M2SUM-DRILL-2425-ASM-SET 1-MATH
Suggested solutions

1. (a) $f(x) = \dfrac{x^2 + kx}{x + 1}$

$= x + (k - 1) + \dfrac{-k + 1}{x + 1}$ 1M

The oblique asymptote is $y = x + (k - 1)$.

Thus, $k = -8$. 1A

(b) $x = -1$ 1A

(c) $f'(x) = 1 + \dfrac{-9}{(x + 1)^2}$ 1M

$= \dfrac{(x + 4)(x - 2)}{(x + 1)^2}$

When $f'(x) = 0$, $x = 2$ or -4.

x	$x < -4$	$-4 < x < -1$	$-1 < x < 2$	$x > 2$
$f'(x)$	+	−	−	+

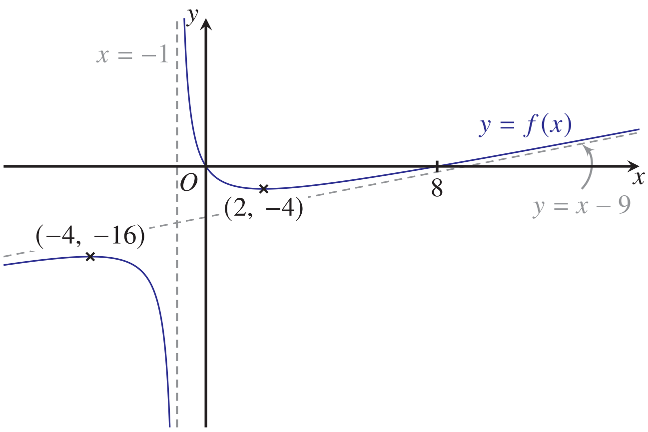
1M

Maximum point is $(-4,\ -16)$. 1A

Minimum point is $(2,\ -4)$. 1A

(d) (Maximum point, minimum point and intercepts) 1A

(All correct) 1A



(e) Required area

$= \displaystyle\int_0^5 -f(x)\,\mathrm{d}x$ 1A

$= \displaystyle\int_0^5 \left(-x + 9 - \frac{9}{x + 1}\right) \mathrm{d}x$

$= \left[-\dfrac{x^2}{2} + 9x - 9\ln|x + 1| \right]_0^5$ 1M

$= \dfrac{65}{2} - 9\ln 6$ 1A

2. (a) Vertical asymptote is $x = -3$. 1A

$$f(x) = \frac{-x^3 + 6x^2}{(x+3)^2}$$

$$= -x + 12 - \frac{63}{x+3} + \frac{81}{(x+3)^2}$$ 1M

Oblique asymptote is $y = -x + 12$. 1A

(b) $f'(x) = -1 + \dfrac{63}{(x+3)^2} - \dfrac{162}{(x+3)^3}$ 1M

$$f''(x) = \frac{-126}{(x+3)^3} + \frac{486}{(x+3)^4}$$

$$= \frac{-18(7x-6)}{(x+3)^4}$$ 1A

(c) $f'(x) = -1 + \dfrac{63}{(x+3)^2} - \dfrac{162}{(x+3)^3}$

$$= \frac{-x(x+12)(x-3)}{(x+3)^3}$$

When $f'(x) = 0$, $x = -12$ or 0 or 3.

x	$x < -12$	$-12 < x < -3$	$-3 < x < 0$	$0 < x < 3$	$x > 3$
$f'(x)$	$-$	$+$	$-$	$+$	$-$

1M

H has two minimum points at $x = -12$ and $x = 0$.

H has one maximum point at $x = 3$.

H has three turning points.

The claim is disagreed. 1A

(d) When $f''(x) = 0$, $x = \dfrac{6}{7}$.

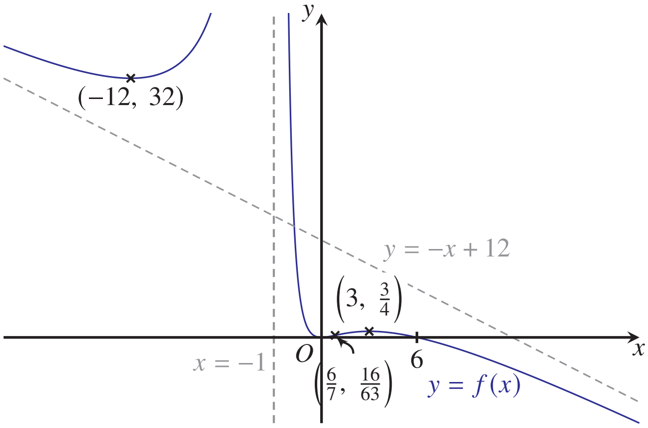
x	$x < -3$	$-3 < x < \frac{6}{7}$	$x > \frac{6}{7}$
$f''(x)$	$+$	$+$	$-$

1M

Point of inflexion is $\left(\dfrac{6}{7}, \dfrac{16}{63}\right)$. 1A

(e) (Maximum point, minimum point, point of inflexion, intercepts) 1A

(All correct) 1A



(f) Required area

$= \int_0^6 \left(-x + 12 - \frac{63}{x+3} + \frac{81}{(x+3)^2}\right) dx$ 1M

$= \left[-\frac{x^2}{2} + 12x - 63 \ln|x+3| - \frac{81}{x+3} \right]_0^6$ 1M

$= 72 - 63 \ln 3$ 1A

3. (a) $f(x) = \frac{2x^3}{(x-4)^2}$

$= 2x + 16 + \frac{96}{x-4} + \frac{128}{(x-4)^2}$

$f'(x) = 2 - \frac{96}{(x-4)^2} - \frac{256}{(x-4)^3}$ 1M

$= \frac{2x^2(x-12)}{(x-4)^3}$ 1A

$f''(x) = \frac{192}{(x-4)^3} + \frac{768}{(x-4)^4}$

$= \frac{192x}{(x-4)^4}$ 1A

(b) When $f'(x) = 0$, $x = 0$ or 12. 1M

x	$x < 0$	$0 < x < 4$	$4 < x < 12$	$x > 12$
$f'(x)$	+	+	−	+

1M

Minimum point is $(12,\ 54)$. 1A

No maximum point.

(c) (i) $f'(2) = \frac{2(2)^2(2-12)}{(2-4)^3} = 10$ 1M

Required equation is

$\frac{y-4}{x-2} = 10$

$y = 10x - 16$ 1A

(ii) Note that $f''(x) > 0$ for $0 < x < 2$.

The curve C is concave upwards in the interval $(0,\ 2)$.

L is the tangent to C at the point $(2,\ 4)$.

Thus, the curve C lies above L in the interval $(0,\ 2)$. 1

(iii) Required area

$= \int_0^2 [f(x) - (10x - 16)]\, dx$ 1M

$= \int_0^2 \left[-8x + 32 + \frac{96}{x-4} + \frac{128}{(x-4)^2}\right] dx$

$= \left[-4x^2 + 32x + 96 \ln|x-4| - \frac{128}{x-4} \right]_0^2$ 1M

$= 80 - 96 \ln 2$ 1A

4. (a) No vertical asymptote.

$$f(x) = x + 6 + \frac{11x + 2}{x^2 + 1}$$ 1M

Oblique asymptote is $y = x + 6$. 1A

(b) $$f'(x) = \frac{3(x+2)^2(x^2+1) - (x+2)^3(2x)}{(x^2+1)^2}$$ 1M

$$= \frac{(x+2)^2(x-1)(x-3)}{(x^2+1)^2}$$

When $f'(x) = 0$, $x = -2$ or 1 or 3. 1M

x	$x < -2$	$-2 < x < 1$	$1 < x < 3$	$x > 3$
$f'(x)$	+	+	−	+

1M

Minimum point is $\left(3,\ \frac{25}{2}\right)$. 1A

Maximum point is $\left(1,\ \frac{27}{2}\right)$. 1A

(c) The coordinates of P and Q are $(0,\ 8)$ and $\left(1,\ \frac{27}{2}\right)$ respectively.

Equation of PQ is

$$\frac{y - \frac{27}{2}}{x - 1} = \frac{8 - \frac{27}{2}}{0 - 1}$$

$$y = \frac{11x}{2} + 8$$ 1M

Required area

$$= \int_0^1 \left[f(x) - \left(\frac{11x}{2} + 8\right)\right] \mathrm{d}x$$ 1M

$$= \int_0^1 \left(-\frac{9x}{2} - 2 + \frac{11x}{x^2+1} + \frac{2}{x^2+1}\right) \mathrm{d}x$$

Let $u = x^2 + 1$. Then $\mathrm{d}u = 2x\,\mathrm{d}x$. 1M

Let $x = \tan\theta$. Then $\mathrm{d}x = \sec^2\theta\,\mathrm{d}\theta$. 1M

Required area

$$= \left[-\frac{9x^2}{4} - 2x\right]_0^1 + \frac{11}{2}\int_1^2 \frac{\mathrm{d}u}{u} + 2\int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{\tan^2\theta + 1}\,\mathrm{d}\theta$$

$$= -\frac{17}{4} + \frac{11}{2}\Big[\ln|u|\Big]_1^2 + 2\Big[\theta\Big]_0^{\frac{\pi}{4}}$$

$$= -\frac{17}{4} + \frac{11}{2}\ln 2 + \frac{\pi}{2}$$ 1A

5. (a) Vertical asymptote is $x = -2$. 1A

$$f(x) = x - 6 + \frac{9}{x+2}$$ 1M

Oblique asymptote is $y = x - 6$. 1A

(b) $f'(x) = 1 - \dfrac{9}{(x+2)^2}$ 1M

$= \dfrac{(x+5)(x-1)}{(x+2)^2}$ 1A

(c) When $f'(x) = 0$, $x = 1$ or -5. 1M

x	$x < -5$	$-5 < x < -2$	$-2 < x < 1$	$x > 1$
$f'(x)$	+	−	−	+

1M

Maximum point is $(-5,\ -14)$. 1A

Minimum point is $(1,\ -2)$. 1A

(d) Required volume

$= \pi \displaystyle\int_0^1 \left(x - 6 + \frac{9}{x+2}\right)^2 \mathrm{d}x$ 1M

$= \pi \displaystyle\int_0^1 \left[(x-6)^2 + \frac{18(x-6)}{x+2} + \frac{81}{(x+2)^2}\right] \mathrm{d}x$

$= \pi \displaystyle\int_0^1 \left[(x-6)^2 + \left(18 - \frac{144}{x+2}\right) + \frac{81}{(x+2)^2}\right] \mathrm{d}x$ 1M

$= \pi \left[\dfrac{(x-6)^3}{3} + 18x - 144\ln|x+2| - \dfrac{81}{x+2}\right]_0^1$ 1M

$= \pi \left(\dfrac{371}{6} - 144\ln\dfrac{3}{2}\right)$ 1A

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