DAYCP-EOC-2425-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. A	3. B	4. A	5. A
6. B	7. B	8. B	9. A	10. D
11. D	12. C	13. D	14. A	15. C
16. D	17. B	18. D	19. B	20. C

1. A

C_1: $x^2 + y^2 - 18x - 24y + 176 = 0$
C_2: $x^2 + y^2 - 36x - 48y + 836 = 0$
The coordinates of G_1 and G_2 are $(9,\ 12)$ and $(18,\ 24)$ respectively.
The coordinates of P are $(18,\ 0)$.

I. ✓.
$PG_1 = \sqrt{(18-9)^2 + (12-0)^2} = 15$
$G_1G_2 = \sqrt{(18-9)^2 + (24-12)^2} = 15 = PG_1$
$\triangle PG_1G_2$ is an isosceles triangle.

II. ✓.
Area of $\triangle PG_1G_2 = \dfrac{(24-0)(18-9)}{2} = 108$
Area of $\triangle OPG_1 = \dfrac{(18-0)(12-0)}{2} = 108$

III. ✗.
Radius of $C_1 = \sqrt{9^2 + 12^2 - 176} = 7$
Radius of $C_2 = \sqrt{18^2 + 24^2 - 836} = 8$
Note that G_1G_2 is equal to the sum of radii of C_1 and C_2.
C_1 and C_2 touch externally.

2. A

$x^2 + y^2 + \dfrac{kx}{2} + 6y = 0$
The coordinates of the centre are $\left(-\dfrac{k}{4},\ -3\right)$.

$$3\left(-\frac{k}{4}\right) - 2(-3) + 6 = 0$$
$$k = 16$$

The coordinates of the centre are $(-4,\ -3)$.
Radius $= \sqrt{4^2 + 3^2} = 5$

3. B

I. ✓.

$$\left(\frac{m}{2}\right)^2 + \left(-\frac{m}{2}\right)^2 + 2m\left(\frac{m}{2}\right) + 2m\left(-\frac{m}{2}\right) + m^2$$
$$= \frac{3m^2}{2}$$
$$> 0$$

The point $\left(\frac{m}{2}, -\frac{m}{2}\right)$ lies outside C.

II. ✗.

The coordinates of the centre are $(-m, -m)$.

$$\text{Circumference} = 2\pi\sqrt{m^2 + m^2 - m^2}$$
$$= 2\pi\sqrt{m^2}$$

Take $m = -1$, the circumference of C is 2π, which does not equal $2m\pi$.

III. ✓.

The coordinates of the centre are $(-m, -m)$.

4. A

$$C\text{: } x^2 + y^2 - kx - ky + \frac{k^2}{4} = 0$$

I. ✗.

The coordinates of the centre of C are $\left(\frac{k}{2}, \frac{k}{2}\right)$.

II. ✓.

$$\text{Area} = \pi\left(\sqrt{\left(\frac{k}{2}\right)^2 + \left(\frac{k}{2}\right)^2 - \frac{k^2}{4}}\right)^2$$
$$= \frac{\pi k^2}{4}$$

III. ✗.

$$(0)^2 + (0)^2 - k(0) - k(0) + \frac{k^2}{4} = \frac{k^2}{4} > 0$$

The origin lies outside C.

5. A

$C: x^2 + y^2 - 4x - ky - 20 = 0$

The coordinates of the centre are $\left(2,\ \frac{k}{2}\right)$.

$$\sqrt{2^2 + \left(\frac{k}{2}\right)^2 + 20} = 5$$

$$24 + \frac{k^2}{4} = 25$$

$$k^2 = 4$$

$$k = \pm 2$$

Since the centre of C lies in quadrant IV, we have $\frac{k}{2} < 0$.

Thus, $k = -2$.

6. B

The coordinates of G_1 and G_2 are $(0,\ 25)$ and $(12,\ 9)$ respectively.

I. ✓.

$G_1G_2 = \sqrt{(12-0)^2 + (25-9)^2} = 20$

II. ✗.

Slope of $OG_2 = \frac{9-0}{12-0} = \frac{3}{4}$

Slope of $G_1G_2 = \frac{25-9}{0-12} = -\frac{4}{3} = -1 \div \frac{3}{4}$

Thus, $\angle OG_2G_1 = 90° \neq 45°$.

III. ✓.

$$\text{Radius} = \frac{OG_1}{2}$$

$$= \frac{25-0}{2}$$

$$= 12.5$$

7. B

The coordinates of the centre are $(-6,\ 4)$.

$$\text{Area} = \pi\left(\sqrt{6^2 + 4^2 - 9}\right)^2$$

$$= 43\pi$$

8. B

I. ✓.

Let G be the centre of the circle.

Since O lies outside the circle, we have $OG > r$, which is $a > r$.

II. ✗.

G lies in the third quadrant, we have $180° < \theta < 270°$.

III. ✓.

When $180° < \theta < 270°$, we have $\tan\theta > 0$.

9. A

$$x^2 + y^2 + 10x - 6y + \frac{55}{4} = 0$$

I. ✓.

The coordinates of the centre are $(-5,\ 3)$.

II. ✓.

Radius $= \sqrt{5^2 + 3^2 - \frac{55}{4}} = \frac{9}{2}$

Required coordinates $= \left(-5 + \frac{9}{2},\ 3\right)$

$= \left(-\frac{1}{2},\ 3\right)$

III. ✗.

Circumference $= 2\pi\left(\frac{9}{2}\right) = 9\pi \neq 81\pi$

10. D

$$x^2 + y^2 - 6x + 8y + \frac{19}{2} = 0$$

I. ✓.

The coordinates of the centre are $(3,\ -4)$.

II. ✗.

Radius $= \sqrt{3^2 + 4^2 - \frac{19}{2}} = \sqrt{\frac{31}{2}} \neq 9$

III. ✓.

$(0)^2 + (0)^2 - 6(0) + 8(0) + \frac{19}{2} = \frac{19}{2} > 0$

The origin lies outside the circle.

11. D

$x^2 + y^2 - 8x + 6y + \frac{26}{3} = 0$

I. ✓.

The coordinates of the centre are $(4,\ -3)$.

II. ✓.

$$\text{Area} = \pi\left(\sqrt{4^2 + 3^2 - \frac{26}{3}}\right)^2$$
$$= \frac{49\pi}{3}$$
$$> 16\pi$$

III. ✓.

$(0)^2 + (0)^2 - 8(0) + 6(0) + \frac{26}{3} = \frac{26}{3} > 0$

The origin lies outside the circle.

12. C

The coordinates of the centre are $\left(-\frac{k}{2},\ 6\right)$.

Note that the centre and the two given points are collinear.

$$\frac{12-0}{0-8} = \frac{6-0}{-\frac{k}{2}-8}$$
$$k = -8$$

13. D

$x^2 + y^2 + 4x - 3y - \frac{3}{2} = 0$

I. ✗.

The coordinates of A are $\left(-2,\ \frac{3}{2}\right)$.

II. ✓.

$(1)^2 + (2)^2 + 4(1) - 3(2) - \frac{3}{2} = \frac{3}{2} > 0$

B lies outside C.

III. ✓.

Let θ_1 and θ_2 be the inclination of OA and OB.

$$\tan\theta_1 = \frac{\frac{3}{2}-0}{-2-0} \quad \text{and} \quad \tan\theta_2 = \frac{2-0}{1-0}$$
$$\theta_1 \approx 143^\circ \qquad\qquad \theta_2 \approx 63.4^\circ$$

$\angle AOB = \theta_1 - \theta_2 \approx 79.7^\circ < 90^\circ$

$\angle AOB$ is an acute angle.

14. A

C_1: $x^2 + y^2 + 2x - 4y - 4 = 0$
C_2: $x^2 + y^2 - 2x - 4y - \frac{41}{2} = 0$
The coordinates of G_1 and G_2 are $(-1,\ 2)$ and $(1,\ 2)$ respectively.

I. ✓.
$OG_1 = \sqrt{1^2 + 2^2} = \sqrt{5}$
$OG_2 = \sqrt{1^2 + 2^2} = \sqrt{5} = OG_1$
Thus, $\triangle OG_1G_2$ is an isosceles triangle.

II. ✓.
Radius of $C_1 = \sqrt{1^2 + 2^2 + 4} = 3$
Radius of $C_2 = \sqrt{1^2 + 2^2 + \frac{41}{2}} = \sqrt{\frac{51}{2}} > 3$

III. ✗.
$(0)^2 + (0)^2 + 2(0) - 4(0) - 4 = -4 < 0$
O lies inside C_1.
$(0)^2 + (0)^2 - 2(0) - 4(0) - \frac{41}{2} = -\frac{41}{2} < 0$
O lies inside C_2.

15. C

C_1: $x^2 + y^2 - 8x - 6y + 20 = 0$
C_2: $x^2 + y^2 + 6x - 8y + \frac{33}{2} = 0$
The coordinates of G_1 and G_2 are $(4,\ 3)$ and $(-3,\ 4)$ respectively.

I. ✓.
Slope of $G_1O = \frac{3-0}{4-0} = \frac{3}{4}$
Slope of $G_2O = \frac{4-0}{-3-0} = -\frac{4}{3} = -1 \div \frac{3}{4}$
Thus, G_1O is perpendicular to G_2O.

II. ✗.
Area of $C_1 = \pi\left(\sqrt{4^3 + 3^2 - 20}\right)^2 = 5\pi$
Area of $C_2 = \pi\left(\sqrt{3^2 + 4^2 - \frac{33}{2}}\right)^2 = \frac{17\pi}{2} > 5\pi$

III. ✓.
$OG_1 = \sqrt{4^2 + 3^2} = 5$
$OG_2 = \sqrt{3^2 + 4^2} = 5$

16. D

$C: x^2 + y^2 - 3x + y - \frac{13}{2} = 0$

I. ✗.

The coordinates of the centre are $\left(\frac{3}{2}, -\frac{1}{2}\right)$.

II. ✓.

$AB = \sqrt{(2-1)^2 + (1+2)^2} = \sqrt{10}$

Radius $= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \frac{13}{2}} = 3 < \sqrt{10}$

III. ✓.

Denote the centre by G.

Slope of $AB = \frac{1+2}{2-1} = 3$

Slope of $BG = \frac{1+\frac{1}{2}}{2-\frac{3}{2}} = 3$

A, B and G are collinear.

Thus, G lies on the straight line passing through A and B.

17. B

The coordinates of the centre are $(5, -4)$.

Suppose L cuts the y-axis at $A\ (0, k)$.

Note that the line joining the centre and the mid-point of PQ is perpendicular to L.

$$\frac{k+8}{0-4} \times \frac{-4+8}{5-4} = -1$$

$$k = -7$$

The y-intercept of L is -7.

18. D

I. ✓. $G_1\ (-2, 6)$, $G_2\ (2, 4)$. Slope of $OG_2 \times$ slope of $G_1G_2 = \frac{4}{2} \times \frac{6-4}{-2-2} = -1$

II. ✓. Distance between centres $= \sqrt{(2+2)^2 + (6-4)^2} = \sqrt{20}$

Radius of $C_1 = \sqrt{2^2 + 6^2 + 40} = \sqrt{80} = 2\sqrt{20}$; radius of $C_2 = \sqrt{2^2 + 4^2} = \sqrt{20}$

Since distance between centres = difference in radii, the circles touch each other internally.

III. ✓. Area ratio $= \left(\frac{\sqrt{80}}{\sqrt{20}}\right)^2 = 4$

19. B

C: $x^2 + y^2 + 10x - 6y + \dfrac{15}{2} = 0$

I. ✓.

The coordinates of the centre are $(-5,\ 3)$.

II. ✗.

Radius $= \sqrt{5^2 + 3^2 - \dfrac{15}{2}} = \sqrt{\dfrac{53}{2}} \neq 11$

III. ✓.

$(2)^2 + (0)^2 + 10(2) - 6(0) + \dfrac{15}{2} = \dfrac{63}{2} > 0$

The point $(2,\ 0)$ lies outside C.

20. C

I. ✓.

The coordinates of the centre of C are $(24,\ 24)$.

Note that $2(24) + 3(24) - 120 = 0$, L passes through the centre of C.

Thus, L is an axis of symmetry of C.

II. ✗.

$(35)^2 + (45)^2 - 48(35) - 48(45) + 576 = -14 < 0$

The point $(35,\ 45)$ lies inside C.

III. ✓.

Circumference $= 2\pi\sqrt{24^2 + 24^2 - 576} = 48\pi$

END OF PAPER