DAYCP-EOC-2425-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a) $(2,\ 3)$ — 1A

(b) (i) Let the coordinates of P be $(a,\ b)$.

We have $3a - 2b - 13 = 0$.

Slope of $PA = \dfrac{3-b}{2-a}$

$\dfrac{3-b}{2-a} \times \dfrac{3}{2} = -1$ — 1M

$2a + 3b - 13 = 0$

Solving, we have $a = 5$ and $b = 1$. — 1A

Required length $= \sqrt{(5-2)^2 + (1-3)^2}$

$= \sqrt{13}$ — 1A

(ii) (1) P, A and Q are collinear. — 1A

(2) Radius $= \sqrt{2^2 + 3^2 - 9}$

$= 2$

Required ratio $= AQ : AP$ — 1M

$= 2 : \sqrt{13}$ — 1A

2. (a) (i) Consider $\triangle ABC$ and $\triangle ODA$,

$AB = AD$ *(given)*

$\angle ACB = \angle ACD$ *(equal chords, equal ∠s)*

$\angle OAD = \angle ACD$ *(∠ in alt. segment)*

$= \angle ACB$

$\angle ADO = \angle ABC$ *(ext. ∠, cyclic quad.)*

$\triangle ABC \sim \triangle ODA$ *(AA)*

$\frac{AB}{BC} = \frac{OD}{AD}$ *(corr. sides, ~ △s)*

$AB^2 = BC \cdot OD$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(ii) Since $\angle ACD = \angle ACB$,

$$\cos\angle ACD = \cos\angle ACB$$

$$\frac{CD^2 + AC^2 - AD^2}{2(CD)(AC)} = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$$ 1M

$$2\left(CD^2 + AC^2 - AD^2\right) = AC^2 + BC^2 - AB^2$$

$$2CD^2 + 2AC^2 - 2AD^2 = AC^2 + (2CD)^2 - AD^2$$ 1M

$$AC^2 = 2CD^2 + AD^2$$ 1

(b) (i) Since $\angle BCD = 90°$, BD is a diameter.

Centre $= \left(\frac{15+27}{2}, \frac{0+24}{2}\right) = (21, 12)$.

The equation of the circle is

$$(x-21)^2 + (y-12)^2 = (15-21)^2 + (0-12)^2$$ 1M

$$(x-21)^2 + (y-12)^2 = 180$$ 1A

(ii) $AB^2 = BC \cdot OD$

$AB = \sqrt{24 \cdot 15}$

$= 6\sqrt{10}$ 1A

$AC^2 = 2CD^2 + AD^2$

$AC^2 = 2(12)^2 + (6\sqrt{10})^2$ 1M

$AC = 18\sqrt{2}$ 1A

3. (a) Let $(h,\ k)$ be the coordinates of A.

$$\begin{cases} \dfrac{k-0}{h-0} = \dfrac{3}{4} \\ 4h + 3k - 50 = 0 \end{cases}$$ 1M

Solving, we have $h = 8$ and $k = 6$. 1A

Required equation is

$$(x-8)^2 + (y-6)^2 = (0-8)^2 + (0-6)^2$$

$$(x-8)^2 + (y-6)^2 = 100$$ 1A

(b) (i) $(x-8)^2 + (0-6)^2 = 100$ 1M

$$x^2 - 16x = 0$$

$$x = 0 \quad \text{or} \quad 16$$

The coordinates of B are $(16,\ 0)$. 1A

(ii) Let θ be the inclination of BD.

$$\tan\theta = \frac{3}{4}$$

$$\theta \approx 36.9^\circ$$

$\angle OAD = 2\angle OBD$ 1M

$= 2\theta$

$\approx 73.7^\circ$ 1A

(iii) $\angle ADM = \angle OAD \approx 73.7^\circ$

Required area

$= \dfrac{1}{2}(DM)(AM)$

$= \dfrac{1}{2}(AD\cos\angle ADM)(AD\sin\angle ADM)$ 1M

≈ 13.4

< 14

The claim is not correct. 1A

4. (a) Let $f(x) = ax + bx^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 36 = 36a + 36^2 b \\ 192 = 48a + 48^2 b \end{cases}$$ 1M

Solving, we have $a = -8$ and $b = \frac{1}{4}$.

Thus, $f(16) = -8(16) + \frac{1}{4}(16)^2 = -64$. 1A

(b) We have $s = f(16) = -64$ and $t = f(32) = 0$. 1M

The coordinates of S, T and U are $(16,\ -64)$, $(32,\ 0)$ and $(16,\ 0)$.

Note that $\angle SUT = 90°$.

ST is a diameter of the circle. 1M

Required area $= \pi\left(\frac{ST}{2}\right)^2$ 1M

$= \pi\left(\frac{\sqrt{16^2 + 64^2}}{2}\right)^2$

$= 1088\pi$ 1A

5. (a) The coordinates of M are $(4,\ 7)$. 1A

Radius $= \sqrt{4^2 + 7^2 - 61} = 2$ 1A

(b) Slope of L is $-\frac{4}{3}$.

Slope of $MN = \frac{3}{4}$ 1M

Required equation is

$$y - 7 = \frac{3}{4}(x - 4)$$ 1M

$$3x - 4y + 16 = 0$$ 1A

(c) (i) Solve $\begin{cases} 3x - 4y + 16 = 0 \\ 4x + 3y - 62 = 0 \end{cases}$, 1M

we have $x = 8$ and $y = 10$.

The coordinates of N are $(8,\ 10)$. 1A

(ii) $MN = \sqrt{(8-4)^2 + (10-7)^2} = 5$

Required distance $= 5 - 2$ 1M

$= 3$ 1A

 END OF PAPER