✓. $\dfrac{4^b}{4^a} = 4^{b-a} = 4^{c-b} = \dfrac{4^c}{4^b}$.

II. ✓. $(4b+4) - (4a+4) = b - a = c - b = (4c+4) - (4b+4)$.

III. ✗. Take $a = 10$, $b = 100$ and $c = 190$. Then $\log a = 1$, $\log b = 2$ and $\log c = \log 190 \neq 3$.

6. $\boxed{\text{A}}$

I. ✓. $r = 2$.

II. ✓. $r = 2$.

III. ✗. Take $k = 2$. The sequence becomes 1, 2, 3, 4, which has no common ratio.

7. $\boxed{\text{C}}$

$$\Delta = (2b)^2 - 4ac = 0$$
$$b^2 = ac$$
$$\frac{b}{a} = \frac{c}{b}$$

a, b, c form a geometric sequence.

I. ✗. Consider the case when $a = 1$, $b = 3$ and $c = 9$.

II. ✓. $\log b - \log a = \log \dfrac{b}{a} = \log \dfrac{c}{b} = \log c - \log b$.

III. ✓. $\dfrac{2018b}{2018a} = \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{2018c}{2018b}$.

8. $\boxed{\text{D}}$

I. ✗. Take $a = 1$, $b = 2$ and $c = 4$.
$a+3$, $b+3$, $c+3$ is not a geometric sequence.

II. ✓. We have $\dfrac{b}{a} = \dfrac{c}{b}$.
Thus, we have $\dfrac{a}{b} = \dfrac{b}{c}$ and c, b, a is a geometric sequence.

III. ✓. We have $\dfrac{1}{b} \div \dfrac{1}{a} = \dfrac{a}{b}$ and $\dfrac{1}{c} \div \dfrac{1}{b} = \dfrac{b}{c} = \dfrac{a}{b}$.

9. D

I. ✗. $a = 1$, $b = -2$, $c = 4$ is a geometric sequence.
We have $b < a < c$.

II. ✓. We have $\frac{b}{a} = \frac{c}{b}$.
Thus, $b^2 = ac$.

III. ✓. We have $\frac{b}{a} = \frac{c}{b}$.
Thus, $\frac{b}{c} = \frac{a}{b}$ and c, b, a is a geometric sequence.

10. C

$$\frac{x-4}{3x} = \frac{x-1}{5} \div (x-4)$$
$$5(x-4)^2 = 3x(x-1)$$
$$2x^2 - 37x + 80 = 0$$
$$x = \frac{5}{2} \quad \text{or} \quad 16$$

11. A

$$\frac{x}{x-4} = \frac{x+12}{x}$$
$$x^2 = (x-4)(x+12)$$
$$0 = 8x - 48$$
$$x = 6$$

Common ratio $= \frac{x}{x-4} = 3$

12. A

$$\frac{4n-3}{2n+3} = \frac{8n+1}{4n-3}$$
$$(4n-3)^2 = (2n+3)(8n+1)$$
$$0 = 50n - 6$$
$$n = \frac{3}{25}$$

13. C

$$\frac{x+2}{x-2} = \frac{18}{x+2}$$
$$(x+2)^2 = 18(x-2)$$
$$x^2 - 14x + 40 = 0$$
$$x = 4 \quad \text{or} \quad 10$$

When $x = 4$, common ratio $= \frac{18}{4+2} = 3$.

When $x = 10$, common ratio $= \frac{18}{10+2} = \frac{3}{2}$.

14. B

$$\frac{a-2}{a-7} = \frac{a+2}{a-2}$$

$$(a-2)^2 = (a-7)(a+2)$$

$$0 = -a - 18$$

$$a = -18$$

Common ratio $= \dfrac{-18-2}{-18-7} = \dfrac{4}{5}$

15. B

$k - 5 = 5 - h$ and $\dfrac{4}{h} = \dfrac{k}{4}$

$h + k = 10$ $\quad$ $hk = 16$

$$h^2 + k^2 = (h+k)^2 - 2hk$$

$$= 10^2 - 2(16)$$

$$= 68$$

16. D

$$\frac{-h}{h-7} = \frac{h+8}{-h}$$

$$(-h)^2 = (h-7)(h+8)$$

$$0 = h - 56$$

$$h = 56$$

Common ratio $= \dfrac{-56}{56-7} = -\dfrac{8}{7}$

17. D

$$\frac{ar^4}{ar^2} = \frac{36}{12}$$

$$r^2 = 3$$

$$r = \pm\sqrt{3}$$

Common ratio is $\sqrt{3}$ or $-\sqrt{3}$.

18. B

We have $(ar)(ar^2) = 18$ and $(ar^2)(ar^3) = 72$.

$$\frac{a^2r^5}{a^2r^3} = \frac{72}{18}$$

$$r^2 = 4$$

$$T_5 \times T_6 = (ar^4)(ar^5)$$

$$= (a^2r^3)(r^6)$$

$$= 18(4)^3$$

$$= 1152$$

19. C

We have $a + ar = 2$ and $ar^5 + ar^6 = -486$.

$$\frac{ar^5 + ar^6}{a + ar} = \frac{-486}{2}$$
$$\frac{ar^5(1+r)}{a(1+r)} = -243$$
$$r^5 = (-3)^5$$
$$r = -3$$

First term $= \dfrac{2}{1+r} = -1$

20. A

First term $= -\dfrac{1}{4}$ and common ratio $= \dfrac{2}{-1} = -2$.

nth term $= -\dfrac{1}{4}(-2)^{n-1}$

$= -(-2)^{-2}(-2)^{n-1}$

$= -(-2)^{n-3}$

21. A

Common ratio $= \dfrac{y}{x}$

First term $= \dfrac{x}{\left(\frac{y}{x}\right)^3}$

$= \dfrac{x^4}{y^3}$

22. B

First term = 3 and common ratio $= \dfrac{1}{2}$.

$(2n+1)$th term $= 3\left(\dfrac{1}{2}\right)^{2n}$

$= \dfrac{3}{2^{2n}}$

23. B

First term $= -2$ and common ratio $= \dfrac{-1}{2}$.

General term $= -2\left(-\dfrac{1}{2}\right)^{n-1}$

$= (-2)(-2)^{1-n}$

$= (-2)^{2-n}$

24. B

$$\frac{ar^6}{ar^3} = \frac{-1458}{2}$$

$$r^3 = -729$$

$$r = -9$$

25. C

Let the common ratio be r.

$$abc = 64$$

$$\left(\frac{b}{r}\right) b(br) = 64$$

$$b^3 = 64$$

$$b = 4$$

$a + c = (a + b + c) - b = 14 - 4 = 10$

26. D

$3r^4 = 27$

$r = \sqrt{3} \quad \text{or} \quad -\sqrt{3} \text{ (rejected)}$

$$\begin{aligned} xyz &= (3r)(3r^2)(3r^3) \\ &= 3r^6 \\ &= 81 \end{aligned}$$

27. B

$$1(k^3)^{n-1} = k^{3m+9}$$

$$k^{3n-3} = k^{3m+9}$$

$$3n - 3 = 3m + 9$$

$$n = m + 4$$

There are $m + 4$ terms.

28. B

$$972\left(\frac{1}{3}\right)^{n-1} = \frac{4}{27}$$

$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{6561}$$

$$(n-1)\log\frac{1}{3} = \log\frac{1}{6561}$$

$$n = 9$$

There are 9 terms.

29. B

$$(-8)\left(\frac{3}{2}\right)^{n-1} > -300$$
$$\left(\frac{3}{2}\right)^{n-1} < \frac{75}{2}$$
$$(n-1)\log\frac{3}{2} < \log\frac{75}{2}$$
$$n < 9.94$$

There are 9 terms.

30. B

$$4(3)^{n-1} > 1800$$
$$3^{n-1} > 450$$
$$(n-1)\log 3 > \log 450$$
$$n > 6.56$$

Required term $= 4(3)^{7-1}$

$= 2916$

Conventional Questions

31. (a) $\dfrac{-16}{32-2k}=\dfrac{-k}{4}\div(-16)$ 1M

$256=\dfrac{k^2}{2}-8k$

$0=\dfrac{k^2}{2}-8k-256$

$k=32 \quad \text{or} \quad -16$ 1A

(b) $k=-16$ and positive terms in the sequence are 64, 4, . . .

Let the required number be n.

$64\left(\dfrac{4}{64}\right)^{n-1}>1\times10^{-30}$ 1M

$(n-1)\log\dfrac{1}{16}>\log\dfrac{1\times10^{-30}}{64}$ 1M

$n<27.4$

Required number is 27. 1A

32. (a) Let the first term and common ratio be a and r respectively.

$\dfrac{ar^4}{ar^2}=\dfrac{400}{900}$ 1M

$r^2=\dfrac{4}{9}$

First term $=\dfrac{400}{r^4}$

$=\dfrac{400}{\left(\frac{4}{9}\right)^2}$

$=2025$ 1A

(b) Common ratio $=\sqrt{\dfrac{4}{9}}=\dfrac{2}{3}$. 1A

$2025\left(\dfrac{2}{3}\right)^n-2025\left(\dfrac{2}{3}\right)^{2n}<2\times10^{-5}$

$-2025\left(\dfrac{2}{3}\right)^{2n}+2025\left(\dfrac{2}{3}\right)^n-2\times10^{-5}<0$

$\left(\dfrac{2}{3}\right)^n<9.8765\times10^{-9} \quad \text{or} \quad \left(\dfrac{2}{3}\right)^n>1.00$ (rejected) 1A

$n\log\dfrac{2}{3}<\log 9.8765\times10^{-9}$ 1M

$n>45.5$

The least value of n is 46. 1A

33. (a) $(x+2)(x-2)=8(x-1)$

$x^2-8x+4=0$

Thus, $p = 8$ and $q = 4$. 1A+1A

(b) Common ratio $= \frac{\log 8}{\log 4} = \frac{3\log 2}{2\log 2} = \frac{3}{2}$. 1M

$$(\log 4)\left(\frac{3}{2}\right)^{\alpha} + (\log 4)\left(\frac{3}{2}\right)^{2\alpha} < \log 2^{2020} \quad \text{1M}$$

$$2(1.5)^{\alpha} + 2(1.5)^{2\alpha} < 2020$$

$$(1.5)^{2\alpha} + 1.5^{\alpha} - 1010 < 0$$

$$\frac{-1-\sqrt{4041}}{2} < 1.5^{\alpha} < \frac{-1+\sqrt{4041}}{2}$$

Since $1.5^{\alpha} > 0$,

$$0 < 1.5^{\alpha} < \frac{-1+\sqrt{4041}}{2}$$

$$\alpha \log 1.5 < \log \frac{-1+\sqrt{4041}}{2} \quad \text{1M}$$

$$\alpha < 8.49$$

The greatest value of α is 8. 1A

34. (a) $T(n) = 1701\left(-\frac{1}{3}\right)^{n-1}$ 1A

(b) $1701\left(-\frac{1}{3}\right)^{k-1} = -\frac{7}{6561}$ 1M

$$\left(-\frac{1}{3}\right)^{k-1} = \left(-\frac{1}{3}\right)^{13} \quad \text{1M}$$

$$k = 14 \quad \text{1A}$$

(c) Consider only the negative terms $-567, -63, -7, \ldots$

Suppose the required number is n.

$$-567\left(\frac{1}{9}\right)^{n-1} < -\frac{1}{100} \quad \text{1M}$$

$$\frac{1}{9^{n-1}} > \frac{1}{56\,700}$$

$$(n-1)\log\frac{1}{9} > \log\frac{1}{56\,700} \quad \text{1M}$$

$$n < 5.98$$

There are 5 terms. 1A

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