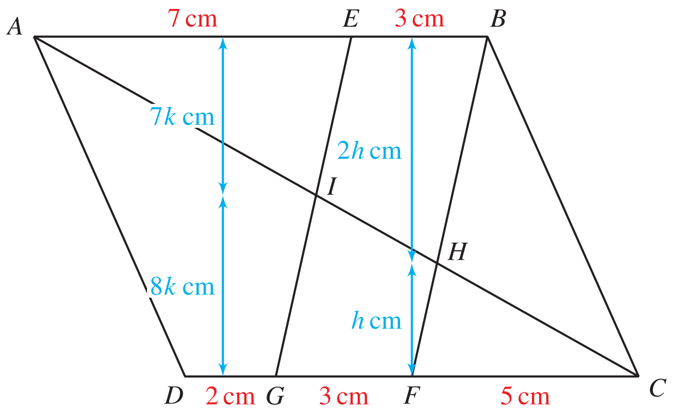
**SUM-MEN-2425-ASM-SET 2-MATH**
**Suggested solutions**

### Multiple Choice Questions

| | | | | |
|---|---|---|---|---|
| 1. A | 2. C | 3. C | 4. A | 5. B |
| 6. D | 7. B | 8. D | 9. C | 10. A |
| 11. D | 12. D | 13. B | 14. B | 15. D |
| 16. B | 17. C | 18. B | 19. C | 20. B |
| 21. D | 22. D | 23. B | 24. D | 25. C |
| 26. B | | | | |

1. $\boxed{\text{A}}$

Let $AE = 7\,\text{cm}$. Then we have the lengths as shown in the figure.




$\boxed{\text{Point } H}$ Note that $\triangle ABH \sim \triangle CFH$ (ratio $10 : 5 = 2 : 1$).

Consider the area of $\triangle BCH$.

$$100 = \frac{5(3h)}{2} - \frac{5(h)}{2}$$
$$h = 20$$

$\boxed{\text{Point } I}$ Note that $\triangle AEI \sim \triangle CGI$ (ratio $7 : 8$).

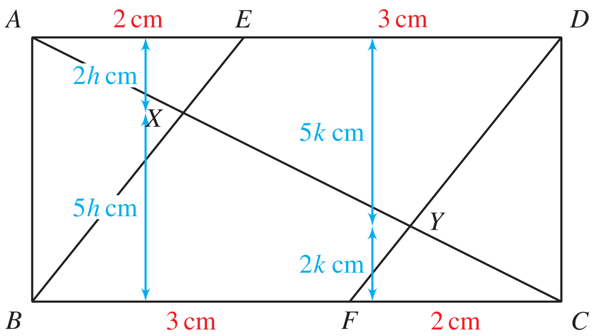
Consider the total height of the parallelogram.

$$7k + 8k = 2h + h$$
$$k = 4$$

$$\text{Required area} = \frac{7(7k)}{2}$$
$$= 98\,\text{cm}^2$$

2. $\boxed{\text{C}}$

Let $AE = 2$ cm. Then we have the lengths as shown in the figure.




$\boxed{\text{Point } X}$ Note that $\triangle AEX \sim \triangle CBX$ (ratio 2 : 5).

Consider the area of $\triangle AEX$.

$$16 = \frac{2(2h)}{2}$$

$$h = 8$$

$\boxed{\text{Point } Y}$ Note that $\triangle ADY \sim \triangle CFY$ (ratio 5 : 2).

Consider the total height of the rectangle.

$$2h + 5h = 5k + 2k$$

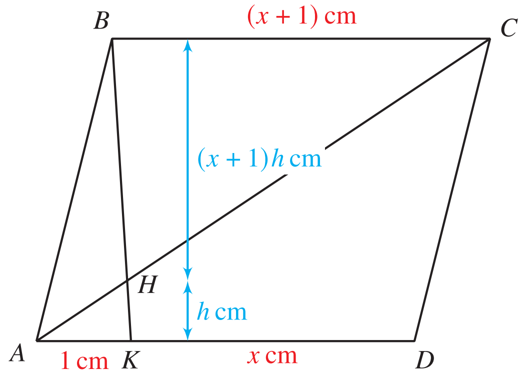
$$k = 8$$

$$\text{Required area} = \frac{5(5h)}{2} - \frac{2(2k)}{2}$$

$$= 84 \text{ cm}^2$$

3. $\boxed{\text{C}}$

Let $AK = 1$ cm and $KD = x$ cm. Then we have the lengths as shown in the figure.




$\boxed{\text{Point } H}$ Note that $\triangle AKH \sim \triangle CBH$ (ratio $1 : (x+1)$).

Consider the area of $\triangle ABH$ and quadrilateral $CDKH$.

$$\begin{cases} 8 = \dfrac{1(x+2)h}{2} - \dfrac{1(h)}{2} = \dfrac{(x+1)h}{2} \\ 38 = \dfrac{(x+1)(x+2)h}{2} - \dfrac{1(h)}{2} = \dfrac{(x^2+3x+1)h}{2} \end{cases}$$

$$\frac{38}{8} = \frac{(x^2+3x+1)h}{2} \div \frac{(x+1)h}{2}$$

$$\frac{19}{4} = \frac{x^2+3x+1}{x+1}$$

$$19x + 19 = 4x^2 + 12x + 4$$

$$0 = 4x^2 - 7x - 15$$

$$x = 3 \quad \text{or} \quad -1.25 \text{ (rejected)}$$

When $x = 3$, $h = 8 \times \dfrac{2}{x+1} = 4$.

Required area $= \dfrac{(x+1)(x+1)h}{2}$

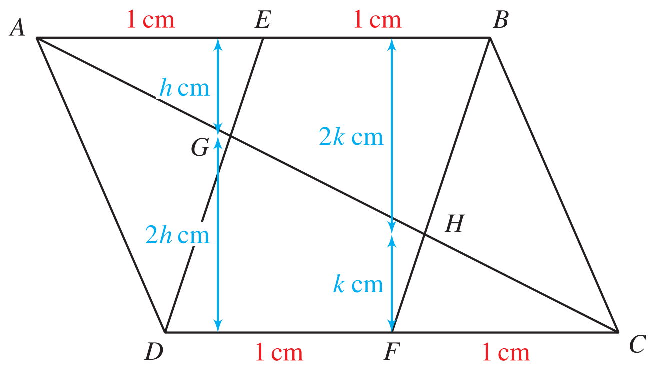
$= 32\text{ cm}^2$

4. $\boxed{\text{A}}$

I. ✓. Note that $EB = DF$ and $BEDF$ is a parallelogram.

$$\begin{aligned} AD &= BC && \textit{(opp. sides of // gram)} \\ \angle DAC &= \angle BCA && \textit{(alt. } \angle s, AD // BC) \\ \angle AGD &= \angle GHF && \textit{(corr. } \angle s, DE // BF) \\ \angle BHC &= \angle GHF && \textit{(vert. opp. } \angle s) \\ &= \angle AGD && \\ \triangle ADG &\cong \triangle CBH && \textit{(AAS)} \end{aligned}$$

II. ✓. Let $AE = 1$ cm. Then we have the lengths as shown in the figure.




$\boxed{\text{Point } G}$ Note that $\triangle AEG \sim \triangle CDG$ (ratio $1:2$).
$\boxed{\text{Point } H}$ Note that $\triangle ABH \sim \triangle CFH$ (ratio $2:1$).
Consider the total height of the parallelogram.

$$\begin{aligned} h + 2h &= 2k + k \\ h &= k \end{aligned}$$

$$\text{Area of } BEGH = \frac{2(2h)}{2} - \frac{1(h)}{2} = \frac{3h}{2}\text{ cm}^2 \quad \text{and} \quad \text{Area of } \triangle ADE = \frac{1(3h)}{2} = \frac{3h}{2}\text{ cm}^2$$

III. ✗. Refer to the figure above.

$$\text{Area of } BCGE = \frac{2(3h)}{2} - \frac{1(h)}{2} = \frac{5h}{2}\text{ cm}^2 \quad \text{and} \quad \text{Area of } \triangle CDG = \frac{2(2h)}{2} = 2h\text{ cm}^2$$

5. $\boxed{\text{B}}$

$AE : EC = 24 : 36 = 2 : 3$

$\boxed{\text{Point } E}$ Note that $\triangle CDE \sim \triangle AFE$ (ratio 3 : 2).

Let $CD = 3$ cm and the heights of two triangles be $3h$ cm and $2h$ cm respectively.

We have $AF = BF = 2$ cm.

Consider $\triangle CDE$.

$$\frac{3(3h)}{2} = 36$$
$$h = 8$$

$$\text{Required area} = \frac{(3+4)(5h)}{2} = 140\text{ cm}^2$$

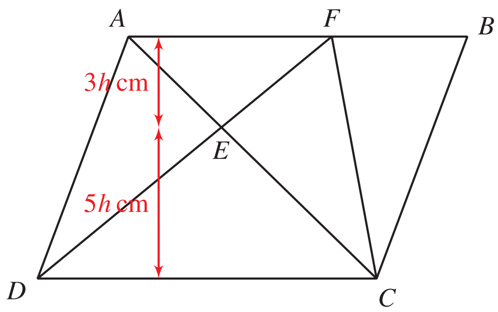
6. $\boxed{\text{D}}$

$\triangle AFE \sim \triangle CDE$ (ratio 3 : 5)

Let $AF = 3$ cm, then $CD = 5$ cm and $BF = 2$ cm.

$$\frac{(2)(3h+5h)}{2} = 16$$
$$h = 2$$

$$\text{Area of } \triangle CDE = \frac{(5)(5\times 2)}{2} = 25\text{ cm}^2$$



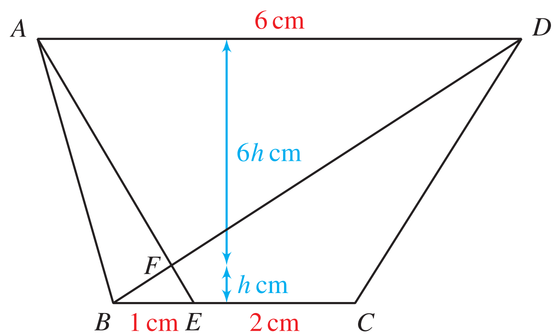

7. $\boxed{\text{B}}$

Let $BE = 1$ cm. Then $CE = 2$ cm and $AD = 6$ cm.

$\triangle ADF \sim \triangle EBF$ (ratio 6 : 1)

$$6 = \frac{(1)(7h)}{2} - \frac{(1)(h)}{2}$$
$$h = 2$$

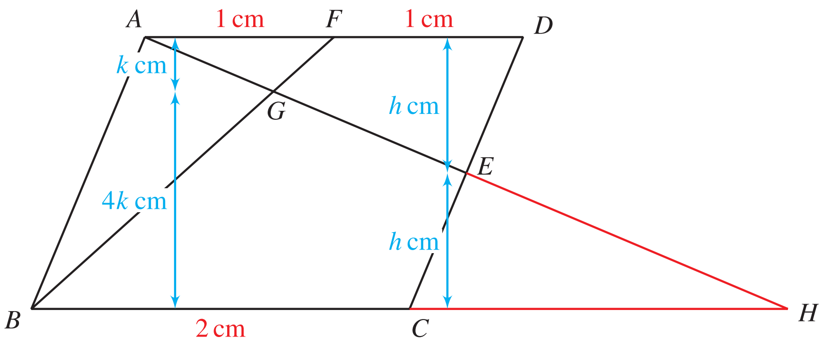
$$\text{Required area} = \frac{(3)(7h)}{2} - \frac{(1)(h)}{2} = 20\text{ cm}^2$$



8. $\boxed{\text{D}}$

Suppose $BC$ produced and $AE$ produced intersect at $H$.

Let $AE = 1$ cm. Then we have the lengths as shown in the figure.




$\boxed{\text{Point } E}$ Note that $\triangle ADE \sim \triangle HCE$ (ratio $1 : 1$).

We have $CH = DA = 2$ cm.

$\boxed{\text{Point } G}$ Note that $\triangle AFG \sim \triangle HBG$ (ratio $1 : 4$).

Consider the total height of the parallelogram.

$$k + 4k = h + h$$
$$k = \frac{2h}{5}$$

Consider the area of quadrilateral $DEGF$.

$$32 = \frac{2(h)}{2} - \frac{1(k)}{2}$$
$$h = 40$$

$$\text{Required area} = \frac{4(4k)}{2} - \frac{2(h)}{2}$$
$$= 88\text{ cm}^2$$

9. $\boxed{\text{C}}$

Let $F$ be the intersection of $AE$ and $CD$ produced.

Let $AB : BC = 1 : r$.

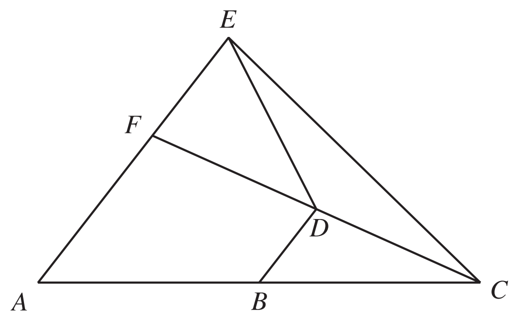
Since $\triangle CBD \sim \triangle CAF$, $FD : DC = 1 : r$.

Area of $\triangle DEF = \frac{8}{r}\text{ cm}^2$

Area of $\triangle CAF = \frac{4(1+r)^2}{r^2}\text{ cm}^2$

$$8 + \frac{8}{r} + \frac{4(1+r)^2}{r^2} = 45$$
$$r = \frac{2}{3} \quad \text{or} \quad -\frac{2}{11} \text{ (rejected)}$$

So, $AB : BC = 3 : 2$.

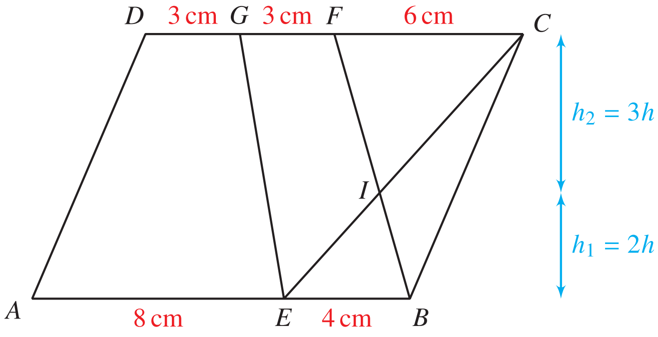


10. $\boxed{\text{A}}$

Let $DG = 3$ cm. Then we have the lengths as shown in the figure.

$\triangle FIC \sim \triangle BIE$

$$\frac{h_2}{h_1} = \frac{FC}{EB}$$
$$= \frac{3}{2}$$




Consider the area of $\triangle BIC$,

$$\frac{(4)(5h)}{2} - \frac{(4)(2h)}{2} = 6$$
$$h = 1$$

Required area

$$= \frac{(9)(5)}{2} - \frac{(6)(3)}{2}$$
$$= 13.5\text{ cm}^2$$

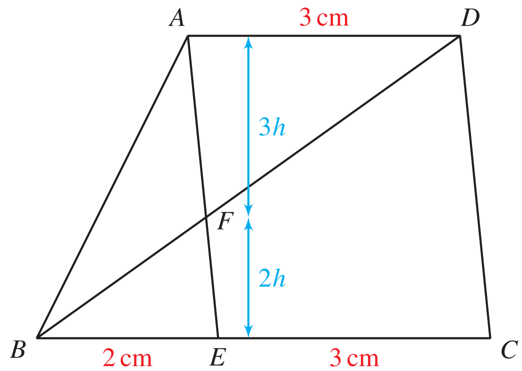
11. $\boxed{\text{D}}$

Let $BE = 2$ cm. Then $CE = AD = 3$ cm.

$\triangle ADF \sim \triangle EBF$ (ratio = 3 : 2)

Required ratio

$$= \frac{(3)(5h)}{2} : \left[(3)(5h) - \frac{(3)(3h)}{2}\right]$$
$$= 5 : 7$$




12. $\boxed{\text{D}}$

$$\frac{\text{area of }\triangle AQC}{\text{area of }\triangle ABC} = \frac{QC}{BC}$$
$$= \frac{4}{5}$$
$$\frac{\text{area of }\triangle BPC}{\text{area of }\triangle ABC} = \frac{1}{4}$$
$$\frac{\text{area of }\triangle BPC}{\text{area of }\triangle AQC} = \frac{1}{4} \div \frac{4}{5}$$
$$= \frac{5}{16}$$

13. $\boxed{\text{B}}$

Let $BF = 1$ cm. Then $FC = 2$ cm and $AD = 3$ cm.

Consider the area of $\triangle BEF$,

$$\frac{(1)(BE)}{2} = 2$$
$$BE = 4 \text{ cm}$$

$AE = BE = 4$ cm

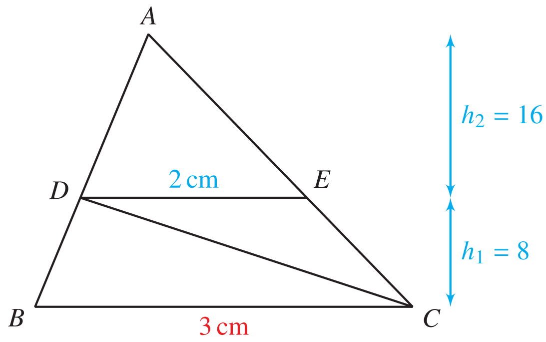
$$\text{Required area} = (3)(8) - \frac{1}{2}(3)(4) - 2 - \frac{1}{2}(8)(2)$$
$$= 8 \text{ cm}^2$$

14. $\boxed{\text{B}}$

Let $BC = 3$ cm.

Consider the area of $\triangle DBC$,

$$\frac{(3)(h_1)}{2} = 12$$
$$h_1 = 8 \text{ cm}$$




Consider the area of $\triangle DEC$,

$$\frac{(DE)(8)}{2} = 8$$
$$DE = 2 \text{ cm}$$

$\triangle ADE \sim \triangle ABC$

$$\frac{h_2}{8 + h_2} = \frac{DE}{BC}$$
$$h_2 = 16 \text{ cm}$$

$$\text{Required area} = \frac{(2)(16)}{2}$$
$$= 16 \text{ cm}^2$$

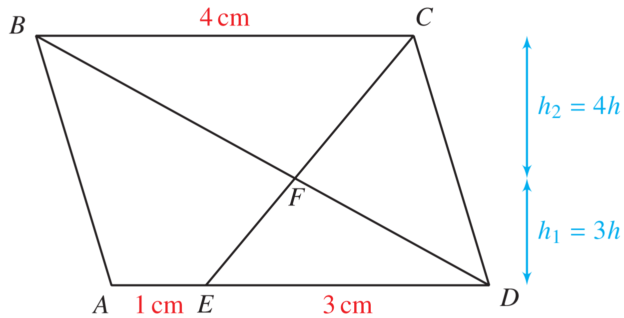
15. $\boxed{\text{D}}$

Rotate the graph such that $BC$ and $AD$ are horizontal.

Let $AE = 1$ cm. Then we have the lengths as shown in the figure.

$\triangle BCF \sim \triangle DEF$

$$\frac{h_1}{h_2} = \frac{DE}{BC}$$
$$= \frac{3}{4}$$




Required ratio

$$= \left(\frac{(4)(7h)}{2} - \frac{(3)(3h)}{2}\right) : \frac{(4)(4h)}{2}$$

$= 19 : 16$

16. $\boxed{\text{B}}$

Rotate the figure such that $BC$ and $AD$ are horizontal.

$\triangle ABF \sim \triangle ECF$ and $BF : FC = AB : CE = 2 : 3$.

Let $BF = 2$ cm and we have the lengths in the figure.

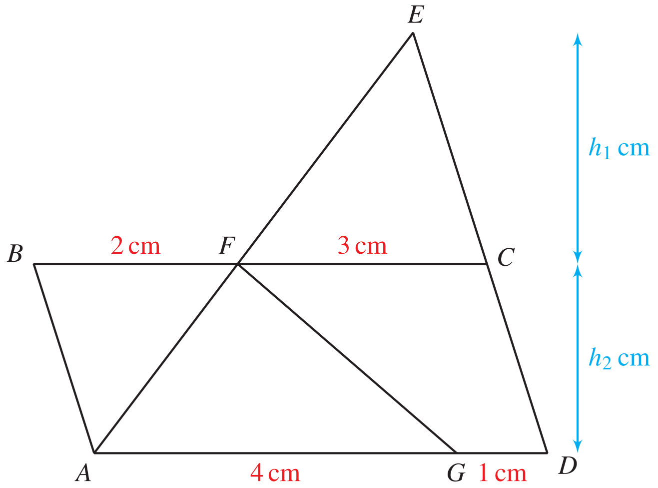
Consider the area of $\triangle ECF$,

$$\frac{(3)(h_1)}{2} = 18$$
$$h_1 = 12$$

$\triangle ECF \sim \triangle EDA$

$$\frac{h_1}{h_1 + h_2} = \frac{FC}{AD}$$
$$h_2 = 8$$

$$\text{Required area} = \frac{(3+1)(8)}{2} = 16\,\text{cm}^2$$



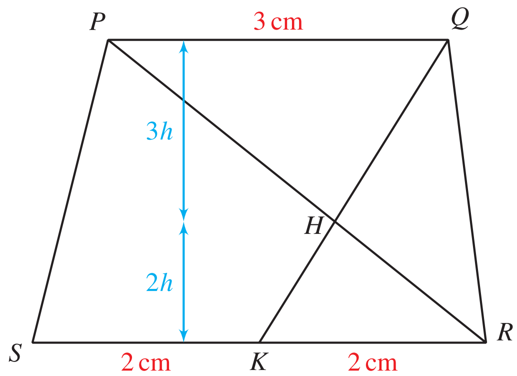

17. $\boxed{\text{C}}$

Let $PQ = 3$ cm. Then $SK = KR = 2$ cm.

$\triangle PQH \sim \triangle RKH$ (ratio 3 : 2)

$$\frac{(3)(3h)}{2} = 90$$
$$h = 20$$

$$\text{Required area} = \frac{(3+4)(100)}{2} = 350\,\text{cm}^2$$




18. $\boxed{\text{B}}$

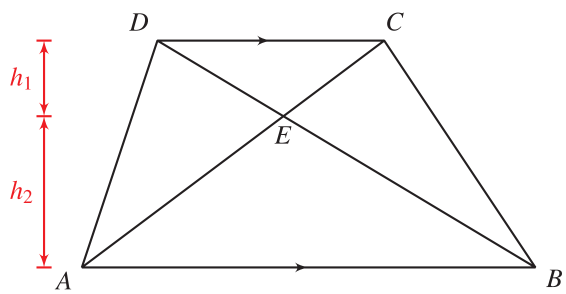
Construct the heights of the upper and lower triangles as follows:

$$\frac{1}{2} = \frac{\text{Area of } \triangle CDE}{\text{Area of } \triangle BCE} = \frac{DE}{BE}$$

Since $\triangle CDE \sim \triangle AEB$, $\dfrac{1}{2} = \dfrac{h_1}{h_2} = \dfrac{CD}{AB}$.

Let $CD = 1$.

$$\frac{\text{Area of } \triangle CDE}{\text{Area of trapezium } ABCD} = \frac{\frac{1}{2}(1)(h_1)}{\frac{1}{2}(1+2)(h_1+h_2)} = \frac{h_1}{3(h_1+2h_1)} = \frac{1}{9}$$

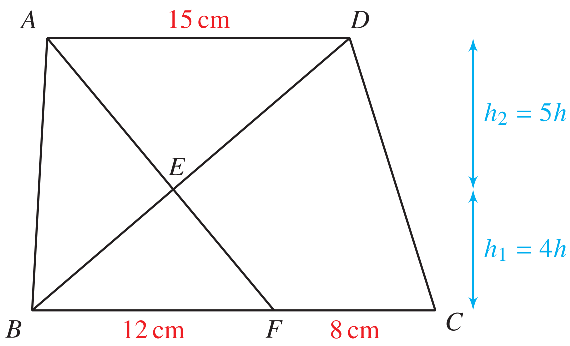


19. $\boxed{\text{C}}$

Let $AD = 15$ cm. Then we have the lengths as shown in the figure.

$\triangle ADE \sim \triangle FBE$

$$\frac{h_1}{h_2} = \frac{BF}{AD} = \frac{4}{5}$$




Consider the area of $ABCD$,

$$\frac{(15+20)(9h)}{2} = 210$$
$$h = \frac{4}{3}$$

Required area

$$= \frac{(20)(12)}{2} - \frac{(12)\left(\frac{16}{3}\right)}{2}$$
$$= 88 \text{ cm}^2$$

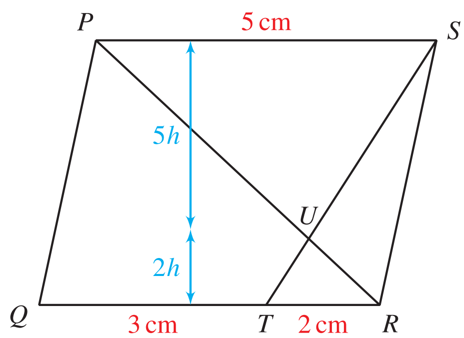
20. $\boxed{\text{B}}$

Let $QT = 3$ cm. Then $TR = 2$ cm and $PS = 5$ cm.

$\triangle PSU \sim \triangle RTU$ (ratio 5 : 2)

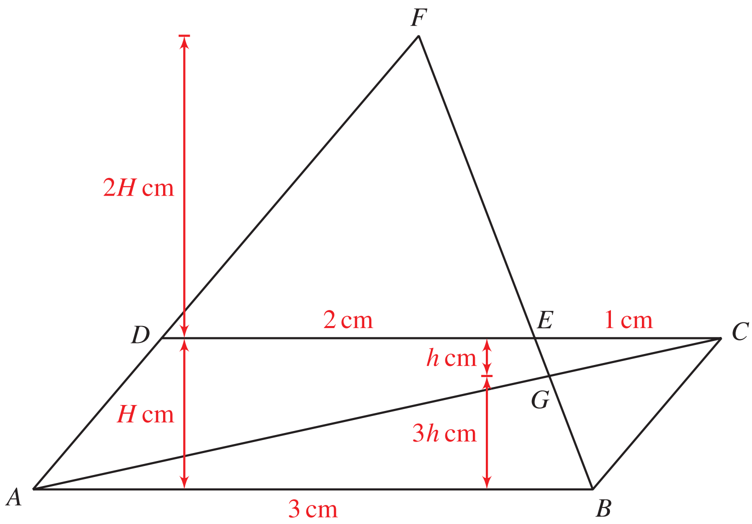
$$\frac{(2)(2h)}{2} = 12$$
$$h = 6$$

$$\text{Required area} = \frac{(5)(7h)}{2} - \frac{(2)(2h)}{2} = 93 \text{ cm}^2$$



21. $\boxed{\text{D}}$

Let $DE = 2$ cm. Then $EC = 1$ cm and $AB = 3$ cm.




Note that $\triangle CEG \sim \triangle ABG$ (ratio 1 : 3).

The heights of two triangles are $h$ cm and $3h$ cm respectively.

Consider the area of $\triangle ABF$.

$$\frac{(1)(4h)}{2} - \frac{(1)(h)}{2} = 6$$

$$h = 4$$

Note that $\triangle DEF \sim \triangle ABF$ (ratio 2 : 3).

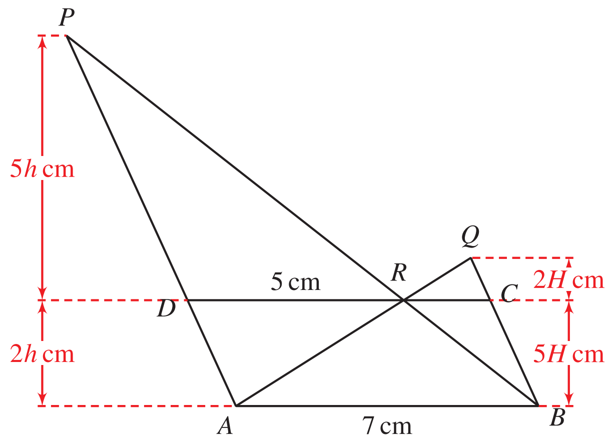
The heights of two triangles are therefore 32 cm and 48 cm respectively $\quad (H = 16)$.

$$\text{Required area} = \frac{(3)(48)}{2}$$

$$= 72\text{ cm}^2$$

22. $\boxed{\text{D}}$

Let $AB = 7$ cm.




$\boxed{\text{Point } P}$

We have $\triangle PDR \sim \triangle PAB$ (ratio 5 : 7).

Let $5h$ cm and $7h$ cm be the heights of two triangles respectively.

We have $DR = AB \times \dfrac{5}{7} = 5$ cm and $RC = 7 - 5 = 2$ cm.

$\boxed{\text{Point } Q}$

We have $\triangle QRC \sim \triangle QAB$ (ratio 2 : 7).

Let $2H$ cm and $7H$ cm be the heights of two triangles respectively.

Consider the area of $\triangle DPR$.

$$\frac{5(5h)}{2} = 50$$
$$h = 4$$

Consider the height of $ABCD$.

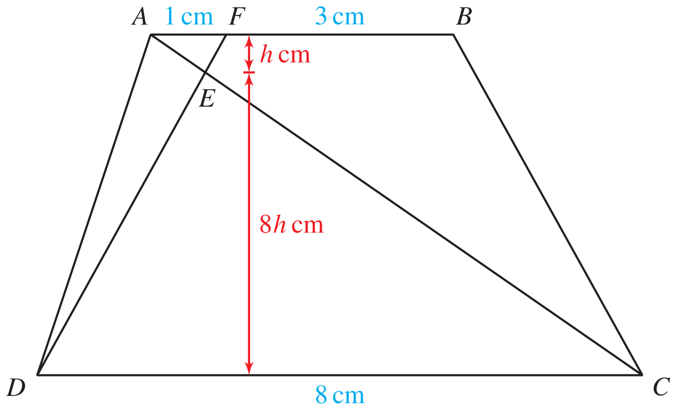
$$2h = 5H$$
$$H = 1.6$$

$$\text{Required area} = \frac{7(7H)}{2} = 39.2\text{ cm}^2$$

23. $\boxed{\text{B}}$

Let $AF = 1$ cm.

Then $BF = 3$ cm and $CD = 8$ cm.




$\boxed{\text{Point } E}$ We have $\triangle AEF \sim \triangle CED$ (ratio 1 : 8).

Let the heights of $\triangle AEF$ and $\triangle AED$ be $h$ cm and $8h$ cm respectively.

Consider the area of $CBFE$.

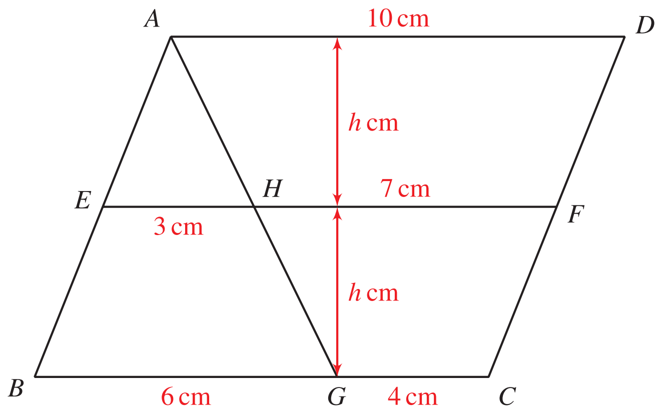
$$105 = \frac{(4)(9h)}{2} - \frac{(1)(h)}{2}$$

$$h = 6$$

$$\text{Required area} = \frac{(4+8)(9h)}{2}$$
$$= 324 \text{ cm}^2$$

24. $\boxed{\text{D}}$

Let $BG = 6\text{ cm}$. Then $GC = 4\text{ cm}$ and $AD = 10\text{ cm}$.




Note that $\triangle AEH \sim \triangle ABG$ (ratio 1 : 2).

We have $EH = 3\text{ cm}$ and $HF = 7\text{ cm}$.

Consider the area of $\triangle AEH$.

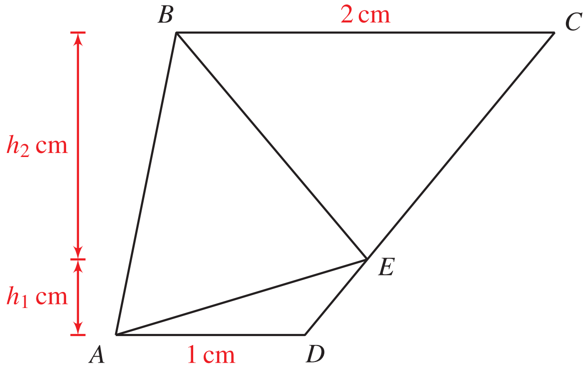
$$\frac{3(h)}{2} = 36$$

$$h = 24$$

$$\text{Required area} = \frac{(7+4)h}{2} = 132\text{ cm}^2$$

25. $\boxed{\text{C}}$

Assume $AD = 1$ cm. Then $BC = 2$ cm.




Note that $h_2 : h_1 = CE : DE = 3 : 1$.

Consider the area of $\triangle ABE$.

$$\frac{(1+2)(h_1+h_2)}{2} - \frac{2h_2}{2} - \frac{1h_1}{2} = 10$$

$$\frac{3}{2}(h_1 + 3h_1) - 3h_1 - \frac{h_1}{2} = 10$$

$$h_1 = 4$$

$$\text{Required area} = \frac{(1+2)(4+12)}{2}$$

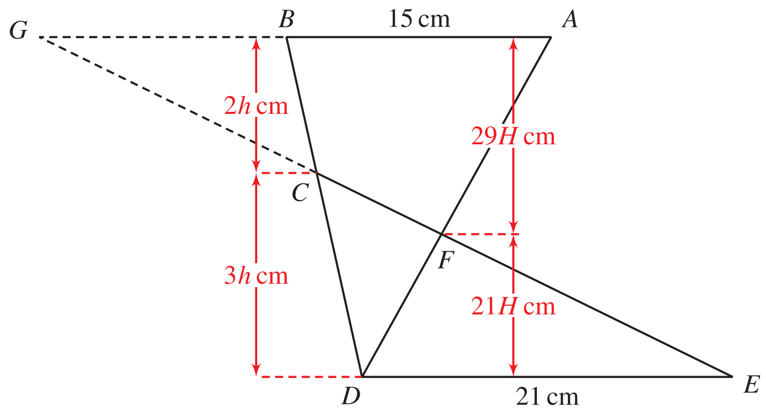
$$= 24\,\text{cm}^2$$

26. $\boxed{\text{B}}$

Let $AB = 15$ cm. Then $DE = 21$ cm.

Let $G$ be the intersection of $AB$ produced and $EC$ produced.

Rotate the figure such that the parallel lines appear horizontal as shown.




$\boxed{\text{Point } C}$

We have $\triangle GBC \sim \triangle EDC$ (ratio 2 : 3).

Let $2h$ cm and $3h$ cm be the heights of two triangles respectively.

$GB = \dfrac{2}{3} \times DE = 14$ cm

$\boxed{\text{Point } F}$

We have $\triangle AFG \sim \triangle DFE$ (ratio 29 : 21).

Let $29H$ cm and $21H$ cm be the heights of two triangles respectively.

Consider the area of $\triangle CDF$.

$$\frac{21(3h)}{2} - \frac{21(21H)}{2} = 63$$
$$3h - 21H = 6$$

Consider the total height of the figure.

$$\begin{cases} 3h - 21H = 6 \\ 2h + 3h = 29H + 21H \end{cases}$$

Solving, we have $h = \dfrac{20}{3}$ and $H = \dfrac{2}{3}$.

$$\text{Required area} = \frac{29(29H)}{2} - \frac{14(2h)}{2}$$
$$= 187 \text{ cm}^2$$

**Conventional Questions**

27. (a) Required area $= \pi(36)(\sqrt{36^2+48^2})$ 1M

$= 2160\pi \text{ cm}^2$ 1A

(b) (i) Let $r$ cm and $h$ cm be the base radius and the height of the part of circular cone that is below the water surface respectively.

$$\left(\frac{r}{36}\right)^2 = \frac{2160\pi - 2025\pi}{2160\pi}$$ 1M

$$r^2 = 81$$

$$r = 9 \quad \text{or} \quad -9 \text{ (rejected)}$$

We have $h = 48 \times \dfrac{r}{36} = 12$.

Required base radius and height are 9 cm and 12 cm respectively. 1A

(ii) Let $H$ cm be the part of the circular cone that is in the vessel.

$$\frac{H}{48} = \frac{27}{36}$$

$$H = 36$$

Volume of water in the vessel

$= \pi(27)^2(80-36+12) - \dfrac{1}{3}\pi(9)^2(12)$ 1M

$= 40\,500\pi \text{ cm}^3$

$\approx 127\,000 \text{ cm}^3$

$\approx 0.127 \text{ m}^3$

$> 0.12 \text{ m}^3$

The claim is disagreed. 1A

28. (a) Required volume $= \dfrac{1}{3}\pi(9)^2(9)$ 1M

$= 243\pi \text{ cm}^3$ 1A

(b) (i) Required volume

$= 243\pi - 243\pi \times \left(\dfrac{9-6}{9}\right)^3$ 1M+1M

$= 234\pi \text{ cm}^3$ 1A

(ii) Capacity of the container $= 18 \times 18 \times 6$

$= 1944 \text{ cm}^3$

Total volume of water and frustum after 4 minutes

$= 5 \times 4 \times 60 + 234\pi$ 1M

$\approx 1940 \text{ cm}^3$

$< 1944 \text{ cm}^3$

The water will not overflow after 4 minutes. 1A

29. (a) Let the radius of the sphere be $r$ cm.

$$4\pi r^2 = 144\pi$$

$$r = 6 \quad \text{or} \quad -6 \text{ (rejected)}$$

Required volume $= \dfrac{4\pi}{3}(6)^3$ 1M

$= 288\pi \text{ cm}^3$ 1A

(b) Volume of water $= \pi(16)^2(14) - 288\pi$ 1M

$= 3296\pi \text{ cm}^3$

Required depth $= \dfrac{3296\pi}{\pi(16)^2}$ 1M

$= \dfrac{103}{8}$ cm 1A

(c) Base radius of cone $= \dfrac{48\pi}{2\pi} = 24$ cm

Slant height of cone $= \dfrac{720\pi}{\pi(24)} = 30$ cm

Volume of cone $= \dfrac{\pi}{3}(24)^2\sqrt{30^2 - 24^2}$ 1M+1M

$= 3456\pi \text{ cm}^3$

$> 3296\pi \text{ cm}^3$

The water will not overflow. 1A

30. (a) $\cos\dfrac{\angle AOB}{2} = \dfrac{10-2}{10}$ 1M

$\angle AOB \approx 73.7°$

Required area

$= \pi(10)^2 \times \dfrac{\angle AOB}{360°}$ 1M

$\approx 64.4 \text{ cm}^2$ 1A

(b) $AB = 2\sqrt{10^2 - 8^2} = 12$ cm

Volume of water

$= \left(\pi(10)^2 \times \dfrac{\angle AOB}{360°} - \dfrac{(12)(8)}{2}\right)(30)$ 1M

$\approx 491 \text{ cm}^3$

$CD = 12 \times \dfrac{5}{4} = 15$ cm 1M

$\sin\dfrac{\angle COD}{2} = \dfrac{\left(\frac{15}{2}\right)}{10}$

$\angle COD \approx 97.2°$

Volume of the oil

$$= \left[\pi(10)^2 \times \frac{\angle COD}{360°} - \frac{(15)(\sqrt{10^2 - 7.5^2})}{2}\right](30) - (\text{volume of water})$$ 1M

$\approx 565\ \text{cm}^3$

$> 491\ \text{cm}^3$

The volume of the oil is greater than the volume of the water. 1A



**END OF PAPER**