Solution	Marks

REG-2425-MOCK-SET 1-MATH-CP 1
Suggested solutions

1. (a) $2a - 3b = 5(b + 4)$

    $2a - 3b = 5b + 20$ | 1M

    $(-3 - 5)b = 20 - 2a$ | 1M

    $b = \dfrac{-10 + a}{4}$ | 1A

    (b) 30 | 1A

2. $\dfrac{4}{2x + 3} + \dfrac{5}{6 - 7x} = \dfrac{4(6 - 7x) + 5(2x + 3)}{(2x + 3)(6 - 7x)}$ | 1M

    $= \dfrac{(-28 + 10)x + (24 + 15)}{(2x + 3)(6 - 7x)}$ | 1M

    $= \dfrac{39 - 18x}{(2x + 3)(6 - 7x)}$ | 1A

3. $21^2 + (25 + a)^2 = (9 - 4a)^2$ | 1M

    $-15a^2 + 122a + 985 = 0$ | 1M

    $a = -5 \quad \text{or} \quad \dfrac{197}{15} \text{ (rejected)}$ | 1A

4. (a) $9x^2 - 25 = (3x + 5)(3x - 5)$ | 1A

    (b) $3x^2y - 7xy - 20y = y(3x^2 - 7x - 20)$

    $= y(3x + 5)(x - 4)$ | 1A

    (c) $9x^2 - 25 - 3x^2y + 7xy + 20y = (3x + 5)(3x - 5) - y(3x + 5)(x - 4)$ | 1M

    $= (3x + 5)(3x - 5 - xy + 4y)$ | 1A

5. Let the marked price be x.

    $\dfrac{x}{1 + 75\%} - x(1 - 60\%) = 1200$ | 2M+1A

    $\dfrac{6x}{35} = 1200$ | 1M

    $x = 7000$ | 1A

6. (a) $2(x - 3) < \dfrac{4x + 21}{4}$

    $x < \dfrac{45}{4}$ | 1A

    $15 - 2x \leq 7$

    $x \geq 4$ | 1A

    Thus, $4 \leq x < \dfrac{45}{4}$. | 1M

    (b) 8 | 1A

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Solution	Marks

7. Let the length be $5k$ cm. Then the breadth is $3k$ cm. — **1A**

$$\frac{5k+8}{3k+3} = \frac{16}{9}$$ — **1M+1A**

$$45k + 72 = 48k + 48$$

$$k = 8$$

The original breadth of the rectangle is 24 cm. — **1A**

8. (a) $\quad 1.36 = \dfrac{0(2) + 1(p) + 2(5) + 3(3)}{2 + p + 5 + 3}$ — **1M**

$\quad 1.36(10 + p) = 19 + p$

$\quad\quad p = 15$ — **1A**

Standard deviation ≈ 0.794 — **1A**

(b) Median $= 1$ and mode $= 1$.

They are equal. — **1A**

9. (a) Let the radius of the smaller hemisphere be r cm.

$$\frac{2}{3}\pi r^3 \left(1 + \frac{125}{64}\right) = 1008\pi$$ — **1M**

$$r^3 = 512$$

$$r = 8$$ — **1A**

(b) Total surface area $= 2\pi(8)^2 + \pi(8)^2$ — **1M**

$$= 192\pi \text{ cm}^2$$ — **1A**

10. (a) Let $C = a + \dfrac{b}{n}$, where a and b are non-zero constants. — **1A**

$$\begin{cases} 1600 = a + \dfrac{b}{200} \\ 1550 = a + \dfrac{b}{400} \end{cases}$$ — **1M**

Solving, we have $a = 1500$ and $b = 20\,000$. — **1A**

Required cost $= 1500 + \dfrac{20\,000}{500} = \$1540$. — **1A**

(b) $2020 = 1500 + \dfrac{20\,000}{n}$ — **1M**

$n \approx 38.5$, which is not an integer

It is not possible. — **1A**

Solution	Marks

11. (a) Let $f(x) = (x^2 + 6x - 7)(Ax + B)$, where A and B are constants. — 1M

$$\begin{cases} f(-1) = 96 = (1 - 6 - 7)(-A + B) \\ f(2) = 9 = (4 + 12 - 7)(2A + B) \end{cases}$$ — 1M

Solving, we have $A = 3$ and $B = -5$.

The quotient is $3x - 5$. — 1A

(b) $\qquad (x^2 + 6x - 7)(3x - 5) = 2x + 14$

$(x + 7)(x - 1)(3x - 5) - 2(x + 7) = 0$ — 1M

$(x + 7)[(x - 1)(3x - 5) - 2] = 0$

$(x + 7)(3x^2 - 8x + 3) = 0$ — 1A

$$x = -7 \quad \text{or} \quad \frac{8 \pm \sqrt{8^2 - 4(3)(3)}}{2(3)}$$

$$= -7 \quad \text{or} \quad \frac{4 \pm \sqrt{7}}{3}$$

Since $\dfrac{4 \pm \sqrt{7}}{3}$ is irrational, the claim is agreed. — 1A

12. (a) Interquartile range $= 2.8 - 1.9$ — 1A

$\qquad\qquad\qquad\quad = 0.9\,\text{h}$ — 1A

(b) (i) $m = 2.4$ — 1A

$n = 1.1 + 2.0 = 3.1$ — 1A

(ii) The interquartile range of John's running time (0.9 hours) is smaller than that of Peter's running time (1.1 hours). — 1M

John should be chosen. — 1A

(iii) According to the past performance,

probability of John to break the record $= \dfrac{5}{19}$; — 1A

probability of Peter to break the record $\leq 0.25 < \dfrac{5}{19}$.

John should be chosen. — 1A

Solution	Marks
13. (a) $AB^2 + BC^2 - AC^2 = 5^2 + 12^2 - 13^2 = 0$	1M
So, $AB^2 + BC^2 = AC^2$ and $\angle ABC = 90°$.	1A
(b) $\angle ADC = 180° - \angle ABC = 90°$	
Let $AD = DC = x$ cm.	
$x^2 + x^2 = 13^2$	1M
$x = \dfrac{13\sqrt{2}}{2}$	
Required perimeter $= 5 + 12 + \dfrac{13\sqrt{2}}{2} \times 2$	1M
$= (17 + 13\sqrt{2})$ cm	1A
(c) Let h be the shortest distance from D to AC.	
$\dfrac{1}{2}(h)(13) = \dfrac{1}{2}\left(\dfrac{13\sqrt{2}}{2}\right)^2$	1M
$h = 6.5$	
There is no such point F.	1A

Solution	Marks

14. (a) (i) $BC = CD$ *(prop. of square)*

        $\angle BCE = 45° = \angle DCE$ *(prop. of square)*

        $CE = CE$ *(common side)*

     $\triangle BCE \cong \triangle DCE$ *(SAS)*

Marking Scheme	
Case 1 Any correct proof with correct reasons.	2
Case 2 Any correct proof without reasons.	1

  (ii) $\angle GEB = \angle BEF$ *(common $\angle$)*

       $AF \parallel CD$ *(prop. of square)*

   $\angle BFE = \angle CDE$ *(alt. $\angle$., $AF \parallel CD$)*

   $\angle GBE = \angle CDE$ *(corr. $\angle s$, $\cong \triangle s$)*

         $= \angle BFE$

  $\triangle BEG \sim \triangle FEB$ *(AA)*

Marking Scheme	
Case 1 Any correct proof with correct reasons.	2
Case 2 Any correct proof without reasons.	1

(b) Since $\triangle BEG \sim \triangle FEB$, $\dfrac{BE}{FE} = \dfrac{EG}{EB} = \dfrac{BG}{FB} = \tan \angle AFD$.   **1M**

Since $BE = DE$, we have $\dfrac{DE}{FE} = \dfrac{EG}{DE} = \tan \angle AFD$ and $EG = DE \tan \angle AFD$.   **1M**

$DE = (EG + FG) \tan \angle AFD$

    $= (DE \tan \angle AFD + FG) \tan \angle AFD$

    $= DE \tan^2 \angle AFD + FG \tan \angle AFD$

    $< DE \left(\dfrac{1}{\sqrt{3}}\right)^2 + FG \left(\dfrac{1}{\sqrt{3}}\right)$ **1M**

$DE < \dfrac{\sqrt{3}}{2} FG$

The claim is agreed.   **1A**

15. (a) Number of ways $= C_2^6 = 15$ **1A**

  (b) Required probability $= 1 - \dfrac{15}{C_4^{12}}$ **1M**

               $= \dfrac{32}{33}$ **1A**

Solution	Marks

16.   (a) Slope of the line $= \dfrac{11-8}{3-0} = 1$.

$$\log_2 y = x + 8$$ 1M

$$y = 2^{x+8}$$

$$= 256(2^x)$$

Thus, $a = 256$ and $b = 2$.   1A+1A

  (b) $256(2^{2t}) - 256(2^t) \geq 32\,768$ 1M

$$256(2^{2t}) - 256(2^t) - 32\,768 \geq 0$$

$$2^t \geq 11.8 \quad \text{or} \quad 2^t \leq -10.8 \text{ (rejected)}$$

$$t \log 2 \geq \log 11.8$$ 1M

$$t \geq 3.56$$

The least integral value of t is 4.   1A

17.   (a) $f(x) = ax^2 + 8a^2x + 16a^3 + a$

$$= a(x^2 + 8ax + 16a^2) + a$$ 1M

$$= a(x + 4a)^2 + a$$ 1M

The coordinates of vertex are $(-4a,\ a)$.   1A

  (b) (i) The graph of $y = f(x)$ is first transformed rightwards by $5a$ units,   1A
          then is enlarged along the y-axis to 4 times the original.   1A

     (ii) $(a,\ 4a)$ 1A

    (iii) Slope of $OP \times$ slope of $OQ = \dfrac{a}{-4a} \times \dfrac{4a}{a} = -1$

So, $OP \perp OQ$.   1M

Thus, the orthocentre of $\triangle OPQ$ is at O.

The coordinates of orthocentre are $(0,\ 0)$.   1A

Solution	Marks
18. (a) $30 + 30r + 30r^2 = 52.5$	1M
$30r^2 + 30r - 22.5 = 0$	
$r = \dfrac{1}{2}$ or $-\dfrac{3}{2}$ (rejected)	1A
(b) Height of the balloon $\le \dfrac{30}{1 - \frac{1}{2}}$	1M
$= 60\,\text{m} < 75\,\text{m}$	
The balloon cannot reach a height of 75 m.	1A
(c) Let the time required be n minutes.	
$\dfrac{30(1 - 0.5^n)}{1 - 0.5} = 58.125$	1M
$0.5^n = 0.03125$	
$n \log 0.5 = \log 0.03125$	1M
$n = 5$	
The time required is 5 minutes.	1A

Solution	Marks

19. (a) Let O be the circumcentre of circle $ABCD$.

$$\angle BAC = \angle DAC \qquad\qquad\qquad\qquad (given)$$

$$\angle BOC = 2\angle BAC \qquad\qquad (\angle\ at\ centre\ twice\ \angle\ at\ \odot^{ce})$$

$$\angle DOC = 2\angle DAC \qquad\qquad (\angle\ at\ centre\ twice\ \angle\ at\ \odot^{ce})$$

$$= \angle BOC$$

$$BC = CD \qquad\qquad\qquad (equal\ \angle s,\ equal\ chords)$$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) (i) Let the coordinates of M be $(a, -a)$ such that it lies on $y = -x$.

$$\sqrt{(a-0)^2 + (-a-0)^2} = \sqrt{(a+200)^2 + (-a+600)^2} \qquad \text{1M}$$

$$2a^2 = 2a^2 - 800a + 400\,000$$

$$a = 500$$

Required equation is

$$(x-500)^2 + (y+500)^2 = (0-500)^2 + (0+500)^2$$

$$(x-500)^2 + (y+500)^2 = 500\,000 \qquad \text{1A}$$

Coordinates of M are $(500, -500)$.   1A

(ii) $(0-500)^2 + (y+500)^2 = 500\,000$

$$(y+500)^2 = 250\,000$$

$$y = -1000 \quad \text{or} \quad 0 \text{ (rejected)}$$

Coordinates of C are $(0, -1000)$.   1A

(c) Let K be a point on VC such that $BK \perp VC$.
Then $DK \perp VC$ and the required angle is $\angle BKD$.

$$BM = CM = DM = \sqrt{(500+200)^2 + (-500+600)^2} = 500\sqrt{2} \qquad \text{1M}$$

$$BC = CD = \sqrt{200^2 + (-1000+600)^2} = 200\sqrt{5}$$

$$VB = VC = VD = \sqrt{MB^2 + 50^2} = 50\sqrt{201}$$

$$MB^2 = MC^2 + BC^2 - 2(MC)(BC)\cos\angle BCM$$

$$\angle BCM \approx 71.6°$$

$$BD = 2 \times BC \sin\angle BCM \approx 849 \qquad \text{1M}$$

$$VB^2 = VC^2 + BC^2 - 2(VC)(BC)\cos\angle VCB$$

$$\angle VCB \approx 71.6°$$

$$DK = BK = BC \sin\angle VCB \approx 424$$

$$BD^2 = BK^2 + DK^2 - 2(BK)(DK)\cos\angle BKD \qquad \text{1M}$$

$$\angle BKD \approx 177° \qquad \text{1A}$$