M2REG-INDINT-2324-ASM-SET 2-MATH
Suggested solutions

1. Let $u = x^6 - 2$. Then $\mathrm{d}u = 6x^5\,\mathrm{d}x$. 1M

$$\int x^5(x^6-2)^5\,\mathrm{d}x = \frac{1}{6}\int u^5\,\mathrm{d}u \quad \text{1M}$$
$$= \frac{u^6}{36} + \text{constant}$$
$$= \frac{(x^6-2)^6}{36} + \text{constant} \quad \text{1A}$$

2. Let $u = 3x + 1$. Then $\mathrm{d}u = 3\,\mathrm{d}x$. 1M

$$\int x^2(3x+1)^8\,\mathrm{d}x = \frac{1}{3}\int \left(\frac{u-1}{3}\right)^2 u^8\,\mathrm{d}u \quad \text{1M}$$
$$= \frac{1}{27}\int (u^{10} - 2u^9 + u^8)\,\mathrm{d}u \quad \text{1M}$$
$$= \frac{u^{11}}{297} - \frac{u^{10}}{135} + \frac{u^9}{243} + \text{constant}$$
$$= \frac{(3x+1)^{11}}{297} - \frac{(3x+1)^{10}}{135} + \frac{(3x+1)^9}{243} + \text{constant} \quad \text{1A}$$

3. Let $u = \sqrt{x} - 2$. Then $\mathrm{d}u = \dfrac{\mathrm{d}x}{2\sqrt{x}}$. 1M

$$\int \frac{5^{\sqrt{x}-2}}{\sqrt{x}}\,\mathrm{d}x = 2\int 5^u\,\mathrm{d}u \quad \text{1M}$$
$$= 2\int e^{u\ln 5}\,\mathrm{d}u \quad \text{1M}$$
$$= \frac{2e^{u\ln 5}}{\ln 5} + \text{constant}$$
$$= \frac{2}{\ln 5}(5^{\sqrt{x}-2}) + \text{constant} \quad \text{1A}$$

4. Let $u = \dfrac{3}{2} - \dfrac{2}{x}$. Then $\mathrm{d}u = \dfrac{2}{x^2}\,\mathrm{d}x$. 1M

$$\int \frac{\mathrm{d}x}{x^2\sqrt{\frac{3}{2} - \frac{2}{x}}} = \frac{1}{2}\int \frac{\mathrm{d}u}{\sqrt{u}} \quad \text{1M}$$
$$= u^{\frac{1}{2}} + \text{constant}$$
$$= \left(\frac{3}{2} - \frac{2}{x}\right)^{\frac{1}{2}} + \text{constant} \quad \text{1A}$$

5. Let $u = \dfrac{1}{x}$. Then $\mathrm{d}u = -\dfrac{1}{x^2}\,\mathrm{d}x$

$$\int \frac{\sqrt[3]{4x^3 - 2}}{x^5}\,\mathrm{d}x = \int \sqrt[3]{4 - \frac{2}{x^3}} \cdot \frac{1}{x^4}\,\mathrm{d}x \qquad \text{1M}$$

$$= -\int \sqrt[3]{4 - 2u^3}u^2\,\mathrm{d}u \qquad \text{1M}$$

Let $w = 4 - 2u^3$. Then $\mathrm{d}w = -6u^2\,\mathrm{d}u$. 1M

$$\int \frac{\sqrt[3]{4x^3 - 2}}{x^5}\,\mathrm{d}x = \frac{1}{6}\int \sqrt[3]{w}\,\mathrm{d}w \qquad \text{1M}$$

$$= \frac{1}{8}w^{\frac{4}{3}} + \text{constant}$$

$$= \frac{1}{8}\left(4 - \frac{2}{x^3}\right)^{\frac{4}{3}} + \text{constant} \qquad \text{1A}$$

6. $\displaystyle\int \sqrt{x^4 + 2x^2}\,\mathrm{d}x = \int x\sqrt{x^2 + 2}\,\mathrm{d}x$

Let $u = x^2 + 2$. Then $\mathrm{d}u = 2x\,\mathrm{d}x$. 1M

$$\int \sqrt{x^4 + 2x^2}\,\mathrm{d}x = \frac{1}{2}\int \sqrt{u}\,\mathrm{d}u \qquad \text{1M}$$

$$= \frac{u^{\frac{3}{2}}}{3} + \text{constant}$$

$$= \frac{(x^2 + 2)^{\frac{3}{2}}}{3} + \text{constant} \qquad \text{1A}$$

7. Let $u = 3 + x^3$. Then $\mathrm{d}u = 3x^2\,\mathrm{d}x$. 1M

$$\int x^2\sqrt{3 + x^3}\,\mathrm{d}x = \frac{1}{3}\int \sqrt{u}\,\mathrm{d}u \qquad \text{1M}$$

$$= \frac{2u^{\frac{3}{2}}}{9} + \text{constant}$$

$$= \frac{2(3 + x^3)^{\frac{3}{2}}}{9} + \text{constant} \qquad \text{1A}$$

8. Let $u = 5 - 2x$. Then $\mathrm{d}u = -2\,\mathrm{d}x$. 1M

$$\int \frac{x^2\,\mathrm{d}x}{\sqrt{5 - 2x}} = -\frac{1}{2}\int \left(\frac{5 - u}{2}\right)^2 \cdot \frac{1}{\sqrt{u}}\,\mathrm{d}u \qquad \text{1M}$$

$$= -\frac{1}{8}\int (25u^{-\frac{1}{2}} - 10u^{\frac{1}{2}} + u^{\frac{3}{2}})\,\mathrm{d}u \qquad \text{1M}$$

$$= -\frac{25}{4}u^{\frac{1}{2}} + \frac{5}{6}u^{\frac{3}{2}} - \frac{1}{20}u^{\frac{5}{2}} + \text{constant}$$

$$= -\frac{25}{4}(5 - 2x)^{\frac{1}{2}} + \frac{5}{6}(5 - 2x)^{\frac{3}{2}} - \frac{1}{20}(5 - 2x)^{\frac{5}{2}} + \text{constant} \qquad \text{1A}$$

9. Let $u = x^3 - 3x^2 + 1$. Then $\mathrm{d}u = (3x^2 - 6x)\,\mathrm{d}x$. 1M

$$\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\,\mathrm{d}x = \frac{1}{3}\int \frac{\mathrm{d}u}{u} \quad \text{1M}$$

$$= \frac{\ln|u|}{3} + \text{constant}$$

$$= \frac{\ln|x^3 - 3x^2 + 1|}{3} + \text{constant} \quad \text{1A}$$

10. Let $y = 1 - t^5$. Then $\mathrm{d}u = -5t^4\,\mathrm{d}t$. 1M

$$\int \frac{t^4}{(1 - t^5)^5}\,\mathrm{d}t = \frac{1}{-5}\int \frac{\mathrm{d}u}{u^5} \quad \text{1M}$$

$$= \frac{1}{20u^4} + \text{constant}$$

$$= \frac{1}{20(1 - t^5)^4} + \text{constant} \quad \text{1A}$$

11. Let $u = e^{2x} - e^{-2x}$. Then $\mathrm{d}u = (2e^{2x} + 2e^{-2x})\,\mathrm{d}x$. 1M

$$\int \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}\,\mathrm{d}x = \frac{1}{2}\int \frac{\mathrm{d}u}{u} \quad \text{1M}$$

$$= \frac{\ln|u|}{2} + \text{constant}$$

$$= \frac{\ln|e^{2x} - e^{-2x}|}{2} + \text{constant} \quad \text{1A}$$

12. Let $u = 1 - e^{-3x}$. Then $\mathrm{d}u = 3e^{-3x}\,\mathrm{d}x$.

$$\int \frac{\mathrm{d}x}{e^{3x} - 1} = \int \frac{e^{-3x}}{1 - e^{-3x}}\,\mathrm{d}x \quad \text{1M}$$

$$= \frac{1}{3}\int \frac{\mathrm{d}u}{u} \quad \text{1M}$$

$$= \frac{\ln|u|}{3} + \text{constant}$$

$$= \frac{\ln|1 - e^{-3x}|}{3} + \text{constant} \quad \text{1A}$$

13. Let $u = \dfrac{x+1}{x-2}$. Then $\mathrm{d}u = -\dfrac{3}{(x-2)^2}\,\mathrm{d}x$. 1A

$$\int \frac{\mathrm{d}x}{x^2 - x - 2} = \int \frac{(x-2)\,\mathrm{d}x}{(x-2)^2(x+1)} \quad \text{1M}$$

$$= -\frac{1}{3}\int \frac{\mathrm{d}u}{u} \quad \text{1M}$$

$$= -\frac{\ln|u|}{3} + \text{constant}$$

$$= \frac{1}{3}\ln\left|\frac{x-2}{x+1}\right| + \text{constant} \quad \text{1A}$$

14. Let $u = 1 + \sin x$. Then $\mathrm{d}u = \cos x \, \mathrm{d}x$. 1M

$$\begin{aligned}
\int \frac{\cos^3 x}{1+\sin x} \, \mathrm{d}x &= \int \frac{(1-\sin^2 x)\cos x}{1+\sin x} \, \mathrm{d}x \\
&= \int \frac{1-(u-1)^2}{u} \, \mathrm{d}u && \text{1M} \\
&= \int (2-u) \, \mathrm{d}u && \text{1M} \\
&= 2u - \frac{u^2}{2} + \text{constant} \\
&= 2(1+\sin x) - \frac{(1+\sin x)^2}{2} + \text{constant} && \text{1A}
\end{aligned}$$

Alternative answer: $\sin x - \dfrac{\sin^2 x}{2} + \text{constant}$

15. Let $u = \cos x$. Then $\mathrm{d}u = -\sin x \, \mathrm{d}x$. 1M

$$\begin{aligned}
\int \frac{\sin x}{\cos^6 x} \, \mathrm{d}x &= -\int \frac{\mathrm{d}u}{u^6} && \text{1M} \\
&= \frac{1}{5u^5} + \text{constant} \\
&= \frac{1}{5\cos^5 x} + \text{constant} && \text{1A}
\end{aligned}$$

16. Let $u = 4\sqrt{x} - 1$. Then $\mathrm{d}u = \dfrac{2}{\sqrt{x}} \, \mathrm{d}x$. 1M

$$\begin{aligned}
\int \frac{\mathrm{d}x}{\sqrt{x}(4\sqrt{x}-1)} &= \frac{1}{2}\int \frac{\mathrm{d}u}{u} && \text{1M} \\
&= \frac{\ln|u|}{2} + \text{constant} \\
&= \frac{\ln|4\sqrt{x}-1|}{2} + \text{constant} && \text{1A}
\end{aligned}$$