

DAYM2-DIFF-2425-ASM-SET 2-MATH**Suggested solutions**

$$\begin{aligned}
 1. \quad y &= \sqrt{(e^x)^3} \\
 &= e^{\frac{3x}{2}} \\
 \frac{dy}{dx} &= e^{\frac{3x}{2}} \left(\frac{3}{2} \right) && 1M \\
 &= \frac{3}{2} e^{\frac{3x}{2}} && 1A
 \end{aligned}$$

$$\begin{aligned}
 2. \quad y &= \ln(4x^3 + 5x^2) \\
 \frac{dy}{dx} &= \frac{12x^2 + 10x}{4x^3 + 5x^2} && 1M \\
 &= \frac{12x + 10}{4x^2 + 5x} && 1A
 \end{aligned}$$

$$\begin{aligned}
 3. \quad y &= \log_2(2 + e^{2x}) \\
 &= \frac{\ln(2 + e^{2x})}{\ln 2} \\
 \frac{dy}{dx} &= \frac{1}{\ln 2} \cdot \frac{2e^{2x}}{2 + e^{2x}} && 1M \\
 &= \frac{2e^{2x}}{(2 + e^{2x}) \ln 2} && 1A
 \end{aligned}$$

$$\begin{aligned}
 4. \quad y &= \sqrt{x}e^{-3x} \\
 &= x^{\frac{1}{2}}e^{-3x} \\
 \frac{dy}{dx} &= \frac{1}{2}x^{-\frac{1}{2}}e^{-3x} + x^{\frac{1}{2}}e^{-3x}(-3) && 1M \\
 &= e^{-3x} \left(\frac{1}{2\sqrt{x}} - 3\sqrt{x} \right) && 1A
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \ln x + \ln y &= 2x^2 + 3x(x + y) \\
 \ln x + \ln y &= 5x^2 + 3xy \\
 \frac{1}{x} + \frac{1}{y} \times \frac{dy}{dx} &= 10x + 3y + 3x \frac{dy}{dx} && 1M \\
 \left(\frac{1}{y} - 3x \right) \frac{dy}{dx} &= 10x + 3y - \frac{1}{x} \\
 \frac{1 - 3xy}{y} \times \frac{dy}{dx} &= \frac{10x^2 + 3xy - 1}{x} \\
 \frac{dy}{dx} &= \frac{y(10x^2 + 3xy - 1)}{x(1 - 3xy)} && 1A
 \end{aligned}$$

6. $\ln(x^2y) = 4x^3 - 10y$
- $$\frac{1}{x^2y} \left(2xy + x^2 \frac{dy}{dx} \right) = 12x^2 - 10 \frac{dy}{dx} \quad 1M$$
- $$\left(\frac{1}{y} + 10 \right) \frac{dy}{dx} = 12x^2 - \frac{2}{x}$$
- $$\left(\frac{1+10y}{y} \right) \frac{dy}{dx} = \frac{12x^3 - 2}{x}$$
- $$\frac{dy}{dx} = \frac{2y(6x^3 - 1)}{x(1+10y)} \quad 1A$$
7. $\ln y = 2 \ln x + 3 \ln(3x - 1) - (-x) - \frac{1}{3} \ln(9 + 4x^2)$ 1M
- $$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{x} + \frac{3(3)}{3x-1} + 1 - \frac{1}{3} \left(\frac{8x}{9+4x^2} \right) \quad 1M$$
- $$\frac{dy}{dx} = \frac{x^2 e^x (3x-1)^3}{\sqrt[3]{9+4x^2}} \left[\frac{2}{x} + 1 + \frac{9}{3x-1} - \frac{8x}{3(9+4x^2)} \right] \quad 1A$$
8. $\ln y = \frac{2}{3} \ln(3x+4) + \frac{4}{3} \ln(5+2x) - \frac{1}{3} \ln(2e^{3x} + 3x)$ 1M
- $$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{3} \left(\frac{3}{3x+4} \right) + \frac{4}{3} \left(\frac{2}{5+2x} \right) - \frac{1}{3} \left(\frac{6e^{3x} + 3}{2e^{3x} + 3x} \right) \quad 1M$$
- $$\frac{dy}{dx} = \sqrt[3]{\frac{(3x+4)^2(5+2x)^4}{2e^{3x} + 3x}} \left[\frac{2}{3x+4} + \frac{8}{3(5+2x)} - \frac{2e^{3x} + 1}{2e^{3x} + 3x} \right] \quad 1A$$
9. $\ln y = \ln x \ln(x \tan x + e^x)$ 1M
- $$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{x} \ln(x \tan x + e^x) + \ln x \times \frac{\tan x + x \sec^2 x + e^x}{x \tan x + e^x} \quad 1M$$
- $$\frac{dy}{dx} = (x \tan x + e^x)^{\ln x} \left[\frac{(x \sec^2 x + \tan x + e^x) \ln x}{x \tan x + e^x} + \frac{\ln(x \tan x + e^x)}{x} \right] \quad 1A$$
10. $\sin xy = \ln(x^2 + 2)$
- $$\cos xy \left(y + x \frac{dy}{dx} \right) = \frac{2x}{x^2 + 2} \quad 1M$$
- $$x \cos xy \frac{dy}{dx} + y \cos xy = \frac{2x}{x^2 + 2}$$
- $$x \cos xy \frac{dy}{dx} = \frac{2x - (x^2 + 2)y \cos xy}{x^2 + 2}$$
- $$\frac{dy}{dx} = \frac{2x - (x^2 + 2)y \cos xy}{(x^2 + 2)x \cos xy} \quad 1A$$
11. $\ln(\sin y) = \cos xy + 3$
- $$\frac{\cos y}{\sin y} \times \frac{dy}{dx} = -\sin xy \left(y + x \frac{dy}{dx} \right) \quad 1M$$
- $$(\cot y + x \sin xy) \frac{dy}{dx} = -y \sin xy$$
- $$\frac{dy}{dx} = -\frac{y \sin xy}{\cot y + x \sin xy} \quad 1A$$

12. $y^3 = 2 \cos(x + y)$
 $3y^2 \frac{dy}{dx} = -2 \sin(x + y) \left(1 + \frac{dy}{dx}\right)$ 1M
 $[3y^2 + 2 \sin(x + y)] \frac{dy}{dx} = -2 \sin(x + y)$
 $\frac{dy}{dx} = -\frac{2 \sin(x + y)}{3y^2 + 2 \sin(x + y)}$ 1A
13. $\sin x^2 y - \cos y = 3x^2$
 $\cos x^2 y \left(2xy + x^2 \frac{dy}{dx}\right) + \sin y \frac{dy}{dx} = 6x$ 1M
 $(x^2 \cos x^2 y + \sin y) \frac{dy}{dx} = 6x - 2xy \cos x^2 y$
 $\frac{dy}{dx} = \frac{6x - 2xy \cos x^2 y}{x^2 \cos x^2 y + \sin y}$ 1A
14. $\frac{dy}{dx} = \frac{1}{2}(5x^2 + 2x)^{-\frac{1}{2}}(10x + 2)$ 1M
 $= (5x + 1)(5x^2 + 2x)^{-\frac{1}{2}}$ 1A
 $\frac{d^2y}{dx^2} = 5(5x^2 + 2x)^{-\frac{1}{2}} + (5x + 1) \left(-\frac{1}{2}\right) (5x^2 + 2x)^{-\frac{3}{2}}(10x + 2)$
 $= (5x^2 + 2x)^{-\frac{3}{2}} [5(5x^2 + 2x) - (5x + 1)^2]$
 $= -(5x^2 + 2x)^{-\frac{3}{2}}$ 1A
15. $\frac{dy}{dx} = 2xe^{x^2} + x^2 e^{x^2} (2x)$ 1M
 $= 2e^{x^2} (x + x^3)$ 1A
 $\frac{d^2y}{dx^2} = 2e^{x^2} (2x)(x + x^3) + 2e^{x^2} (1 + 3x^2)$
 $= 2e^{x^2} [(2x^2 + 2x^4) + (1 + 3x^2)]$
 $= 2e^{x^2} (2x^4 + 5x^2 + 1)$ 1A
16. $\frac{dy}{dx} = e^{-\frac{x}{2}} \left(-\frac{1}{2}\right) \cos \frac{x}{2} + e^{-\frac{x}{2}} \left(-\sin \frac{x}{2}\right) \left(\frac{1}{2}\right)$ 1M
 $= -\frac{1}{2} e^{-\frac{x}{2}} \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)$ 1A
 $\frac{d^2y}{dx^2} = -\frac{1}{2} e^{-\frac{x}{2}} \left(-\frac{1}{2}\right) \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) - \frac{1}{2} e^{-\frac{x}{2}} \left(-\frac{1}{2} \sin \frac{x}{2} + \frac{1}{2} \cos \frac{x}{2}\right)$
 $= \frac{1}{4} e^{-\frac{x}{2}} \left[\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) + \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)\right]$
 $= \frac{1}{2} e^{-\frac{x}{2}} \sin \frac{x}{2}$ 1A

17. $\frac{dy}{dx} = \cos x \ln(\cos x) + \sin x \cdot \frac{-\sin x}{\cos x}$ 1M
 $= \cos x \ln(\cos x) - \sin x \tan x$ 1A
- $\frac{d^2y}{dx^2} = -\sin x \ln(\cos x) + \cos x \cdot \frac{-\sin x}{\cos x} - \cos x \tan x - \sin x \sec^2 x$
 $= -\sin x [\ln(\cos x) + 1 + 1 + \sec^2 x]$
 $= -\sin x [\ln(\cos x) + \sec^2 x + 2]$ 1A
18. (a) $\frac{dy}{dx} = 5 \left(\frac{2x+3}{2x-1} \right)^4 \cdot \frac{2(2x-1) - (2x+3)(2)}{(2x-1)^2}$ 1M
 $= 5 \left(\frac{2x+3}{2x-1} \right)^4 \cdot \frac{-8}{(2x-1)^2}$
 $= \frac{-40(2x+3)^4}{(2x-1)^6}$ 1
- (b) $\frac{dy}{dx} = -40(2x+3)^4(2x-1)^{-6}$
 $\frac{d^2y}{dx^2} = -160(2x+3)^3(2)(2x-1)^{-6} - 40(2x+3)^4(-6)(2x-1)^{-7}(2)$ 1M
 $= 160(2x+3)^3(2x-1)^{-7} [-2(2x-1) + 3(2x+3)]$
 $= 160(2x+3)^3(2x-1)^{-7}(2x+11)$ 1A
19. (a) $y = \ln(1+x^2) - \ln(1-x^2)$
 $\frac{dy}{dx} = \frac{2x}{1+x^2} - \frac{-2x}{1-x^2}$ 1M
 $= \frac{2x(1-x^2) + 2x(1+x^2)}{(1-x^2)(1+x^2)}$
 $= \frac{4x}{1-x^4}$ 1
- (b) $\frac{d^2y}{dx^2} = \frac{4(1-x^4) - 4x(-4x^3)}{(1-x^4)^2}$ 1M
 $= \frac{4(3x^4+1)}{(1-x^4)^2}$ 1A
20. $\frac{dy}{dx} = e^{-4x} + xe^{-4x}(-4)$ 1M
 $= e^{-4x}(1-4x)$ 1A
- $\frac{d^2y}{dx^2} = e^{-4x}(-4)(1-4x) + e^{-4x}(-4)$
 $= -4e^{-4x}[(1-4x) + 1]$
 $= 8e^{-4x}(2x-1)$ 1A
- $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y$
 $= [8e^{-4x}(2x-1)] + 3[e^{-4x}(1-4x)] - 4xe^{-4x}$
 $= e^{-4x} [8(2x-1) + 3(1-4x) - 4x]$
 $= -5e^{-4x}$ 1

$$21. \quad \frac{dy}{dx} = \frac{1}{x} \cdot \frac{1}{x^2} + \ln x(-2x^{-3}) \quad 1M$$

$$= x^{-3}(1 - 2 \ln x) \quad 1A$$

$$\frac{d^2y}{dx^2} = -3x^{-4}(1 - 2 \ln x) + x^{-3} \left(-\frac{2}{x} \right)$$

$$= -x^{-4} [3(1 - 2 \ln x) + 2]$$

$$= -x^{-4}(5 - 6 \ln x) \quad 1A$$

$$x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 4y$$

$$= x^2 [-x^{-4}(5 - 6 \ln x)] + 5x [x^{-3}(1 - 2 \ln x)] + 4x^{-2} \ln x$$

$$= x^{-2} [-(5 - 6 \ln x) + 5(1 - 2 \ln x) + 4 \ln x]$$

$$= 0 \quad 1$$

$$22. \quad \frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} \ln x + x^{\frac{3}{2}} \cdot \frac{1}{x} \quad 1M$$

$$= \frac{1}{2}x^{\frac{1}{2}}(3 \ln x + 2) \quad 1A$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}x^{-\frac{1}{2}}(3 \ln x + 2) + \frac{1}{2}x^{\frac{1}{2}} \left(\frac{3}{x} \right)$$

$$= \frac{1}{4}x^{-\frac{1}{2}} [(3 \ln x + 2) + 6]$$

$$= \frac{1}{4}x^{-\frac{1}{2}}(3 \ln x + 8) \quad 1A$$

$$4x^2 \frac{d^2y}{dx^2} - 8x \frac{dy}{dx} + 9y$$

$$= 4x^2 \left[\frac{1}{4}x^{-\frac{1}{2}}(3 \ln x + 8) \right] - 8x \left[\frac{1}{2}x^{\frac{1}{2}}(3 \ln x + 2) \right] + 9x^{\frac{3}{2}} \ln x$$

$$= x^{\frac{3}{2}} [(3 \ln x + 8) - 4(3 \ln x + 2) + 9 \ln x]$$

$$= 0 \quad 1$$

$$23. \quad \frac{dy}{dx} = -2x^{-3} \cos x + x^{-2}(-\sin x) \quad 1M$$

$$= -x^{-3}(2 \cos x + x \sin x) \quad 1A$$

$$\frac{d^2y}{dx^2} = 3x^{-4}(2 \cos x + x \sin x) - x^{-3} [-2 \sin x + \sin x + x \cos x]$$

$$= x^{-4} [3(2 \cos x + x \sin x) - x(-2 \sin x + \sin x + x \cos x)]$$

$$= x^{-4}(4x \sin x - x^2 \cos x + 6 \cos x) \quad 1A$$

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + (x^2 + 2)y$$

$$= x^2 [x^{-4}(4x \sin x - x^2 \cos x + 6 \cos x)] + 4x [-x^{-3}(2 \cos x + x \sin x)] + (x^2 + 2)x^{-2} \cos x$$

$$= x^{-2} [(4x \sin x - x^2 \cos x + 6 \cos x) - 4(2 \cos x + x \sin x) + (x^2 + 2) \cos x]$$

$$= 0 \quad 1$$

$$24. \quad \frac{dy}{dx} = 9x^2 - 8x \quad 1A$$

$$\frac{d^2y}{dx^2} = 18x - 8 \quad 1A$$

$$\begin{aligned} 0 &= x^2 \frac{d^2y}{dx^2} + kx \frac{dy}{dx} + my \\ &= x^2(18x - 8) + kx(9x^2 - 8x) + m(3x^3 - 4x^2) \\ &= (18 + 9k + 3m)x^3 + (-8 - 8k - 4m)x^2 \end{aligned}$$

$$\text{We have } 18 + 9k + 3m = 0 \text{ and } -8 - 8k - 4m = 0. \quad 1M$$

$$\text{Solving, we have } k = -4 \text{ and } m = 6. \quad 1A$$

$$25. \quad (a) \quad \frac{dy}{dx} = e^x \sin x + e^x \cos x \quad 1M$$

$$= e^x (\sin x + \cos x) \quad 1A$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= e^x (\sin x + \cos x) + e^x (\cos x - \sin x) \\ &= 2e^x \cos x \quad 1A \end{aligned}$$

$$\begin{aligned} (b) \quad 0 &= \frac{d^2y}{dx^2} - k \frac{dy}{dx} + 2y \\ 0 &= 2e^x \cos x - k e^x (\sin x + \cos x) + 2e^x \sin x \\ &= e^x [(2 - k) \cos x + (-k + 2) \sin x] \end{aligned}$$

$$\text{We have } 2 - k = 0. \quad 1M$$

$$\text{Thus, } k = 2. \quad 1A$$

$$26. \quad (a) \quad (i) \quad \frac{dy}{dx} = k \left(\frac{x+1}{x-1} \right)^{k-1} \cdot \frac{1(x-1) - (x+1)(1)}{(x-1)^2} \quad 1M$$

$$= k \left(\frac{x+1}{x-1} \right)^k \left(\frac{x+1}{x-1} \right)^{-1} \cdot \frac{-2}{(x-1)^2}$$

$$= ky \frac{-2}{(x+1)(x-1)}$$

$$= \frac{2ky}{1-x^2} \quad 1$$

$$(ii) \quad \frac{d^2y}{dx^2} = \frac{2k \frac{dy}{dx} - 2ky(-2x)}{(1-x^2)^2}$$

$$= \frac{2k}{1-x^2} \frac{dy}{dx} + \frac{4kxy}{(1-x^2)^2} \quad 1A$$

$$(x^2-1) \frac{d^2y}{dx^2} + 2(x+k) \frac{dy}{dx}$$

$$= (x^2-1) \left[\frac{2k}{1-x^2} \frac{dy}{dx} + \frac{4kxy}{(1-x^2)^2} \right] + 2(x+k) \left[\frac{2ky}{1-x^2} \right]$$

$$= -2k \left(\frac{2ky}{1-x^2} \right) - \frac{4kxy}{1-x^2} + \frac{4kxy}{1-x^2} + \frac{4k^2y}{1-x^2}$$

$$= 0 \quad 1$$

$$(b) \quad y = \left(\frac{0+1}{0-1} \right)^k$$

$$= (-1)^k$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{2k(-1)^k}{1-0}$$

$$= 2k(-1)^k$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=0} = \frac{2k}{1} [2k(-1)^k] + 0$$

$$16 = 4k^2(-1)^k \quad 1A$$

We have k being a positive even number and $4k^2 = 16$. 1M

Thus, $k = 2$. 1A