

DAYM2-DIFF-2425-ASM-SET 1-MATH

建議題解

$$1. \frac{dy}{dx} = \frac{4(2x^2 + x) - (4x - 1)(4x + 1)}{(2x^2 + x)^2} \quad 1M$$

$$= \frac{-8x^2 + 4x + 1}{(2x^2 + x)^2} \quad 1A$$

$$2. \frac{dy}{dx} = -x^{-2} - 4(-2x^{-3}) + 9(-3x^{-4})$$

$$= -x^{-2} + 8x^{-3} - 27x^{-4} \quad 1A$$

$$3. \quad (a) \quad x^2 + y^2 = 30$$

$$2x + 2y \frac{dy}{dx} = 0 \quad 1M$$

$$\frac{dy}{dx} = -\frac{x}{y} \quad 1A$$

$$\left. \frac{dy}{dx} \right|_{(p, q)} = \frac{-p}{q} = -\frac{1}{2}$$

$$q = 2p \quad 1$$

$$(b) \quad p^2 + (2p)^2 = 30 \quad 1M$$

$$p^2 = 6$$

$$p = \pm\sqrt{6}$$

因此， $(p, q) = (\sqrt{6}, 2\sqrt{6})$ 或 $(-\sqrt{6}, -2\sqrt{6})$ 。 1A+1A

$$4. \frac{dy}{dx} = 3(2x + 1)^2(2)(x - 4)^{\frac{1}{5}} + (2x + 1)^3 \left(\frac{1}{5} \right) (x - 4)^{-\frac{4}{5}} \quad 1M$$

$$= \frac{1}{5} (2x + 1)^2 (x - 4)^{-\frac{4}{5}} [30(x - 4) + (2x + 1)]$$

$$= \frac{(2x + 1)^2 (32x - 119)}{5(x - 4)^{\frac{4}{5}}} \quad 1A$$

$$5. \quad y = \frac{1}{x} - \sqrt{9x^3} + \sqrt[4]{x^5} + 4x^{\frac{3}{7}}$$

$$= x^{-1} - 3x^{\frac{3}{2}} + x^{\frac{5}{4}} + 4x^{\frac{3}{7}} \quad 1M$$

$$\frac{dy}{dx} = -x^{-2} - \frac{9}{2}x^{\frac{1}{2}} + \frac{5}{4}x^{\frac{1}{4}} + \frac{12}{7}x^{-\frac{4}{7}} \quad 1A$$

$$\begin{aligned}
6. \quad f(x) &= \sqrt{(x-4)^3(x^2+4x+16)^3} \\
&= \sqrt{[(x-4)(x^2+4x+16)]^3} \\
&= (x^3-64)^{\frac{3}{2}} \\
f'(x) &= \frac{3}{2}(x^3-64)^{\frac{1}{2}}(3x^2) & 1M \\
&= \frac{9}{2}x^2(x^3-64)^{\frac{1}{2}} & 1A \\
18a^2 &= \frac{9}{2}a^2(a^3-64)^{\frac{1}{2}} & 1M \\
\frac{9}{2}a^2[(a^3-64)^{\frac{1}{2}}-4] &= 0 \\
(a^3-64)^{\frac{1}{2}} &= 4 \quad \text{或} \quad a^2 = 0 \quad (\text{捨去}) \\
a^3 &= 80 \\
a &= 2\sqrt[3]{10} & 1A
\end{aligned}$$

$$\begin{aligned}
7. \quad y &= \frac{x^2 \cdot \sqrt{x^3} - 3x\sqrt{x} + x^{\frac{9}{8}}}{\sqrt[4]{x^{11}}} \\
&= x^{\frac{3}{4}} - 3x^{-\frac{5}{4}} + x^{-\frac{13}{8}} & 1M \\
\frac{dy}{dx} &= \frac{3}{4}x^{-\frac{1}{4}} + \frac{15}{4}x^{-\frac{9}{4}} - \frac{13}{8}x^{-\frac{21}{8}} & 1A
\end{aligned}$$

$$\begin{aligned}
8. \quad (a) \quad \frac{d}{dy}(\sqrt{x}) &= \frac{d}{dx} \left(y^2 - 5\sqrt{y} - \frac{12}{y} \right) \\
\frac{1}{2}x^{-\frac{1}{2}} \frac{dx}{dy} &= 2y - \frac{5}{2}y^{-\frac{1}{2}} + 12y^{-2} & 1M \\
\frac{dx}{dy} &= x^{\frac{1}{2}}(4y - 5y^{-\frac{1}{2}} + 24y^{-2}) & 1A \\
(b) \quad \frac{dy}{dx} &= \frac{1}{\left(\frac{dx}{dy}\right)} \\
&= \frac{1}{x^{\frac{1}{2}}(4y^2 - 5y^{-\frac{1}{2}} + 24y^{-2})} & 1A \\
\left. \frac{dy}{dx} \right|_{(9, 4)} &= \frac{1}{9^{\frac{1}{2}} [4(4)^2 - 5(4)^{-\frac{1}{2}} + 24(4)^{-2}]} \\
&= \frac{1}{45} & 1A
\end{aligned}$$

$$\begin{aligned}
9. \quad \frac{dy}{dx} &= \frac{4(1-5x) - (2+4x)(-5)}{(1-5x)^2} & 1M \\
&= \frac{14}{(1-5x)^2} & 1A
\end{aligned}$$

10. $y = (2x^3 - 3x)^{\frac{2}{3}}(x^2 - 1)^{\frac{1}{2}}$
- $$\frac{dy}{dx} = \frac{3}{2}(2x^3 - 3x)^{-\frac{1}{3}}(6x^2 - 3)(x^2 - 1)^{\frac{1}{2}} + (2x^3 - 3x)^{\frac{2}{3}}\left(\frac{1}{2}\right)(x^2 - 1)^{-\frac{1}{2}}(2x)$$
- $$= (2x^3 - 3x)^{-\frac{1}{3}}(x^2 - 1)^{-\frac{1}{2}}[2(x^2 - 1)(2x^2 - 1) + x(2x^3 - 3x)]$$
- $$= \frac{6x^4 - 9x^2 + 2}{(x^2 - 1)^{\frac{1}{2}}(2x^3 - 3x)^{\frac{1}{3}}}$$
11. $y = 4x^3 \cdot \sqrt{x} + 9x^2 \cdot \sqrt[3]{x^4} - 5x \cdot \sqrt[5]{x^2}$
- $$= 4x^{\frac{7}{2}} + 9x^{\frac{10}{3}} - 5x^{\frac{7}{5}}$$
- $$\frac{dy}{dx} = 14x^{\frac{5}{2}} + 30x^{\frac{7}{3}} - 7x^{\frac{2}{5}}$$
12. $y = \left(\frac{7}{x} - 6x\sqrt{x}\right)^{-12}$
- $$= (7x^{-1} - 6x^{\frac{3}{2}})^{-12}$$
- $$\frac{dy}{dx} = -12(7x^{-1} - 6x^{\frac{3}{2}})^{-13}(-7x^{-2} - 9x^{\frac{1}{2}})$$
- $$= \frac{12(7x^{-2} + 9x^{\frac{1}{2}})}{(7x^{-1} - 6x^{\frac{3}{2}})^{13}}$$
13. $y = \frac{x^2 + 10x + 25}{x - 11}$
- $$\frac{dy}{dx} = \frac{(2x + 10)(x - 11) - (x^2 + 10x + 25)(1)}{(x - 11)^2}$$
- $$= \frac{x^2 - 22x - 135}{(x - 11)^2}$$
14. $y = (x^2 - 4x + 4)^3(\sqrt{8} - \sqrt{2}x)^8$
- $$= [(x - 2)^2]^3[\sqrt{2}(2 - x)]^8$$
- $$= 16(x - 2)^{14}$$
- $$\frac{dy}{dx} = 224(x - 2)^{13}$$
- $$\left.\frac{dy}{dx}\right|_{x=3} = 224(3 - 2)^{13}$$
- $$= 224$$
15. $y = 4\left(\frac{2x^2 + 1}{5 - 3x^2}\right)^3 \cdot \frac{4x(5 - 3x^2) - (2x^2 + 1)(-6x)}{(5 - 3x^2)^2}$
- $$= 4\left(\frac{2x^2 + 1}{5 - 3x^2}\right)^3 \cdot \frac{26x}{(5 - 3x^2)^2}$$
- $$= \frac{104x(2x^2 + 1)^3}{(5 - 3x^2)^5}$$

$$16. \quad y = \frac{3x^{-1} + 4x^5 - 7x^6}{2x^2}$$

$$= \frac{3}{2}x^{-3} + 2x^3 - \frac{7}{2}x^4$$

1M

$$\frac{dy}{dx} = -\frac{9}{2}x^{-4} + 6x^2 - 14x^3$$

1A

$$17. \quad y = (\sqrt[3]{x} + 2)^8$$

$$= (x^{\frac{1}{3}} + 2)^8$$

$$\frac{dy}{dx} = 8(x^{\frac{1}{3}} + 2)^7 \left(\frac{1}{3}x^{-\frac{2}{3}} \right)$$

1M

$$= \frac{8}{3}x^{-\frac{2}{3}}(x^{\frac{1}{3}} + 2)^7$$

1A

$$18. \quad \frac{dy}{dx} = \frac{3x^2(5x^6 + 6)^{\frac{1}{2}} - (x^3 + 1)\left(\frac{1}{2}\right)(5x^6 + 6)^{-\frac{1}{2}}(30x^5)}{5x^6 + 6}$$

1M

$$= \frac{3x^2(5x^6 + 6) - 15x^5(x^3 + 1)}{(5x^6 + 6)^{\frac{3}{2}}}$$

$$= \frac{-15x^5 + 18x^2}{(5x^6 + 6)^{\frac{3}{2}}}$$

1A

$$19. \quad (a) \quad \frac{4x}{y^2} + 2xy = 25$$

$$\frac{d}{dx}(4x + 2xy^3) = \frac{d}{dx}(25y^2)$$

$$4 + 2y^3 + 6xy^2 \frac{dy}{dx} = 50y \frac{dy}{dx}$$

1M

$$(6xy^2 - 50y) \frac{dy}{dx} = -2y^3 - 4$$

$$\frac{dy}{dx} = \frac{y^3 + 2}{25y - 3xy^2}$$

1A

$$(b) \quad \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{25y - 3xy^2}{y^3 + 2}$$

1A

$$\left. \frac{dx}{dy} \right|_{(5, 2)} = \frac{25(2) - 3(5)(2)^2}{2^3 + 2}$$

$$= -1$$

1A