M2REG-INDINT-2324-ASM-SET 5-MATH
Suggested solutions

1. $a\left(\frac{1}{a}\right)^2 - 3\left(\frac{1}{a}\right) - 6 = -5$ 1M

$$-\frac{2}{a} = 1$$

$$a = -2$$ 1A

$f(x) = \int (-2x^2 - 3x - 6)\,\mathrm{d}x$ 1M

$$= -\frac{2x^3}{3} - \frac{3x^2}{2} - 6x + C$$

$120 = -\frac{2}{3}(-6)^3 - \frac{3}{2}(-6)^2 - 6(-6) + C$ 1M

$C = -6$

Thus, $f(x) = -\frac{2}{3}x^3 - \frac{3}{2}x^2 - 6x - 6$. 1A

2. $\frac{\mathrm{d}y}{\mathrm{d}x} = \int \frac{8}{e^{2x}}\,\mathrm{d}x$ 1M

$$= -4e^{-2x} + C_1$$

$y = \int (-4e^{-2x} + C_1)\,\mathrm{d}x$

$$= 2e^{-2x} + C_1 x + C_2$$ 1A

When $x = 0$,

$$\begin{cases} -4e^{-2(0)} + C_1 = 1 \\ 2e^{-2(0)} + C_1(0) + C_2 = 7 \end{cases}$$ 1M

Solving, we have $C_1 = 5$ and $C_2 = 5$. 1A

Thus, the equation of the curve is $y = \frac{2}{e^{2x}} + 5x + 5$. 1A

3. Let $u = x^2$. Then $\mathrm{d}u = 2x\,\mathrm{d}x$. 1M

$y = \int 2x^3 e^{x^2}\,\mathrm{d}x$ 1M

$$= \int ue^u\,\mathrm{d}u$$

$$= ue^u - \int e^u\,\mathrm{d}u$$

$$= ue^u - e^u + C$$ 1A

$$= x^2 e^{x^2} - e^{x^2} + C$$

$4 = e^1 - e^1 + C$

$C = 4$

Required equation is $y = x^2 e^{x^2} - e^{x^2} + 4$. 1A

4. $\frac{dy}{dx} = \int (4e^{2x} - e^{-x})\,dx$ 1M

$= 2e^{2x} + e^{-x} + C$

At $(0,\ 2)$, $\frac{dy}{dx} = 2$ and

$2 = 2 + 1 + C$ 1M

$C = -1$

$y = \int \left(2e^{2x} + e^{-x} - 1\right) dx$

$= e^{2x} - e^{-x} - x + C_1$

When $x = 0$, $y = 2$, we have $C_1 = 2$. 1M

The equation of the curve is $y = e^{2x} - e^{-x} - \frac{5x}{2} + 2$. 1A

5. (a) Let $u = x^2$, then $du = 2x\,dx$.

$\int 2x^3 \cos\left(x^2\right) dx = \int u \cos u\,du$ 1M

$= u \sin u - \int \sin u\,du$ 1M

$= x^2 \sin\left(x^2\right) + \cos\left(x^2\right) + \text{constant}$ 1A

(b) (i) $y = \int 2x^3 \cos\left(x^2\right) dx = x^2 \sin\left(x^2\right) + \cos\left(x^2\right) + C$

Substitute $\left(\sqrt{\pi},\ -1\right)$, we have $C = 0$ 1M

The equation of Γ is $y = x^2 \sin\left(x^2\right) + \cos\left(x^2\right)$ 1A

(ii) Slope of tangent $= 2\pi^{\frac{3}{2}} \cos \pi = -2\pi^{\frac{3}{2}}$ 1M

The equation is

$y + 1 = -2\pi^{\frac{3}{2}}(x - \sqrt{\pi})$

$y = -2\pi^{\frac{3}{2}}x + 2\pi^2 - 1$ 1A

6. (a) Let $u = 1 + x^3$. Then $\mathrm{d}u = 3x^2\,\mathrm{d}x$. 1M

$$\int \frac{x^5}{\sqrt{1+x^3}}\,\mathrm{d}x = \frac{1}{3}\int \frac{u-1}{\sqrt{u}}\,\mathrm{d}u \quad \text{1M}$$

$$= \frac{1}{3}\int (u^{\frac{1}{2}} - u^{-\frac{1}{2}})\,\mathrm{d}u$$

$$= \frac{2}{9}u^{\frac{3}{2}} - \frac{2}{3}u^{\frac{1}{2}} + \text{constant}$$

$$= \frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}(1+x^3)^{\frac{1}{2}} + \text{constant} \quad \text{1A}$$

(b) $y = \int \frac{9x^5}{2\sqrt{1+x^3}}\,\mathrm{d}x$ 1M

$$= \frac{9}{2}\left[\frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}(1+x^3)^{\frac{1}{2}}\right] + C \quad \text{1M}$$

$$= (1+x^3)^{\frac{3}{2}} - 3\sqrt{1+x^3} + C$$

$3 = 1 - 3 + C$ 1M

$C = 5$

Required equation is $y = (1+x^3)^{\frac{3}{2}} - 3\sqrt{1+x^3} + 5$. 1A

7. (a) Suppose $x > 0$.

$$f'(x) = \frac{2}{x}\left[\left[x^2 - 2(x)\left(\frac{5}{4}\right) + \left(\frac{5}{4}\right)^2\right] + \frac{55}{16}\right] \qquad \text{1M}$$

$$= \frac{2}{x}\left[\left(x - \frac{5}{4}\right)^2 + \frac{55}{16}\right]$$

$$> 0$$

Thus, $f(x)$ is not a decreasing function. 1A

(b) (i) $y = \int \left(2x - 5 + \frac{10}{x}\right) \mathrm{d}x$ 1M

$= x^2 - 5x + 10 \ln x + C$

Γ passes through the point $(1,\ 5)$.

$5 = 1 - 5 + 0 + C$ 1M

$C = 9$

Required equation is $y = x^2 - 5x + 10 \ln x + 9$. 1A

(ii) $f''(x) = \dfrac{x(4x-5) - (2x^2 - 5x + 10)}{x^2}$ 1M

$= \dfrac{2(x^2 - 5)}{x^2}$

When $f''(x) = 0$, $x = \sqrt{5}$.

x	$0 < x < \sqrt{5}$	$x > \sqrt{5}$
$f''(x)$	$-$	$+$

1M

The point of inflexion of Γ is $\left(\sqrt{5},\ 14 - 5\sqrt{5} + 5 \ln 5\right)$. 1A

(iii) $x > \sqrt{5}$ 1A

8. (a) $\int xe^x \, dx = xe^x - \int e^x \, dx$ 1M

$= xe^x - e^x + \text{constant}$ 1A

(b) Let $u = 4 - x^2$. Then $du = -2x \, dx$.

$y = \int (x^3 - 4x^2)e^{4-x^2} \, dx$ 1M

$= \frac{1}{2} \int ue^u \, du$ 1A

$= \frac{ue^u}{2} - \frac{e^u}{2} + C$

$= \frac{(3 - x^2)e^{4-x^2}}{2} + C$

The curve passes through $(2, 5)$,

$5 = \frac{-e^0}{2} + C$ 1M

$C = \frac{11}{2}$

The equation of the curve is $y = \frac{(3 - x^2)e^{4-x^2}}{2} + \frac{11}{2}$. 1A

9. (a) Let $x = 3\tan\theta$. Then $dx = 3\sec^2\theta \, d\theta$. 1M

$\int \frac{dx}{x^2 + 9} = \int \frac{3\sec^2\theta}{9\tan^2\theta + 9} \, d\theta$ 1M

$= \frac{1}{3} \int d\theta$

$= \frac{\theta}{3} + \text{constant}$

$= \frac{1}{3} \tan^{-1} \frac{x}{3} + \text{constant}$ 1A

(b) $y = \int \frac{x^2}{x^2 + 9} \, dx$

$= \int \left(1 - \frac{9}{x^2 + 9}\right) dx$ 1M

$= x - 3\tan^{-1} \frac{x}{3} + C$, where C is a constant. 1A

$6 = x - 3\tan^{-1} 0 + C$ 1M

$C = 6$

The equation is $y = x - 3\tan^{-1} \frac{x}{3} + 6$. 1A

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