Solution	Marks
REG-2324-MOCK-SET 1-MATH-EP(M2)	
Suggested solutions	
1. $f'(2) = \lim_{h\to 0} \dfrac{(2+h)\ln(2+h-1) - 2\ln 1}{h}$	1M
$= \lim_{h\to 0} (2+h)\ln(1+h)^{\frac{1}{h}}$	1M
$= (2+0)(\ln e)$	1A
$= 2$	1A
2. (a) $P(x) = (x-\lambda)^2 \begin{vmatrix} 4 & x-\lambda \\ (x-\lambda)^3 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 2 \\ 4 & x-\lambda \end{vmatrix}$	
$= 20(x-\lambda)^2 - (x-\lambda)^6 + 3(x-\lambda) - 8$	1A
$-1195 = 20 - C^6_2(-\lambda)^4$	1M
$\lambda^4 = 81$	
$\lambda = 3$ or -3 (rejected)	1A
(b) $P'(x) = 40(x-3) - 6(x-3)^5 + 3$	1A
$P''(x) = 40 - 30(x-3)^4$	
$P''(\lambda) = 40$	1A
3. (a) $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$	
$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$	1M
$= \dfrac{\cos 2\theta}{1}$	
$= \cos 2\theta$	1
(b) $\cos\dfrac{3\pi}{4} = \dfrac{1-\tan^2\frac{3\pi}{8}}{1+\tan^2\frac{3\pi}{8}}$	1M
$-\dfrac{1}{\sqrt{2}} = \dfrac{1-\tan^2\frac{3\pi}{8}}{1+\tan^2\frac{3\pi}{8}}$	1A
$-1-\tan^2\dfrac{3\pi}{8} = \sqrt{2}\left(1-\tan^2\dfrac{3\pi}{8}\right)$	
$\tan^2\dfrac{3\pi}{8} = \dfrac{\sqrt{2}+1}{\sqrt{2}-1}$	1A
$= \dfrac{\sqrt{2}+1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1}$	1M
$= \dfrac{3+2\sqrt{2}}{2-1}$	
$= 3+2\sqrt{2}$	1

Solution	Marks
4. (a) $A^2 - mA = nI$	
$A\left(\frac{1}{n}A - \frac{m}{n}I\right) = I$	1M
Thus, $A^{-1} = \frac{1}{n}A - \frac{m}{n}I$.	1A
(b) $A^3 = A^2A$	
$= (mA + nI)A$	1M
$= m(mA + nI) + nA$	
$= (m^2 + n)A + mnI$	1
(c) $A^3 = -A^{-1}$ when $m^2 + n = -\frac{1}{n}$ and $mn = \frac{m}{n}$.	1M
$mn = \frac{m}{n}$	
$0 = m\left(\frac{1}{n} - n\right)$	
$n = 1$ (rejected) or -1 or $m = 0$ (rejected)	1A
For $n = 1$, we have $m = \sqrt{2}$, $A^3 = A - \sqrt{2}I$ and $A^{-1} = -A + \sqrt{2}I = -A^3$.	1
5. (a) $W = \int (-10 - t)\,\mathrm{d}t$	1M
$= -10t - \frac{t^2}{2} + C$, where C is a constant	
Since $W = 540$ when $t = 0$, we have $C = 540$.	1M
So, we have $W = -10t - \frac{t^2}{2} + 540$.	
When $t = 24$, we have $W = 12 > 0$.	
Thus, the claim is incorrect.	1A
(b) When $t = 20$, $\frac{\mathrm{d}W}{\mathrm{d}t} = -10 - 20 = -30$ and $W = -10(20) - \frac{20^2}{2} + 540 = 140$.	
$140 = 5x^2 + 3x$	
$0 = 5x^2 + 3x - 140$	
$x = 5$ or $-\frac{28}{5}$ (rejected)	1A
$\frac{\mathrm{d}W}{\mathrm{d}t} = (10x + 3)\frac{\mathrm{d}x}{\mathrm{d}t}$	1M
$-10 - 20 = [10(5) + 3]\frac{\mathrm{d}x}{\mathrm{d}t}$	
$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{30}{53}$	1A

Solution	Marks

6. (a) When $n = 1$,

L.H.S. $= \dfrac{1}{(1)(3)} + \dfrac{1}{(3)(5)} = \dfrac{2}{5}$ and R.H.S. $= \dfrac{1+1}{(1)(5)} = \dfrac{2}{5} =$ L.H.S.

The statement is true for $n = 1$. 1

Assume that $\displaystyle\sum_{k=p}^{2p} \frac{1}{(2k-1)(2k+1)} = \frac{k+1}{(2k-1)(4k+1)}$ for some $k \in \mathbb{Z}^+$. 1

When $n = p + 1$,

$$\sum_{k=p+1}^{2p+2} \frac{1}{(2k-1)(2k+1)}$$

$$= \sum_{k=p}^{2p} \frac{1}{(2k-1)(2k+1)} - \frac{1}{(2p-1)(2p+1)} + \frac{1}{(4p+1)(4p+3)} + \frac{1}{(4p+3)(4p+5)} \quad \text{1M}$$

$$= \frac{p+1}{(2p-1)(4p+1)} - \frac{1}{(2p-1)(2p+1)} + \frac{(4p+5)+(4p+1)}{(4p+1)(4p+3)(4p+5)} \quad 1$$

$$= \frac{(p+1)(2p+1)-(4p+1)}{(2p-1)(2p+1)(4p+1)} + \frac{2}{(4p+1)(4p+5)}$$

$$= \frac{p(2p-1)}{(2p-1)(2p+1)(4p+1)} + \frac{2}{(4p+1)(4p+5)}$$

$$= \frac{p(4p+5)+2(2p+1)}{(2p+1)(4p+1)(4p+5)}$$

$$= \frac{(p+2)(4p+1)}{(2p+1)(4p+1)(4p+5)}$$

$$= \frac{p+2}{(2p+1)(4p+5)} \quad 1$$

The statement is true for $n = p + 1$.

By MI, the statement is true $\forall n \in \mathbb{Z}^+$. 1

(b) $$\sum_{k=505}^{2020} \frac{1}{(2k-1)(2k+1)}$$

$$= \sum_{k=505}^{1010} \frac{1}{(2k-1)(2k+1)} + \sum_{k=1010}^{2020} \frac{1}{(2k-1)(2k+1)} - \frac{1}{(2019)(2021)}$$

$$= \frac{506}{(1009)(2021)} + \frac{1011}{(2019)(4041)} - \frac{1}{(2019)(2021)}$$

$$\approx 3.72 \times 10^{-4} \quad \text{1A}$$

Solution	Marks
7. (a) $y = \int xe^{-2x}\,dx$	1M
$= \dfrac{xe^{-2x}}{-2} + \dfrac{1}{2}\int e^{-2x}\,dx$	1M
$= -\dfrac{xe^{-2x}}{2} - \dfrac{e^{-2x}}{4} + C$, where C is a constant.	
$\dfrac{1}{4} = 0 - \dfrac{1}{4} + C$	1M
$C = \dfrac{1}{2}$	
Equation of Γ is $y = -\dfrac{xe^{-2x}}{2} - \dfrac{e^{-2x}}{4} + \dfrac{1}{2}$.	1A
(b) $f''(x) = (1-2x)e^{-2x}$	
When $f''(x) = 0$, $x = \dfrac{1}{2}$.	

x	$x < \dfrac{1}{2}$	$x > \dfrac{1}{2}$
$f''(x)$	$+$	$-$

1M

Γ has one point of inflexion at $x = \dfrac{1}{2}$.	
The claim is disagreed.	1A

Solution	Marks
8. (a) $g'(x) = e^x(\cos x - \sin x)$	1A
When $g'(x) = 0$,	
$\cos x - \sin x = 0$	
$\tan x = 1$	
$x = \dfrac{\pi}{4}$	
<table><tr><td>x</td><td>$0 < x < \frac{\pi}{4}$</td><td>$\frac{\pi}{4} < x < \pi$</td></tr><tr><td>$g'(x)$</td><td>$+$</td><td>$-$</td></tr></table>	
Thus, G has only one maximum point.	1
(b) $\displaystyle\int e^{2x}\cos 2x\,\mathrm{d}x = \frac{e^{2x}\cos 2x}{2} + \int e^{2x}\sin 2x\,\mathrm{d}x$	1M
$\displaystyle= \frac{e^{2x}\cos 2x}{2} + \frac{e^{2x}\sin 2x}{2} - \int e^{2x}\cos 2x\,\mathrm{d}x$	
$\displaystyle 2\int e^{2x}\cos 2x\,\mathrm{d}x = \frac{e^{2x}(\sin 2x + \cos 2x)}{2} + \text{constant}$	1M
$\displaystyle\int e^{2x}\cos 2x\,\mathrm{d}x = \frac{e^{2x}(\sin 2x + \cos 2x)}{4} + \text{constant}$	1A
(c) Volume $\displaystyle= \pi\int_0^{\frac{\pi}{4}} (e^x\cos x)^2\,\mathrm{d}x$	1M
$\displaystyle= \frac{\pi}{2}\int_0^{\frac{\pi}{4}} e^{2x}\cos 2x\,\mathrm{d}x + \frac{\pi}{2}\int_0^{\frac{\pi}{4}} e^{2x}\,\mathrm{d}x$	1M
$\displaystyle= \frac{\pi}{2}\left[\frac{e^{2x}(\sin 2x + \cos 2x)}{4}\right]_0^{\frac{\pi}{4}} + \frac{\pi}{2}\left[\frac{e^{2x}}{2}\right]_0^{\frac{\pi}{4}}$	
$\displaystyle= \frac{3\pi(e^{\frac{\pi}{2}} - 1)}{8}$	1A

Solution	Marks
9. (a) Vertical asymptote is $x = -\frac{1}{2}$.	1A
$f(x) = \frac{x^2 - 4x}{2x + 1}$	
$= \frac{x}{2} - \frac{9}{4} + \frac{9}{4(2x+1)}$	1M
Oblique asymptote is $y = \frac{x}{2} - \frac{9}{4}$.	1A
(b) $f'(x) = \frac{1}{2} - \frac{9}{4}(2x+1)^{-2}(2)$	1M
$= \frac{1}{2} - \frac{9}{2(2x+1)^2}$	1A
(c) $\frac{1}{2} - \frac{9}{2(2x+1)^2} = 0$	1M
$(2x+1)^2 = 9$	
$x = -2 \quad \text{or} \quad 1$	
$\begin{array}{c\|c\|c\|c\|c} x & x < -2 & -2 < x < -\frac{1}{2} & -\frac{1}{2} < x < 1 & x > 1 \\ \hline f'(x) & + & - & - & + \end{array}$	1M
Maximum point is $(-2, -4)$ and minimum point is $(1, -1)$.	1A+1A
(d) $0 = \frac{x^2 - 4x}{2x + 1}$	
$x = 0 \quad \text{or} \quad 4$	
Area $= \int_0^4 [-f(x)]\,dx$	1M+1A
$= \int_0^4 \left(\frac{9}{4} - \frac{x}{2} - \frac{9}{4(2x+1)} \right) dx$	
$= \left[\frac{9x}{4} - \frac{x^2}{4} - \frac{9}{8}\ln\|2x+1\| \right]_0^4$	1M
$= 5 - \frac{9}{8}\ln 9$	1A

Solution	Marks
10. (a) $\dfrac{A}{u^2+1}+\dfrac{B}{u^2+4}=\dfrac{A(u^2+4)+B(u^2+1)}{(u^2+1)(u^2+4)}$	
$=\dfrac{(A+B)u^2+(4A+B)}{(u^2+1)(u^2+4)}$	
Comparing like terms, we have $A+B=0$ and $4A+B=1$.	1M
Solving, we have $A=\dfrac{1}{3}$ and $B=-\dfrac{1}{3}$.	
Thus, $\dfrac{1}{(u^2+1)(u^2+4)}=\dfrac{1}{3(u^2+1)}-\dfrac{1}{3(u^2+4)}$.	1A
(b) Let $u=-x$. Then $\mathrm{d}u=-\mathrm{d}x$.	1M
$\displaystyle\int_{-a}^{a} f(x)\,\mathrm{d}x=\int_{-a}^{0} f(x)\,\mathrm{d}x+\int_{0}^{a} f(x)\,\mathrm{d}x$	
$\displaystyle=-\int_{a}^{0} f(-u)\,\mathrm{d}u+\int_{0}^{a} f(x)\,\mathrm{d}x$	1M
$\displaystyle=\int_{0}^{a} f(u)\,\mathrm{d}u+\int_{0}^{a} f(x)\,\mathrm{d}x$	
$\displaystyle=2\int_{0}^{a} f(x)\,\mathrm{d}x$	1
(c) (i) $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}}\frac{\mathrm{d}u}{(u^2+1)(u^2+4)}$	
$\displaystyle=\int_{-\sqrt{3}}^{\sqrt{3}}\left(\frac{1}{3(u^2+1)}-\frac{1}{3(u^2+4)}\right)\mathrm{d}u$	1M
$\displaystyle=\frac{1}{3}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{\sec^2\theta}{\tan^2\theta+1}\,\mathrm{d}\theta-\frac{1}{3}\int_{-\tan^{-1}\frac{\sqrt{3}}{2}}^{\tan^{-1}\frac{\sqrt{3}}{2}}\frac{2\sec^2\phi}{4\tan^2\phi+4}\,\mathrm{d}\phi$, where $u=\tan\theta$ and $u=2\tan\phi$	1M+1A
$\displaystyle=\frac{1}{3}\Big[\theta\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}-\frac{1}{6}\Big[\phi\Big]_{-\tan^{-1}\frac{\sqrt{3}}{2}}^{\tan^{-1}\frac{\sqrt{3}}{2}}$	
$\displaystyle=\frac{2\pi}{9}-\frac{1}{3}\tan^{-1}\frac{\sqrt{3}}{2}$	1A
(ii) $\dfrac{1}{\tan^2(-x)+4}=\dfrac{1}{\tan^2 x+4}$	
Let $u=\tan x$. Then $\mathrm{d}u=\sec^2\theta\,\mathrm{d}x$.	
$\displaystyle\int_{0}^{\frac{\pi}{3}}\frac{\mathrm{d}x}{\tan^2 x+4}=\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{\mathrm{d}x}{\tan^2 x+4}$	1M
$\displaystyle=\frac{1}{2}\int_{-\sqrt{3}}^{\sqrt{3}}\frac{\mathrm{d}u}{(u^2+1)(u^2+4)}$	1M
$\displaystyle=\frac{\pi}{9}-\frac{1}{6}\tan^{-1}\frac{\sqrt{3}}{2}$	1A

Solution	Marks
11. (a) (i) $\begin{vmatrix} 1 & 1 & 1 \\ -1 & a & 1 \\ 4 & 1 & a \end{vmatrix} = a^2 - 3a + 2$	1A
$= (a-1)(a-2)$	
Since (E) has a unique solution, $a^2 - 3a + 2 \neq 0$.	
Thus, we have $a \neq 1$ and $a \neq 2$.	1A
(ii) When $a = 2$, the augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ -1 & 2 & 1 & 3 \\ 4 & 1 & 2 & b \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 2 & 7 \\ 0 & -3 & -2 & b-16 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 2 & 7 \\ 0 & 0 & 0 & b-9 \end{array}\right)$	1M
When (E) is consistent, $b = 9$.	1A
The solution set of (E) is $\left\{\left(\frac{5-t}{3}, \frac{7-2t}{3}, t\right) : t \in \mathbb{R}\right\}$.	1A
(b) The solution set is $\left\{\left(\frac{5-t}{3}, \frac{7-2t}{3}, t\right) : t \in \mathbb{R}\right\}$.	
$p\left(\frac{5-t}{3}\right) = \left(\frac{7-2t}{3}\right)t$	1M
$0 = -\frac{2t^2}{3} + \left(\frac{7+p}{3}\right)t - \frac{5p}{3}$	
$\Delta = \left(\frac{7+p}{3}\right)^2 - 4\left(-\frac{2}{3}\right)\left(-\frac{5p}{3}\right) \geq 0$	1M+1A
$\frac{p^2}{9} - \frac{26p}{9} + \frac{49}{9} \geq 0$	
$p \leq 13 - 2\sqrt{30}$ or $p \geq 13 + 2\sqrt{30}$	1A
(c) $\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ -1 & 1 & 1 & 3 \\ 8 & 2 & 2 & 11 \\ 3 & 5 & 5 & 19 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 2 & 7 \\ 0 & -6 & -6 & -21 \\ 0 & 2 & 2 & 7 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 2 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$	1M
So, (F) is consistent.	1A
The solution set is $\left\{\left(\frac{1}{2}, \frac{7-2s}{2}, s\right) : s \in \mathbb{R}\right\}$.	1A

Solution	Marks
12. (a) $\overrightarrow{AB} = \overrightarrow{DC}$	
$(16-a)\mathbf{i} + (b-6)\mathbf{j} - 6\mathbf{k} = 10\mathbf{i} - 3\mathbf{j} + (c+3)\mathbf{k}$	1M
We have $a = 6$, $b = 3$ and $c = -9$.	1A+1A
(b) $\overrightarrow{OA} \cdot \overrightarrow{AD} = 6 - 6 + 0 = 0$	1A
(c) $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & -3 & -6 \\ 1 & -1 & 0 \end{vmatrix}$	
$= -6\mathbf{i} - 6\mathbf{j} - 7\mathbf{k}$	1A
(d) (i) Let $\mathbf{n} = -\overrightarrow{AB} \times \overrightarrow{AD} = 6\mathbf{i} + 6\mathbf{j} + 7\mathbf{k}$.	
$\overrightarrow{OE} = \dfrac{\overrightarrow{OA} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}}\mathbf{n}$	1M
$= \dfrac{36+36-21}{36+36+49}(6\mathbf{i} + 6\mathbf{j} + 7\mathbf{k})$	
$= \dfrac{51}{121}(6\mathbf{i} + 6\mathbf{j} + 7\mathbf{k})$	1A
Required distance $= \dfrac{51}{121}\sqrt{6^2 + 6^2 + 7^2}$	1M
$= \dfrac{51}{11}$	1A
(ii) $OA = \sqrt{6^2 + 6^2 + 3^2} = 9$	
Let θ be the angle between OA and the plane $ABCD$.	
$\sin\theta = \dfrac{\left(\frac{51}{11}\right)}{9}$	1M
$\theta = \sin^{-1}\dfrac{51}{99}$	1A
(iii) Since $\overrightarrow{OA} \cdot \overrightarrow{AD} = 0$ and E is the projection of O on $ABCD$, $\angle OAD = \angle EAD = 90°$.	1M
The angle between $\triangle OAD$ and the plane $ABCD$ is $\angle OAE$.	
The claim is agreed.	1A

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