解	分

REG-2324-MOCK-SET 4-MATH-EP(M2)

建議題解

1. $f'(1) = \lim_{h\to 0} \dfrac{(1+h+1)\ln(1+h) - 0}{h}$ 1M

$= \lim_{h\to 0} \dfrac{h+2}{h}\ln(1+h)$

$= \lim_{h\to 0} (2+h)\ln(1+h)^{\frac{1}{h}}$ 1M+1A

$= 2\ln e$

$= 2$ 1A

2. $(1+x)^m(1-2x)^n = \left[1 + mx + \dfrac{m(m-1)}{2}x^2 + \dots\right]\left[1 - 2nx + \dfrac{n(n-1)}{2}4x^2 + \dots\right]$ 1M

$= 1 + (m-2n)x + \left[\dfrac{m(m-1)}{2} - 2mn + \dfrac{n(n-1)}{2}\right]x^2 + \dots$

$\begin{cases} m - 2n = 0 \\ \dfrac{m(m-1)}{2} - 2mn + 2n(n-1) = -9 \end{cases}$ 1M

$\dfrac{2n(2n-1)}{2} - 4n^2 + 2n(n-1) = -9$ 1M

$9 - 3n = 0$

$n = 3$

當 $n = 3$ 時，$m = 2(3) = 6$。 1A

解	分
3. (a) 當 $n = 1$，	
左式 $= \sum_{k=1}^{1} (-1)^k (2k-1)^2 = (-1)(1)^2 = -1$	
右式 $= \dfrac{(-1)^1(2-1)(2+1)+1}{2} = -1 =$ 左式	
對 $n = 1$，命題為真。	1
假設 $\sum_{k=1}^{p} (-1)^k (2k-1)^2 = \dfrac{(-1)^p(2p-1)(2p+1)+1}{2}$，其中 $p \in \mathbb{Z}^+$。	1
$\sum_{k=1}^{p+1} (-1)^k (2k-1)^2$	
$= \sum_{k=1}^{p} (-1)^k (2k-1)^2 + (-1)^{p+1}(2p+1)^2$	
$= \dfrac{(-1)^p(2p-1)(2p+1)+1}{2} + (-1)^{p+1}(2p+1)^2$	1
$= \dfrac{(-1)^{p+1}(2p+1)[-(2p-1)+2(2p+1)]+1}{2}$	
$= \dfrac{(-1)^{p+1}(2p+1)(2p+3)+1}{2}$	
對 $n = p + 1$，命題為真。	
藉數學歸納法，$\forall n \in \mathbb{Z}^+$，命題為真。	1
(b) $\sum_{k=99}^{200} (-1)^{k+1} (2k-1)^2$	
$= -\sum_{k=1}^{200} (-1)^k (2k-1)^2 + \sum_{k=1}^{98} (-1)^k (2k-1)^2$	1M
$= -\dfrac{(-1)^{200}(400-1)(400+1)+1}{2} + \dfrac{(-1)^{98}(196-1)(196+1)+1}{2}$	
$= -60\,792$	1A
4. (a) $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \dfrac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$	1M
$= \tan\dfrac{x+y}{2}$	1
(b) (i) $\dfrac{\tan\left[2\left(\frac{\theta}{2}-\frac{\pi}{4}\right)\right]}{\tan\left(\frac{\theta}{2}-\frac{\pi}{4}\right)} = \dfrac{\tan\left(\theta-\frac{\pi}{2}\right)}{\tan\frac{\theta-\frac{\pi}{2}}{2}} = \dfrac{-\frac{1}{\tan\theta}}{\frac{\sin\theta+\sin\frac{-\pi}{2}}{\cos\theta+\cos\frac{-\pi}{2}}}$	1M
$= -\dfrac{\cos^2\theta}{\sin\theta(\sin\theta-1)} = \dfrac{(\sin\theta+1)(\sin\theta-1)}{\sin\theta(\sin\theta-1)}$	1M
$= \dfrac{\sin\theta+1}{\sin\theta} = \csc\theta + 1$	1
(ii) $\dfrac{\tan\left[2\left(\frac{\theta}{2}-\frac{\pi}{4}\right)\right]}{\tan\left(\frac{\theta}{2}-\frac{\pi}{4}\right)} = 3$	
$\csc\theta + 1 = 3$	
$\sin\theta = \dfrac{1}{2}$	1M
$\theta = \dfrac{\pi}{6}$ 或 $\dfrac{5\pi}{6}$	1A

解	分
5. (a) $\ell^2 = (OC + OP\cos\theta)^2 + (OB + OP\sin\theta)^2$	
$= (1+\cos\theta)^2 + (1+\sin\theta)^2$	1A+1A
$= 2 + (\sin^2\theta + \cos^2\theta) + 2\sin\theta + 2\cos\theta$	
$= 3 + 2(\sin\theta + \cos\theta)$	1
(b) $\frac{d\ell^2}{dt} = 2(\cos\theta - \sin\theta)\frac{d\theta}{dt}$	
$2\ell\frac{d\ell}{dt} = 2(\cos\theta - \sin\theta)\frac{d\theta}{dt}$	1M
$\frac{d\ell}{dt} = \frac{\cos\theta - \sin\theta}{\ell}\frac{d\theta}{dt}$	1A
當 $\theta = \frac{\pi}{3}$，	
$\ell^2 = 3 + 2\left(\sin\frac{\pi}{3} + \cos\frac{\pi}{3}\right)$	
$\ell = \sqrt{4+\sqrt{3}}$	1A
$\frac{d\ell}{dt} = \frac{\cos\frac{\pi}{3} - \sin\frac{\pi}{3}}{\ell}(3)$	
≈ -0.46	
所求之率為 -0.46 cm/s。	1A
6. (a) $P(P^3 + P^2 + P + I) = P(\mathbf{0})$	1M
$P^4 + (P^3 + P^2 + P) = \mathbf{0}$	
$P^4 + (-I) = \mathbf{0}$	1M
$P^4 = I$	1
(b) $A^2 = \begin{pmatrix} 3 & 2 \\ -5 & -3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -5 & -3 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	1A
$A^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -5 & -3 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ 5 & 3 \end{pmatrix}$	
$A^3 + A^2 + A + I = \begin{pmatrix} -3 & -2 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ -5 & -3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	
$= \mathbf{0}$	1A
$A^{2021} = (A^4)^{505}A$	
$= I^{505}A$	1M
$= A$	
$= \begin{pmatrix} 3 & 2 \\ -5 & -3 \end{pmatrix}$	1A

解	分
7. (a) $\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$	
$\overrightarrow{AC} = 4\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$	
$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & -1 \\ 4 & 2 & -5 \end{vmatrix}$	
$= -3\mathbf{i} + 11\mathbf{j} + 2\mathbf{k}$	1A
(b) $\overrightarrow{AM} = (\alpha - 3)\mathbf{i} - 22\mathbf{j} + (\beta + 2)\mathbf{k}$	
$\overrightarrow{AM}$ 平行於 $\overrightarrow{AB} \times \overrightarrow{AC}$。	
可得 $\dfrac{\alpha - 3}{-3} = \dfrac{-22}{11} = \dfrac{\beta + 2}{2}$。	1M
求解後，可得 $\alpha = 9$ 及 $\beta = -6$。	1A+1A
(c) 可得 $\overrightarrow{AP} = \dfrac{t}{1+t}\overrightarrow{AB}$ 及 $\overrightarrow{AQ} = \dfrac{1}{1+t}\overrightarrow{AC}$。	
$\dfrac{3}{16} = \dfrac{MAPQ \text{ 的體積}}{MABC \text{ 的體積}}$	
$= \dfrac{\frac{1}{3}(AM)(\triangle APQ \text{ 的面積})}{\frac{1}{3}(AM)(\triangle ABC \text{ 的面積})}$	
$= \dfrac{\left\lvert \overrightarrow{AP} \times \overrightarrow{AQ} \right\rvert}{\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert}$	1M
$= \dfrac{\frac{t}{(1+t)^2}\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert}{\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert}$	
$= \dfrac{t}{(1+t)^2}$	1A
$3(1+t)^2 = 16t$	
$3t^2 - 10t + 3 = 0$	
$t = 3$ 或 $\dfrac{1}{3}$	1A

解	分
8. (a) $g'(x) = e^x \cos x - e^x \sin x$	1M
當 $g'(x) = 0$，	
$e^x(\cos x - \sin x) = 0$	
$\tan x = 1$	
$x = \frac{\pi}{4}$	
<table><tr><td>x</td><td>$0 < x < \frac{\pi}{4}$</td><td>$\frac{\pi}{4} < x < \pi$</td></tr><tr><td>$g'(x)$</td><td>$+$</td><td>$-$</td></tr></table>	1M
因此，G 只在 $x = \frac{\pi}{4}$ 時達至極大值，且 G 只有一個極大點。	1
(b) $\int e^{2x} \cos 2x \, dx = \frac{1}{2}e^{2x} \cos 2x + \int e^{2x} \sin 2x \, dx$	1M
$= \frac{1}{2}e^{2x} \cos 2x + \frac{1}{2}e^{2x} \sin 2x - \int e^{2x} \cos 2x \, dx$	1M
$2\int e^{2x} \cos 2x \, dx = \frac{1}{2}e^{2x} \cos 2x + \frac{1}{2}e^{2x} \sin 2x +$ 常數	
$\int e^{2x} \cos 2x \, dx = \frac{1}{4}e^{2x} \cos 2x + \frac{1}{4}e^{2x} \sin 2x +$ 常數	1A
(c) 體積 $= \pi \int_0^{\frac{\pi}{4}} e^{2x} \cos^2 x \, dx$	1M
$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} e^{2x}(\cos 2x + 1) \, dx$	
$= \frac{\pi}{2}\left[\frac{1}{4}e^{2x} \cos 2x + \frac{1}{4}e^{2x} \sin 2x\right]_0^{\frac{\pi}{4}} + \frac{\pi}{2}\left[\frac{1}{2}e^{2x}\right]_0^{\frac{\pi}{4}}$	
$= \frac{3\pi}{8}(e^{\frac{\pi}{2}} - 1)$	1A

解	分
9. (a) 垂直漸近線為 $x = -\dfrac{1}{2}$。	1A
$f(x) = \dfrac{x}{2} - \dfrac{9}{4} + \dfrac{9}{4(2x+1)}$	1M
因此，斜漸近線為 $y = \dfrac{x}{2} - \dfrac{9}{4}$。	1A
(b) $f'(x) = \dfrac{1}{2} - \dfrac{9}{2(2x+1)^2}$	1M+1A
(c) $f''(x) = \dfrac{18}{(2x+1)^3}$.	
當 $f'(x) = 0$，	
$\dfrac{1}{2} - \dfrac{9}{2(2x+1)^2} = 0$	
$(2x+1)^2 = 9$	
$x = -2$ 或 1	1A
當 $x = -2$，$f(-3) = \dfrac{-2}{2} - \dfrac{9}{4} + \dfrac{9}{4(-4+1)} = -4$ 及 $f''(-2) = \dfrac{18}{(-4+1)^3} = -\dfrac{2}{3} < 0$。	1M
當 $x = 1$，$f(1) = -1$ 及 $f''(1) = \dfrac{2}{3} > 0$。	
極大點為 $(-2,\ -4)$ 及極小點為 $(1,\ -1)$。	1A+1A
(d) $\dfrac{x^2 - 4x}{2x+1} = 0$	
$x(x-4) = 0$	
$x = 0$ 或 4	1A
所求面積 $= \displaystyle\int_0^4 -\left(\frac{x^2 - 4x}{2x+1}\right) dx$	1M
$= \displaystyle\int_0^4 \left[-\frac{x}{2} + \frac{9}{4} - \frac{9}{4(2x+1)}\right]$	
$= \left[-\dfrac{x^2}{4} + \dfrac{9x}{4} - \dfrac{9}{8}\ln\lvert 2x+1\rvert\right]_0^4$	1M
$= 5 - \dfrac{9}{8}\ln 9$	1A

解	分

10. (a) (i) (1) $\begin{vmatrix} h & -2 & 2h-1 \\ 3 & -1 & 5 \\ 1 & -3 & 3-h \end{vmatrix} = \begin{vmatrix} h-6 & 0 & 2h-11 \\ 3 & -1 & 5 \\ -8 & 0 & -h-12 \end{vmatrix}$

$= (h-6)(-h-12) + 8(2h-11)$

$= h^2 - 10h + 16$ 1A

$= (h-2)(h-8)$

(E) 有唯一解當且僅當 $\begin{vmatrix} h & -2 & 2h-1 \\ 3 & -1 & 5 \\ 1 & -3 & 3-h \end{vmatrix} \neq 0$。 1M

故此，$h \neq 2$ 及 $h \neq 8$。 1

(2) $x = \dfrac{\begin{vmatrix} -2 & -2 & 2h-1 \\ -2k & -1 & 5 \\ k+1 & -3 & 3-h \end{vmatrix}}{(h-2)(h-8)} = \dfrac{18hk - 29k - 35}{(h-2)(h-8)}$ 1M+1A

$y = \dfrac{\begin{vmatrix} h & -2 & 2h-1 \\ 3 & -2k & 5 \\ 1 & k+1 & 3-h \end{vmatrix}}{(h-2)(h-8)} = \dfrac{2h^2k - hk - 5h - 5k + 5}{(h-2)(h-8)}$

$z = \dfrac{\begin{vmatrix} h & -2 & -2 \\ 3 & -1 & -2k \\ 1 & -3 & k+1 \end{vmatrix}}{(h-2)(h-8)} = \dfrac{-7hk - h + 10k + 22}{(h-2)(h-8)}$ 1A

(ii) (1) (E) 的增廣矩陣為

$$\left(\begin{array}{ccc|c} 2 & -2 & 3 & -2 \\ 3 & -1 & 5 & -2k \\ 1 & -3 & 1 & k+1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & \frac{3}{2} & -1 \\ 0 & 2 & \frac{1}{2} & 3-2k \\ 0 & -2 & -\frac{1}{2} & k+2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & \frac{3}{2} & -1 \\ 0 & 2 & \frac{1}{2} & 3-2k \\ 0 & 0 & 0 & 5-k \end{array}\right)$$ 1M

由於 (E) 有解，$k = 5$。 1A

(2) (E) 的增廣矩陣為

$$\left(\begin{array}{ccc|c} 1 & -1 & \frac{3}{2} & -1 \\ 0 & 2 & \frac{1}{2} & -7 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

設 $z = t$，其中 $t \in \mathbb{R}$。則 $y = \dfrac{-t-14}{4}$ 及 $x = \dfrac{-7t-18}{4}$。

解集為 $\left\{\left(\dfrac{-7t-18}{4}, \dfrac{-t-14}{4}, t\right) : t \in \mathbb{R}\right\}$。 1A

(b) 方程組等價於 (E) 當 $h = 2$ 及 $k = 5$。

解	分
解為 $\left\{\left(\dfrac{-7t-18}{4},\ \dfrac{-t-14}{4},\ t\right) : t \in \mathbb{R}\right\}$。	1M
$2x + y^2 + z < 1$	
$\dfrac{-7t-18}{2} + \left(\dfrac{-t-14}{4}\right)^2 + t < 1$	
$\dfrac{t^2}{16} - \dfrac{3t}{4} + \dfrac{9}{4} < 0$	
$\dfrac{1}{6}(t-6)^2 < 0$	1M
因對所有實數值 t，$\dfrac{1}{6}(t-6)^2$ 為非負數，該不等式無解。	
因此，方程組沒有實根滿足 $2x + y^2 + z < 1$。	1A
11. (a) $\dfrac{d}{d\theta}\ln(\tan\theta) = \dfrac{\sec^2\theta}{\tan\theta} = \dfrac{2}{2\sin\theta\cos\theta} = 2\csc 2\theta$	1M
$\displaystyle\int \csc 2\theta \, d\theta = \frac{1}{2}\ln(\tan\theta) +$ 常數	1A+1
(b) (i) $\sqrt{2}\sin\left(\dfrac{\pi}{4} + \theta\right) = \sqrt{2}\left[\sin\dfrac{\pi}{4}\cos\theta + \cos\dfrac{\pi}{4}\sin\theta\right]$	1M
$= \sin\theta + \cos\theta$	1
(ii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sin\theta + \cos\theta} = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \csc\left(\frac{\pi}{4} + \theta\right) d\theta$	
設 $u = \dfrac{\pi}{8} + \dfrac{\theta}{2}$。則 $du = \dfrac{1}{2}d\theta$。	
$\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sin\theta + \cos\theta} = \sqrt{2}\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \csc 2u \, du$	1M
$= \dfrac{1}{\sqrt{2}}\Big[\ln(\tan u)\Big]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$	
$= \dfrac{1}{\sqrt{2}}\left[\ln\left(\tan\left(\dfrac{\pi}{2} - \dfrac{\pi}{8}\right)\right) - \ln\left(\tan\dfrac{\pi}{8}\right)\right]$	1M
$= -\sqrt{2}\ln\tan\dfrac{\pi}{8}$	
$= -\sqrt{2}\ln\left(\sqrt{2} - 1\right)$	1A
(c) 設 $\phi = \dfrac{\pi}{2} - \theta$。則 $d\phi = -d\theta$	1M
$\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta = -\int_{\frac{\pi}{2}}^{0} \frac{\sin\left(\frac{\pi}{2} - \phi\right)}{\sin\left(\frac{\pi}{2} - \phi\right) + \cos\left(\frac{\pi}{2} - \phi\right)} d\phi$	
$\displaystyle= \int_0^{\frac{\pi}{2}} \frac{\cos\phi}{\sin\phi + \cos\phi} d\phi = \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta$	1
(d) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta = \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta + \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta\right]$	1M
$= \dfrac{1}{2}\Big[\theta\Big]_0^{\frac{\pi}{2}} = \dfrac{\pi}{4}$	
$\displaystyle\int_0^{\frac{\pi}{2}} \frac{2\sin\theta + 5}{\sin\theta + \cos\theta} d\theta = 2\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta + 5\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sin\theta + \cos\theta}$	1M
$= \dfrac{\pi}{2} - 5\sqrt{2}\ln\left(\sqrt{2} - 1\right)$	1A

解	分
12. (a) (i) $\overrightarrow{OA}\cdot\overrightarrow{OC} = c\left(\frac{\mathbf{a}\cdot\mathbf{a}}{\lvert\mathbf{a}\rvert}+\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{b}\rvert}\right)$	1M
$\lvert\mathbf{a}\rvert\lvert\mathbf{c}\rvert\cos\angle AOC = c\lvert\mathbf{a}\rvert\left(1+\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)$	1M
$\angle AOC = \frac{c}{\lvert\mathbf{c}\rvert}\left(1+\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)$	
$\overrightarrow{OB}\cdot\overrightarrow{OC} = c\left(\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert}+\lvert\mathbf{b}\rvert\right)$	
$\lvert\mathbf{b}\rvert\lvert\mathbf{c}\rvert\cos\angle BOC = c\lvert\mathbf{b}\rvert\left(\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}+1\right)$	
$\cos\angle BOC = \frac{c}{\lvert\mathbf{c}\rvert}\left(1+\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)$	
$= \cos\angle AOC$	
由於 $0 \le \angle AOC \le \pi$ 及 $0 \le \angle BOC \le \pi$，可得 $\angle AOC = \angle COB$。	1
(ii) $\overrightarrow{BC} = c\left(\frac{\mathbf{a}}{\lvert\mathbf{a}\rvert}+\frac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right)-\mathbf{b}$	1M
$= \frac{\lvert\mathbf{b}\rvert\mathbf{a}+\lvert\mathbf{a}\rvert\mathbf{b}}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}-\mathbf{b}$	
$= \frac{\lvert\mathbf{b}\rvert\mathbf{a}-\lvert\mathbf{b}\rvert\mathbf{b}-\lvert\mathbf{a}-\mathbf{b}\rvert\mathbf{b}}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}$	
$= \frac{\lvert\mathbf{a}-\mathbf{b}\rvert\lvert-\mathbf{b}\rvert}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}\left(\frac{\mathbf{a}-\mathbf{b}}{\lvert\mathbf{a}-\mathbf{b}\rvert}+\frac{-\mathbf{b}}{\lvert-\mathbf{b}\rvert}\right)$	
$= l\left(\frac{\mathbf{a}-\mathbf{b}}{\lvert\mathbf{a}-\mathbf{b}\rvert}+\frac{-\mathbf{b}}{\lvert-\mathbf{b}\rvert}\right)$ 其中 $l = \frac{\lvert\mathbf{a}-\mathbf{b}\rvert\lvert-\mathbf{b}\rvert}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}$	1A
留意 $\overrightarrow{BA} = \mathbf{a}-\mathbf{b}$ 及 $\overrightarrow{BO} = -\mathbf{b}$。利用 (a)(i)，可得 $\angle ABC = \angle CBO$。	1M
由於 $\angle ABC = \angle CBO$ 及 $\angle AOC = \angle COB$，C 為 $\triangle OAB$ 的內心。	1
(b) $\overrightarrow{PQ} = (3\mathbf{i}-5\mathbf{k})-(\mathbf{i}+\mathbf{j}-3\mathbf{k}) = 2\mathbf{i}-\mathbf{j}-2\mathbf{k}$ 及 $\overrightarrow{PR} = 4\mathbf{i}-6\mathbf{j}+12\mathbf{k}$。	1M
設 $\mathbf{a} = \overrightarrow{PQ}$ 及 $\mathbf{b} = \overrightarrow{PR}$。	
$\mathbf{a}-\mathbf{b} = -2\mathbf{i}+5\mathbf{j}-14\mathbf{k}$、$\lvert\mathbf{a}\rvert = \sqrt{2^2+1^2+2^2} = 3$、$\lvert\mathbf{b}\rvert = 14$ 及 $\lvert\mathbf{a}-\mathbf{b}\rvert = 15$。	1M
$\overrightarrow{PI} = \frac{(3)(14)}{3+14+15}\left(\frac{2\mathbf{i}-\mathbf{j}-2\mathbf{k}}{3}+\frac{4\mathbf{i}-6\mathbf{j}+12\mathbf{k}}{14}\right)$	1M
$= \frac{5}{4}\mathbf{i}-\mathbf{j}+\frac{1}{4}\mathbf{k}$	1A
$\overrightarrow{OI} = \overrightarrow{OP}+\overrightarrow{PI} = \frac{9}{4}\mathbf{i}-\frac{11}{4}\mathbf{k}$	1A