

M2REG-MAD-2425-ASM-SET 2-MATH

建議題解

$$\begin{aligned}
 1. \quad \left| \begin{array}{cc} x-1 & x+\sqrt{2} \\ x-\sqrt{2} & 2x \end{array} \right| &= 2x(x-1) - (x+\sqrt{2})(x-\sqrt{2}) & 1M \\
 &= 2x^2 - 2x - (x^2 - 2) \\
 &= x^2 - 2x + 2 \\
 &= (x-1)^2 + 1 & 1M \\
 &> 0 & 1
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \left| \begin{array}{cc} 2 & \frac{1}{x} \\ -1 & x \end{array} \right| &= 3 \\
 2x + \frac{1}{x} &= 3 & 1M \\
 2x^2 - 3x + 1 &= 0 & 1M \\
 x &= \frac{1}{2} \quad \text{或} \quad 1 & 1A
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \left| \begin{array}{cc} \cos x & -\tan x \\ \sin x \cos x + \cos^2 x & \cos x - \sin x \end{array} \right| \\
 = \cos x(\cos x - \sin x) + \tan x(\sin x \cos x + \cos^2 x) & 1M \\
 = \cos^2 x - \sin x \cos x + \sin^2 x + \sin x \cos x \\
 = 1 & 1
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \left| \begin{array}{cc} (-1)^n & -1 \\ 1 & (-1)^n \end{array} \right| &= (-1)^{2n} - (-1) & 1M \\
 &= 1 + 1 \\
 &= 2 & 1A
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \left| \begin{array}{cc} \sin \theta & 1 \\ \sin 2\theta & \cos \theta \end{array} \right| &= \sin \theta \cos \theta - \sin 2\theta & 1M \\
 &= \sin \theta \cos \theta - 2 \sin \theta \cos \theta \\
 &= -\sin \theta \cos \theta & 1A
 \end{aligned}$$

$$\begin{aligned}
 \left| \begin{array}{cc} (\sin \theta - \cos \theta)^2 & \sin 2\theta - 2 \\ -\frac{1}{2} & 1 \end{array} \right| &= (\sin \theta - \cos \theta)^2 + \frac{1}{2}(\sin 2\theta - 2) \\
 &= \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta + \frac{1}{2}(2 \sin \theta \cos \theta) - 1 \\
 &= -\sin \theta \cos \theta
 \end{aligned}$$

$$\text{因此, } \left| \begin{array}{cc} \sin \theta & 1 \\ \sin 2\theta & \cos \theta \end{array} \right| = \left| \begin{array}{cc} (\sin \theta - \cos \theta)^2 & \sin 2\theta - 2 \\ -\frac{1}{2} & 1 \end{array} \right|. \quad 1$$

$$\begin{aligned}
6. \quad & \begin{vmatrix} x & x+1 & x(1+yz) \\ y & y+1 & y(1+zx) \\ z & z+1 & z(1+xy) \end{vmatrix} \\
&= x(1+yz) \begin{vmatrix} y & y+1 \\ z & z+1 \end{vmatrix} - y(1+zx) \begin{vmatrix} x & x+1 \\ z & z+1 \end{vmatrix} + z(1+xy) \begin{vmatrix} x & x+1 \\ y & y+1 \end{vmatrix} & 1M \\
&= x(1+yz)[y(z+1) - z(y+1)] - y(1+zx)[x(z+1) - z(x+1)] \\
&\quad + z(1+xy)[x(y+1) - y(x+1)] \\
&= x(1+yz)(y-z) - y(1+zx)(x-z) + z(1+xy)(x-y) \\
&= xy - xz + xy^2z - xyz^2 - xy + yz - x^2yz + xyz^2 + xz - yz + x^2yz - xy^2z \\
&= 0 & 1A
\end{aligned}$$

$$\begin{aligned}
7. \quad & \begin{vmatrix} x & 0 & x \\ y & x & x \\ x & y & y \end{vmatrix} = x \begin{vmatrix} x & x \\ y & y \end{vmatrix} + x \begin{vmatrix} y & x \\ x & y \end{vmatrix} & 1M \\
&= x(xy - xy) + x(y^2 - x^2) \\
&= x(y + x)(y - x) & 1A
\end{aligned}$$

$$\begin{aligned}
8. \quad & \begin{vmatrix} \cos \theta & -\sin \theta & \tan 2\theta \\ \sin \theta & \cos \theta & -1 \\ \cos \theta & \sin \theta & 1 \end{vmatrix} \\
&= \tan 2\theta \begin{vmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{vmatrix} - (-1) \begin{vmatrix} \cos \theta & -\sin \theta \\ \cos \theta & \sin \theta \end{vmatrix} + (1) \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} & 1M \\
&= \tan 2\theta(\sin^2 \theta - \cos^2 \theta) + (\sin \theta \cos \theta + \sin \theta \cos \theta) + (\cos^2 \theta + \sin^2 \theta) \\
&= \tan 2\theta(-\cos 2\theta) + \sin 2\theta + 1 & 1M \\
&= -\sin 2\theta + \sin 2\theta + 1 \\
&= 1 & 1A
\end{aligned}$$

$$\begin{aligned}
9. \quad (a) \quad & 0 = \begin{vmatrix} 8-x^2 & 0 & 4-x \\ 4-x & 1 & x^2 \\ 2-3x & 0 & x-4 \end{vmatrix} \\
&= \begin{vmatrix} 8-x^2 & 4-x \\ 2-3x & x-4 \end{vmatrix} & 1M \\
&= (8-x^2)(x-4) - (4-x)(2-3x) \\
&= (x-4)(-x^2-3x+10) & 1M \\
&= -(x-4)(x-2)(x+5) \\
&x = 4 \quad \text{或} \quad 2 \quad \text{或} \quad -5 & 1A
\end{aligned}$$

$$\begin{aligned}
10. \quad 0 &= \begin{vmatrix} x-1 & -1 & x+1 \\ 2x+1 & 1 & x \\ -1 & 2 & 2x \end{vmatrix} \\
&= (x-1) \begin{vmatrix} 1 & x \\ 2 & 2x \end{vmatrix} - (-1) \begin{vmatrix} 2x+1 & x \\ -1 & 2x \end{vmatrix} + (x+1) \begin{vmatrix} 2x+1 & 1 \\ -1 & 2 \end{vmatrix} & 1M \\
&= (x-1)(2x-2x) + [2x(2x+1) + x] + (x+1)[2(2x+1) + 1] \\
&= 8x^2 + 10x + 3 & 1A \\
x &= -\frac{3}{4} \quad \text{或} \quad -\frac{1}{2} & 1A
\end{aligned}$$

$$\begin{aligned}
11. \quad (a) \quad \det B &= \begin{vmatrix} 1 & x & x+1 \\ 1 & x-1 & x+2 \\ 1 & 2x & x^2 \end{vmatrix} \\
&= \begin{vmatrix} x-1 & x+2 \\ 2x & x^2 \end{vmatrix} - \begin{vmatrix} x & x+1 \\ 2x & x^2 \end{vmatrix} + \begin{vmatrix} x & x+1 \\ x-1 & x+2 \end{vmatrix} & 1M \\
&= [x^2(x-1) - 2x(x+2)] - [x^3 - 2x(x+1)] + [x(x+2) - (x+1)(x-1)] \\
&= 1 - x^2 \\
&= (1+x)(1-x) & 1A \\
(b) \quad \det B &= 0 \\
1 - x^2 &= 0 \\
x &= \pm 1 & 1A
\end{aligned}$$

$$\begin{aligned}
12. \quad (a) \quad M + N &= \begin{pmatrix} 6 & 2 & k+1 \\ k-1 & 3 & k \\ k & 1 & -1 \end{pmatrix} & 1A \\
(b) \quad -14 &= \begin{vmatrix} 6 & 2 & k+1 \\ k-1 & 3 & k \\ k & 1 & -1 \end{vmatrix} \\
&= 6 \begin{vmatrix} 3 & k \\ 1 & -1 \end{vmatrix} - 2 \begin{vmatrix} k-1 & k \\ k & -1 \end{vmatrix} + (k+1) \begin{vmatrix} k-1 & 3 \\ k & 1 \end{vmatrix} & 1M \\
&= 6(-3-k) - 2(-k+1-k^2) + (k+1)(k-1-3k) \\
&= -7k - 21 \\
k &= -1 & 1A
\end{aligned}$$

$$\begin{aligned}
 13. \quad 0 &= \begin{vmatrix} x & x+1 & 1 \\ x-1 & 2 & -3 \\ 1 & -1 & 4 \end{vmatrix} \\
 &= (1) \begin{vmatrix} x-1 & 2 \\ 1 & -1 \end{vmatrix} - (-3) \begin{vmatrix} x & x+1 \\ 1 & -1 \end{vmatrix} + (4) \begin{vmatrix} x & x+1 \\ x-1 & 2 \end{vmatrix} & 1M \\
 &= [(1-x)-2] + 3[-x-(x+1)] + 4[2x-(x+1)(x-1)] \\
 &= -4x^2 + x & 1A \\
 x &= 0 \quad \text{或} \quad \frac{1}{4} & 1A
 \end{aligned}$$

$$\begin{aligned}
 14. \quad (a) \quad \det(A^2) &= \begin{vmatrix} 8 & 6 \\ -2 & 3 \end{vmatrix} \\
 &= 36 & 1M \\
 (\det A)^2 &= 36 & 1M \\
 \det A &= \pm 6 & 1A \\
 (b) \quad \det B &= \begin{vmatrix} b & 1 \\ 2b & b \end{vmatrix} \\
 &= b^2 - 2b \\
 18 &= \det(AB) \\
 &= (\det A)(\det B) & 1M \\
 &= \pm 6(b^2 - 2b) \\
 6b^2 - 12b &= 18 & \text{或} & 6b^2 - 12b = -18 & 1M \\
 6b^2 - 12b - 18 &= 0 & 6b^2 - 12b + 18 = 0 \quad (\text{捨去}) \\
 b &= -1 \quad \text{或} \quad 3 & 1A
 \end{aligned}$$

$$15. \quad (a) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (1) \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - (1) \begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + (1) \begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix} \quad 1M$$

$$\begin{aligned} &= (bc^2 - b^2c) - (ac^2 - a^2c) + (ab^2 - a^2b) \\ &= c^2(b - a) - c(b^2 - a^2) + ab(b - a) \\ &= (a - b)(-c^2) + c(a - b)(a + b) - ab(a - b) \end{aligned}$$

$$= (a - b)[-c^2 + c(a + b) - ab] \quad 1M$$

$$= (a - b)[c(b - c) - a(b - c)]$$

$$= (a - b)(b - c)(c - a) \quad 1$$

$$(b) \quad 20 = \begin{vmatrix} 1 & x-1 & (x-1)^2 \\ 1 & 3 & 9 \\ 1 & x-5 & (x-5)^2 \end{vmatrix}$$

$$20 = [(x-1) - 3][3 - (x-5)][(x-5) - (x-1)] \quad 1M$$

$$20 = (x-4)(8-x)(-4)$$

$$0 = -x^2 + 12x - 27 \quad 1M$$

$$x = 3 \quad \text{或} \quad 9 \quad 1A$$

$$16. \quad (a) \quad P(x) = -2(x + \lambda)^2 - (x + \lambda)^3 - 4 \quad 1M$$

考慮 x 的係數。

$$-2(2\lambda) - C_1^3 \lambda^2 = 1 \quad 1M$$

$$-3\lambda^2 - 4\lambda - 1 = 0$$

$$\lambda = -1 \quad \text{或} \quad -\frac{1}{3} \quad 1A$$

$$(b) \quad P'(x) = -4(x + \lambda) - 3(x + \lambda)^2 \quad 1A$$

$$\text{當 } \lambda = -1 \text{ 時, } P'(1) = -4(1 - 1) - 3(1 - 1)^2 = 0. \quad 1A$$

$$\text{當 } \lambda = -\frac{1}{3} \text{ 時, } P'(1) = -4\left(1 - \frac{1}{3}\right) - 3\left(1 - \frac{1}{3}\right)^2 = -4. \quad 1A$$