DAYM2-AOD-2425-ASM-SET 1-MATH
Suggested solutions

1. $\frac{\mathrm{d}y}{\mathrm{d}x} = e^x \ln x + e^x \left(\frac{1}{x}\right)$ 1M

$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(1,\ 0)} = 0 + e(1)$

$= e$ 1M

Required equation is

$$y - 0 = e(x - 1)$$

$ex - y - e = 0$ 1A

2. $x = y^3 - 3y^2$

$1 = 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} - 6y\frac{\mathrm{d}y}{\mathrm{d}x}$ 1M

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2 - 6y}$

$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(-2,\ 1)} = \frac{1}{3 - 6} = -\frac{1}{3}$ 1M

Required equation is

$$y - 1 = -\frac{1}{3}(x + 2)$$

$x + 3y - 1 = 0$ 1A

3. $x^2 - 2\sin y = 3$

$2x - 2\cos y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ 1M

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{\cos y}$

$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(2,\ \frac{\pi}{6})} = \frac{2}{\cos\frac{\pi}{6}} = \frac{4}{\sqrt{3}}$ 1M

Required equation is

$$y - \frac{\pi}{6} = \frac{4}{\sqrt{3}}(x - 2)$$

$$6y - \pi = 8\sqrt{3}(x - 2)$$

$8\sqrt{3}x - 6y + \pi - 16\sqrt{3} = 0$ 1A

4. $x^2(0)+3x-4(0)-9=0$

$$x=3$$

The curve cuts the x-axis at $(3,\ 0)$. 1A

$$x^2y+3x-4y-9=0$$

$$2xy+x^2\frac{dy}{dx}+3-4\frac{dy}{dx}=0$$ 1M

$$\frac{dy}{dx}=\frac{2xy+3}{4-x^2}$$

$$\left.\frac{dy}{dx}\right|_{(3,\ 0)}=\frac{0+3}{4-3^2}$$

$$=-\frac{3}{5}$$ 1M

Required equation is

$$y-0=-\frac{3}{5}(x-3)$$

$$3x+5y-9=0$$ 1A

5. $y=\sqrt{10-(0+1)^2}$

$$=3$$

The curve cuts the y-axis at $(0,\ 3)$. 1A

$$\frac{dy}{dx}=\frac{1}{2}[10-(x+1)^2]^{-\frac{1}{2}}(-2)(x+1)$$ 1M

$$=-\frac{x+1}{\sqrt{10-(x+1)^2}}$$

$$\left.\frac{dy}{dx}\right|_{(0,\ 3)}=-\frac{1}{\sqrt{10-1}}=-\frac{1}{3}$$ 1M

Required equation is

$$y-3=-\frac{1}{3}(x-0)$$

$$x+3y-9=0$$ 1A

6. $\cos^2 x - 2y + 1 = 0$

$$2\cos x(-\sin x) - 2\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \qquad \text{1M}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sin x \cos x$$

Let the coordinates of the point of contact be $(a,\ b)$.

$$-\sin a \cos a = -\frac{1}{2} \qquad \text{1M}$$

$$-\frac{\sin 2a}{2} = -\frac{1}{2}$$

$$\sin 2a = 1$$

$$2a = \frac{\pi}{2} \quad \text{or} \quad \frac{5\pi}{2}$$

$$a = \frac{\pi}{4} \quad \text{or} \quad \frac{5\pi}{4}$$

We have $(a,\ b) = \left(\frac{\pi}{4},\ \frac{3}{4}\right)$ or $\left(\frac{5\pi}{4},\ \frac{3}{4}\right)$.

The equation of tangent to the curve at $\left(\frac{\pi}{4},\ \frac{3}{4}\right)$ is

$$y - \frac{3}{4} = -\frac{1}{2}\left(x - \frac{\pi}{4}\right)$$

$$4x + 8y - \pi - 6 = 0 \qquad \text{1A}$$

The equation of tangent to the curve at $\left(\frac{5\pi}{4},\ \frac{3}{4}\right)$ is

$$y - \frac{3}{4} = -\frac{1}{2}\left(x - \frac{5\pi}{4}\right)$$

$$4x + 8y - 5\pi - 6 = 0 \qquad \text{1A}$$

7. $\dfrac{dy}{dx} = \dfrac{(3x+1)-(x-1)(3)}{(3x+1)^2}$ 1M

$= \dfrac{4}{(3x+1)^2}$

Let the coordinates of the point of contact be $(a,\ b)$.

$\dfrac{4}{(3a+1)^2} = \dfrac{1}{4}$ 1M

$(3a+1)^2 = 16$

$3a+1 = \pm 4$

$a = 1 \quad \text{or} \quad -\dfrac{5}{3}$

We have $(a,\ b) = (1,\ 0)$ or $\left(-\dfrac{5}{3},\ \dfrac{2}{3}\right)$. The equation of tangent to the curve at $(1,\ 0)$ is

$y - 0 = \dfrac{1}{4}(x-1)$

$x - 4y - 1 = 0$ 1A

The equation of tangent to the curve at $\left(-\dfrac{5}{3},\ \dfrac{2}{3}\right)$ is

$y - \dfrac{2}{3} = \dfrac{1}{4}\left(x + \dfrac{5}{3}\right)$

$3x - 12y + 13 = 0$ 1A

8. $\dfrac{dy}{dx} = \ln x + x\left(\dfrac{1}{x}\right)$ 1M

$= \ln x + 1$

$\ln a + 1 = 1$ 1M

$\ln a = 0$

$a = 1$

We have $a = 1$ and $b = 0$. 1A

9. $\dfrac{dy}{dx} = 6x^2 - 4$ 1M

$\left.\dfrac{dy}{dx}\right|_{(-1,\ 11)} = 6(-1)^2 - 4 = 2$ 1M

The equation of tangent to the curve at A is

$$y - 11 = 2(x + 1)$$

$$y = 2x + 13 \quad \text{1A}$$

$2x + 13 = 2x^3 - 4x + 9$ 1M

$$0 = 2x^3 - 6x - 4$$

$$0 = 2(x + 1)(x^2 - x - 2)$$

$$0 = 2(x + 1)^2(x - 2)$$

$$x = -1 \quad \text{or} \quad 2$$

The coordinates of B are $(2,\ 17)$. 1A

10. $x^2 + y^2 = 16$

$2x + 2y\dfrac{dy}{dx} = 0$ 1M

$$\frac{dy}{dx} = -\frac{x}{y}$$

Let the coordinates of the point of contact be $(a,\ b)$.

$-\dfrac{a}{b} = \dfrac{b - 0}{a - 8}$ 1M

$$-a^2 + 8a = b^2$$

$-a^2 + 8a = 16 - a^2$ 1M

$$a = 2$$

When $a = 2$, $b = \pm\sqrt{16 - 2^2} = \pm 2\sqrt{3}$.

The equation of tangent to the curve at $\left(2,\ 2\sqrt{3}\right)$ is

$$y - 0 = -\frac{2}{2\sqrt{3}}(x - 8)$$

$x + \sqrt{3}y - 8 = 0$ 1A

The equation of tangent to the curve at $\left(2,\ -2\sqrt{3}\right)$ is

$$y - 0 = -\frac{2}{-2\sqrt{3}}(x - 8)$$

$x - \sqrt{3}y - 8 = 0$ 1A

11. $\frac{\mathrm{d}y}{\mathrm{d}x} = 9x^2 - 2$ 1M

Let the coordinates of the point of contact be $(a,\ b)$.

$$9a^2 - 2 = \frac{b+1}{a-2} \quad \text{1M}$$

$$(9a^2 - 2)(a - 2) = b + 1$$

$$9a^3 - 18a^2 - 2a + 4 = 3a^3 - 2a + 4 \quad \text{1M}$$

$$6a^3 - 18a^2 = 0$$

$$6a^2(a - 3) = 0$$

$$a = 0 \quad \text{or} \quad 3$$

We have $(a,\ b) = (0,\ 3)$ or $(3,\ 78)$.

The equation of tangent to the curve at $(0,\ 3)$ is

$$y - 3 = [9(0)^2 - 2](x - 0)$$

$$2x + y - 3 = 0 \quad \text{1A}$$

The equation of tangent to the curve at $(3,\ 78)$ is

$$y - 78 = [9(3)^2 - 2](x - 3)$$

$$79x - y - 159 = 0 \quad \text{1A}$$

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