

M2REG-MI-2425-ASM-SET 1-MATH

Suggested solutions

$$\begin{aligned}
 1. \quad (a) \quad \sum_{i=1}^5 (a_i^2 + 2a_i + 1) &= \sum_{i=1}^5 a_i^2 + 2 \sum_{i=1}^5 a_i + \sum_{i=1}^5 1 \\
 &= 8 + 2(4) + 5(1) && 1M \\
 &= 21 && 1A
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \sum_{i=1}^5 (a_i^2 + 1)(2a_i - 3) &= 2 \sum_{i=1}^5 a_i^3 - 3 \sum_{i=1}^5 a_i^2 + 2 \sum_{i=1}^5 a_i + \sum_{i=1}^5 (-3) \\
 &= 2(-3) - 3(8) + 2(4) + 5(-3) && 1M \\
 &= -37 && 1A
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \sum_{i=1}^8 (3a_i + 2)(2b_i - 5) &= 6 \sum_{i=1}^8 a_i b_i - 15 \sum_{i=1}^8 a_i + 4 \sum_{r=1}^8 b_i + \sum_{i=1}^8 (-10) \\
 &= 6(12) - 15(-4) + 4(6) + 8(-10) && 1M \\
 &= 76 && 1A
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (a) \quad \sum_{i=5}^9 a_i (a_i^2 + 2) &= \sum_{i=5}^9 a_i^3 + 2 \sum_{i=5}^9 a_i \\
 &= 5 + 2(1) && 1M \\
 &= 7 && 1A
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \sum_{j=5}^9 \frac{a_j^6 - 1}{a_j^3 + 1} &= \sum_{i=5}^9 \frac{a_i^6 - 1}{a_i^3 + 1} \\
 &= \sum_{i=5}^9 (a_i^3 - 1) \\
 &= \sum_{i=5}^9 a_i^3 - \sum_{i=5}^9 (1) \\
 &= 5 - (9 - 5 + 1)(1) && 1M \\
 &= 0 && 1A
 \end{aligned}$$

$$\begin{aligned}
4. \quad (a) \quad \sum_{i=3}^7 m_i^2(1 - m_i^3) &= \sum_{i=3}^7 m_i^2 - \sum_{i=3}^7 m_i^5 & 1M \\
&= 2 - 6 \\
&= -4 & 1A
\end{aligned}$$

$$\begin{aligned}
(b) \quad \sum_{k=3}^7 \frac{m_k^{10} - 1}{m_k^5 + 1} &= \sum_{i=3}^7 \frac{m_i^{10} - 1}{m_i^5 + 1} & 1M \\
&= \sum_{i=3}^7 m_i^5 - \sum_{i=3}^7 (1) \\
&= 6 - (7 - 3 + 1)(1) \\
&= 1 & 1A
\end{aligned}$$

5. When $n = 1$,

$$\text{L.H.S.} = 1 \text{ and R.H.S.} = \frac{3^1 - 1}{2} = 1 = \text{L.H.S.}$$

The statement is true for $n = 1$. 1

Assume $1 + 3 + 9 + \dots + 3^{k-1} = \frac{3^k - 1}{2}$, where $k \in \mathbb{Z}^+$. 1

$$\begin{aligned}
&1 + 3 + 9 + \dots + 3^{k-1} + 3^{(k+1)-1} \\
&= \frac{3^k - 1}{2} + 3^k & 1 \\
&= \frac{3^k - 1 + 2 \times 3^k}{2} \\
&= \frac{3^{k+1} - 1}{2} & 1
\end{aligned}$$

The statement is true for $n = k + 1$.

By M.I., the statement is true $\forall n \in \mathbb{Z}^+$. 1

6. For $n = 1$,

$$\text{L.H.S.} = 1 \times 11 = 11 \text{ and R.H.S.} = \frac{1}{6}(1)(1+1)(2(1+31)) = 11 = \text{L.H.S.}$$

The statement is true for $n = 1$. 1

Assume $1 \times 11 + 2 \times 12 + 3 \times 13 + \dots + k(k+10) = \frac{1}{6}k(k+1)(2k+31)$, where $k \in \mathbb{Z}^+$. 1

$$\begin{aligned}
&1 \times 11 + 2 \times 12 + 3 \times 13 + \dots + k(k+10) + (k+1)[(k+1)+10] \\
&= \frac{1}{6}k(k+1)(2k+31) + (k+1)(k+11) & 1 \\
&= \frac{k+1}{6}[2k^2 + 31k + 6(k+11)] \\
&= \frac{k+1}{6}(2k^2 + 37k + 66) \\
&= \frac{k+1}{6}(k+2)(2k+33) \\
&= \frac{1}{6}(k+1)[(k+1)+1][2(k+1)+31] & 1
\end{aligned}$$

The statement is true for $n = k + 1$.

By the principle of M.I., the statement is true $\forall n \in \mathbb{Z}^+$. 1

7. For $n = 1$,

$$\text{L.H.S.} = 1 \times 2 \times 5 = 10 \text{ and R.H.S.} = \frac{1}{3}(1)(2)(3)(5) = 10 = \text{L.H.S.}$$

The statement is true for $n = 1$.

1

Assume $1 \times 2 \times 5 + 2 \times 3 \times 13 + \dots + k(k+1)(8k-3) = \frac{1}{3}k(k+1)(k+2)(6k-1)$ for some $k \in \mathbb{Z}^+$.

1M

$$1 \times 2 \times 5 + 2 \times 3 \times 13 + 3 \times 4 \times 21 + \dots + k(k+1)(8k-3) + (k+1)(k+2)(8k+5)$$

$$= \frac{1}{3}k(k+1)(k+2)(6k-1) + (k+1)(k+2)(8k+5)$$

1M

$$= \frac{1}{3}(k+1)(k+2)[(6k^2 - k) + 3(8k+5)]$$

$$= \frac{1}{3}(k+1)(k+2)(6k^2 + 23k + 15)$$

$$= \frac{1}{3}(k+1)(k+2)(k+3)(6k+5)$$

1

The statement is true for $n = k + 1$.

By mathematical induction, the statement is true $\forall n \in \mathbb{Z}^+$.

1

8. For $n = 1$,

$$\text{L.H.S.} = (2)(2)(3) = 12 \text{ and R.H.S.} = 1(2)^2(3) = 12 = \text{L.H.S.}$$

The statement is true for $n = 1$.

1

Assume $\sum_{i=1}^k 2i(i+1)(2i+1) = k(k+1)^2(k+2)$ for some $k \in \mathbb{Z}^+$.

1M

$$\sum_{i=1}^{k+1} 2i(i+1)(2i+1) = \sum_{i=1}^k 2i(i+1)(2i+1) + 2(k+1)(k+2)(2k+3)$$

$$= k(k+1)^2(k+2) + 2(k+1)(k+2)(2k+3)$$

1M

$$= (k+1)(k+2)(k^2 + k + 4k + 6)$$

$$= (k+1)(k+2)(k^2 + 5k + 6)$$

$$= (k+1)(k+2)^2(k+3)$$

1

The statement is true for $n = k + 1$.

By mathematical induction, the statement is true $\forall n \in \mathbb{Z}^+$.

1

9. For $n = 1$,

$$\text{L.H.S.} = \frac{1^2}{1 \cdot 3} = \frac{1}{3} \text{ and R.H.S.} = \frac{1(1+1)}{2(2+1)} = \frac{1}{3} = \text{L.H.S.}$$

The statement is true for $n = 1$.

1

$$\text{Assume } \sum_{i=1}^k \frac{i^2}{(2i-1)(2i+1)} = \frac{k(k+1)}{2(2k+1)} \text{ for some } k \in \mathbb{Z}^+.$$

1M

$$\begin{aligned} \sum_{i=1}^{k+1} \frac{i^2}{(2i-1)(2i+1)} &= \sum_{i=1}^k \frac{i^2}{(2i-1)(2i+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)} \\ &= \frac{k(k+1)}{2(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)} \\ &= \frac{k+1}{2(2k+1)(2k+3)} [k(2k+3) + 2(k+1)] \\ &= \frac{k+1}{2(2k+1)(2k+3)} (2k^2 + 5k + 2) \\ &= \frac{k+1}{2(2k+1)(2k+3)} (2k+1)(k+2) \\ &= \frac{(k+1)(k+2)}{2(2k+3)} \end{aligned}$$

1M

1

The statement is true for $n = k + 1$.

By mathematical induction, the statement is true $\forall n \in \mathbb{Z}^+$.

1

10. When $n = 1$,

$$\text{L.H.S.} = 4 \text{ and R.H.S.} = \frac{2^2 + 3^2 - 5}{2} = 4 = \text{L.H.S.}$$

It is true for $n = 1$.

1

$$\text{Assume } 4 + 11 + 31 + \dots + (2^{k-1} + 3^k) = \frac{2^{k+1} + 3^{k+1} - 5}{2}, \text{ where } k \in \mathbb{Z}^+.$$

1

$$\begin{aligned} &4 + 11 + 31 + \dots + (2^{k-1} + 3^k) + (2^k + 3^{k+1}) \\ &= \frac{2^{k+1} + 3^{k+1} - 5}{2} + 2^k + 3^{k+1} \\ &= \frac{2^{k+1} + 3^{k+1} - 5 + 2^{k+1} + 2 \cdot 3^{k+1}}{2} \\ &= \frac{2^{k+2} + 3^{k+2} - 5}{2} \end{aligned}$$

1

1

It is true for $n = k + 1$.

By M.I., it is true $\forall n \in \mathbb{Z}^+$.

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