SUM-LP-2425-ASM-SET 2-MATH

Suggested solutions

Multiple Choice Questions

1. C	2. A	3. C	4. D	5. A
6. D	7. A	8. B	9. A	10. C
11. B	12. B	13. C	14. B	15. C
16. D	17. D	18. D	19. D	20. D
21. C	22. C	23. B	24. B	

1. $\boxed{\text{C}}$

   Greatest value of $x + 3y + 4$ occurs at top right corner.

   Compare points A, B and C:

$(x,\ y)$	$(4,\ 0)$	$(3,\ 2)$	$(0,\ 4)$
$x + 3y + 4$	8	13	16

   Required value is 16.

2. $\boxed{\text{A}}$

   $3x + y - 1$ is smaller when x and y are smaller.

   $3x + y - 1$ attains its minimum at the bottom left corner.

   Coordinates of the bottom left corners are $(2,\ 3)$ and $(3,\ 1)$.

$(x,\ y)$	$(2,\ 3)$	$(3,\ 1)$
$3x + y - 1$	8	9

   Required value is 8.

3. $\boxed{\text{C}}$

   $x - 2y + 10$ is greater when x is larger and y is smaller.

   $x - 2y + 10$ attains its greatest value at the bottom right corner.

   Coordinates of the bottom right corner (A) are $(40,\ 0)$.

   Required value $= (40) - 2(0) + 10$

   $\qquad\qquad\qquad = 50$

4. $\boxed{\text{D}}$

   The value of $7y - 5x + 3$ is larger when x is small and y is large, i.e., top left corner S.

5. $\boxed{\text{A}}$

We have $P\,(0,\ 9)$, $Q\,(9,\ 6)$ and $R\,(12,\ 0)$.

$(x,\ y)$	$(0,\ 9)$	$(9,\ 6)$	$(12,\ 0)$	$(0,\ 0)$
$x + 2y - 5$	13	16	7	-5

Required value is 16.

6. $\boxed{\text{D}}$

Maximum value of $3x - 2y + 15$ occurs at the bottom right corners, $B\,(3,\ 3)$ or $C\,(2,\ 0)$.

$(x,\ y)$	$B\,(3,\ 3)$	$C\,(2,\ 0)$
$3x - 2y + 15$	18	21

Maximum value = 21

7. $\boxed{\text{A}}$

$x + 3y$ is larger when x and y are larger.

$x + 3y$ attains its greatest value at the top right corners.

Coordinates of the top right corners are $(3,\ 4)$ and $(5,\ 3)$.

$(x,\ y)$	$(3,\ 4)$	$(5,\ 3)$
$x + 3y$	15	14

Required value is 15.

8. $\boxed{\text{B}}$

$(x,\ y)$	$(-4,\ 3)$	$(-1,\ 2)$	$(2,\ -1)$	$(3,\ -3)$
$2x + 3y + 5$	6	9	6	2

Required value is 9.

9. $\boxed{\text{A}}$

The value of $2x - 3y + 35$ is larger when x is large and y is small, i.e., bottom right corners $P\,(0,\ -7)$ or $Q\,(6,\ -1)$.

$(x,\ y)$	$P\,(0,\ -7)$	$Q\,(6,\ -1)$
$2x - 3y + 35$	56	50

Required point is P.

10. C

    $3x + 2y$ is larger when x and y are larger.

    $3x + 2y$ attains its greatest value at the top right corners.

    Coordinates of the top right corners are $(0, 4)$ and $(4, 1)$.

(x, y)	$(0, 4)$	$(4, 1)$
$3x + 2y$	8	14

Required value is 14.

11. B

    $2x - y + 10$ is smaller when x is smaller and y is larger.

    $2x - y + 10$ attains its least value at the top left corners.

    Coordinates of the top left corners are $C(0, 30)$.

    Required value $= 2(0) - (30) + 10$

                     $= -20$

12. B

    $32 - 2x - 3y$ is smaller when x and y are larger.

    $32 - 2x - 3y$ attains its least value at the top right corners.

    Coordinates of the top right corners are $B(6, 6)$.

    Required value $= 32 - 2(6) - 3(6)$

                     $= 2$

13. C

    The value of $3x - y + 16$ is larger when x is large and y is small, i.e., bottom right corner $C(8, 0)$.

    Required value $= 3(8) - 0 + 16 = 40$

14. B

    $2x - y + 4$ is larger when x is larger and y is smaller.

    $2x - y + 4$ attains its greatest value at the bottom right corners.

    Coordinates of the bottom right corners are $X(9, -2)$ and $Y(3, -4)$.

(x, y)	$(9, -2)$	$(3, -4)$
$2x - y + 4$	24	14

Required point is X.

15. $\boxed{\text{C}}$

The value of $x + 3y + 4$ is larger when x and y are large, i.e., top right corners A, B or C.

The coordinates of A, B and C are $(4, 0)$, $(3, 2)$ and $(0, 4)$ respectively.

(x, y)	A $(4, 0)$	B $(3, 2)$	C $(0, 4)$
$x + 3y + 4$	8	13	16

Required value = 16

16. $\boxed{\text{D}}$

Note that $q > p > 0$.

$px - qy$ is smaller when x is smaller and y is larger.

$px - qy$ attains its least value at the top left corner.

Coordinates of the top left corners are $(-p, q)$ and $(-q, -p)$.

(x, y)	$(-p, q)$	$(-q, -p)$
$px - qy$	$-p^2 - q^2$	0

Since $-p^2 - q^2 < 0$, required point is $(-p, q)$.
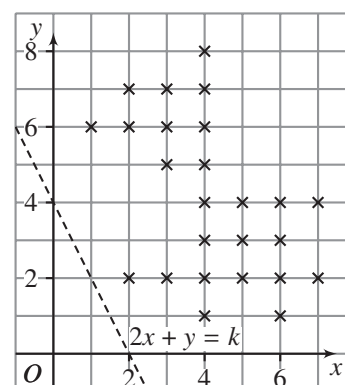
17. $\boxed{\text{D}}$

From the graph, we have $b > a > 0$.

The value of $bx - ay + 3$ is larger when x is large and y is small, i.e., bottom right corner $(b, -a)$.

18. $\boxed{\text{D}}$

Draw the line $2x + y = k$, where k is a constant.

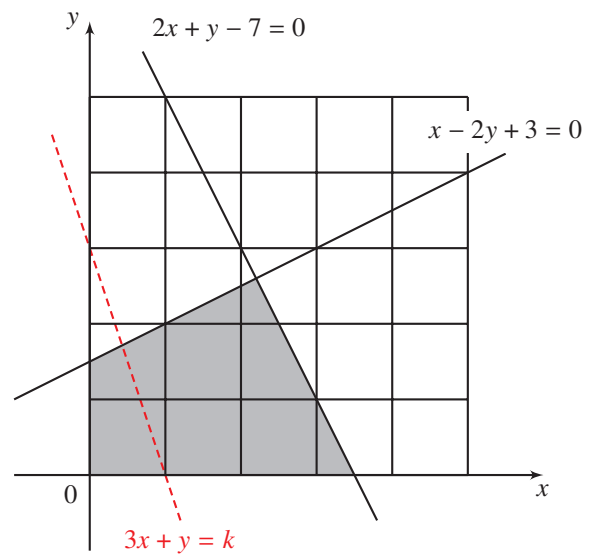$2x + y + 4$ attains its maximum at $(7, 4)$.

19. D

Draw the line $3x + y = k$, where k is a constant.
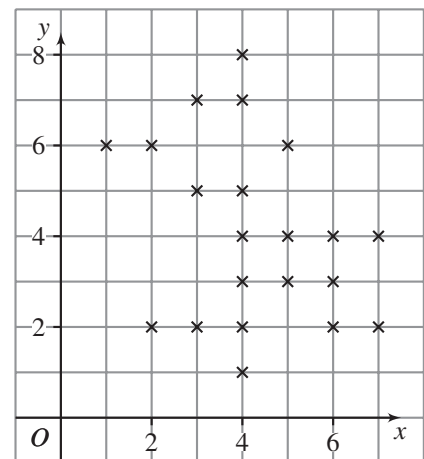
The value of $3x + y$ is maximum at $(3, \ 1)$.

Required value $= 3(3) + 1 = 10$

20. D

Draw the straight lines $x + y = k_1$, $x - y = k_2$ and $5x + y = k_3$, where k_1, k_2 and k_3 are constants.

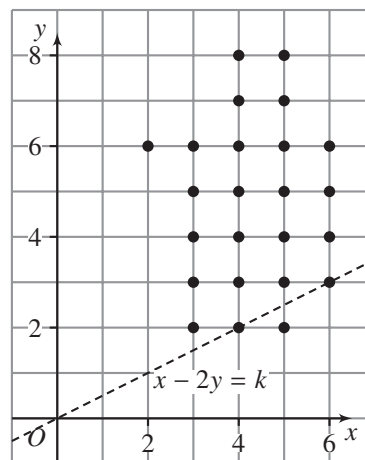I. ✗. $x + y$ attains its minimum at $(2, \ 2)$.

II. ✓.

III. ✓.

21. C

Draw the line $x - 2y = k$, where k is a constant.

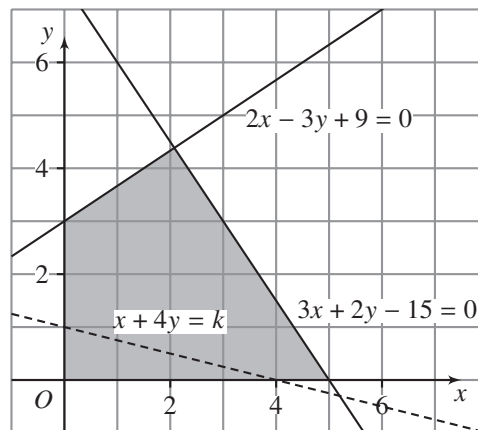$x - 2y$ attains its maximum at $(5,\ 2)$.
Required value $= 5 - 2(2)$
$\qquad\qquad\quad = 1$

22. C

Draw the line $x + 4y = k$, where k is a constant.

$x + 4y$ attains its maximum at $(2,\ 4)$.
Required value $= 2 + 4(4)$
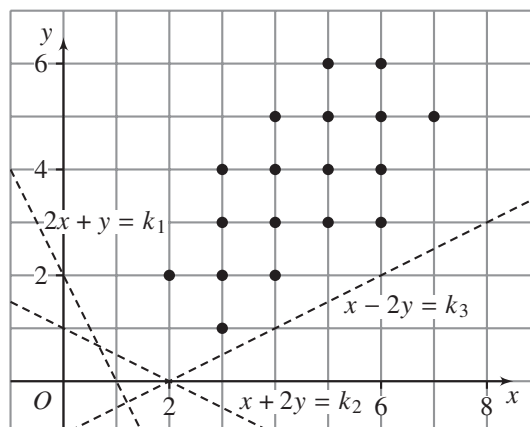$\qquad\qquad\quad = 18$

23. B

Draw the lines $2x + y = k_1$, $x + 2y = k_2$ and $x - 2y = k_3$, where k_1, k_2 and k_3 are constants.

   I. ✗. $2x + y$ attains its least value at $(2,\ 2)$.

  II. ✓.

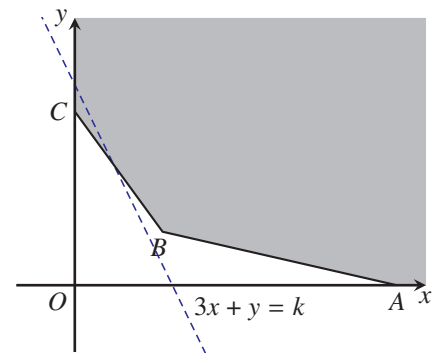 III. ✗. $x - 2y$ attains its least value at $(5,\ 6)$.

24. B

Draw the line $3x + y = k$, where k is a constant.

$3x + y$ attains its least value at $C\ (0,\ 26)$.

Required value $= 3(0) + 26$

$\qquad\qquad\qquad = 26$

Conventional Questions

25. (a) Slope of $L_2 = \dfrac{1-0}{2-0} = \dfrac{1}{2}$.　　　　　　　　　　　　1M

　　　　The equation of L_2 is $y = \dfrac{x}{2}$.　　　　　　　　　　　1A

　　(b) Consider the line $x + y = 9$. When $x = 2$, $y = 7$.　　　　1M

　　　　The coordinates of B are $(2,\ 7)$.　　　　　　　　　　　1A

　　　　Solving $\begin{cases} y = \dfrac{x}{2} \\ x + y = 9 \end{cases}$, we have $x = 6$ and $y = 3$.　　1M

　　　　The coordinates of C are $(6,\ 3)$.　　　　　　　　　　　1A

　　(c)

$(x,\ y)$	$(2,\ 1)$	$(2,\ 7)$	$(6,\ 3)$
P	1	-41	3

　　　　　　　　　　　　　　　　　　　　　　　　　　　　1M

　　　　Thus, the least value of P is -41.　　　　　　　　　　1A

26. (a) Solve $\begin{cases} 4x + y - 5 = 0 \\ x - 2y + 7 = 0 \end{cases}$,

　　　　we have $x = \dfrac{1}{3}$ and $y = \dfrac{11}{3}$.

　　　　The coordinates of C are $\left(\dfrac{1}{3},\ \dfrac{11}{3}\right)$.　　　　　　　　1A

　　(b) $4x + y - 5 \le 0$　　　　　　　　　　　　　　　　　　1A

　　　　$3x + 2y + 5 \ge 0$　　　　　　　　　　　　　　　　　　1A

　　　　$x - 2y + 7 \ge 0$　　　　　　　　　　　　　　　　　　1A

　　(c) Coordinates of the corners are $(3,\ -7)$, $(-3,\ 2)$ and $\left(\dfrac{1}{3},\ \dfrac{11}{3}\right)$.

$(x,\ y)$	$(3,\ -7)$	$(-3,\ 2)$	$\left(\dfrac{1}{3},\ \dfrac{11}{3}\right)$
$3x - 12y + 7$	100	-26	-36

　　　　　　　　　　　　　　　　　　　　　　　　　　　　1M

　　　　Required value is -36.　　　　　　　　　　　　　　　1A

27. (a) Equation of L_2 is

$$y - 0 = 2(x - 3)$$

$$2x - y - 6 = 0$$　　　　　　　　　　　　　　　　　　　1A

　　　　Equation of L_3 is

$$y - 6 = \dfrac{0 - 6}{6 - 0}(x - 0)$$

$$x + y - 6 = 0$$　　　　　　　　　　　　　　　　　　　　1A

(b) $y \le 6$ 1A

   $2x - y \le 6$ 1A

   $x + y \ge 6$ 1A

(c) (i) Coordinates of the vertices are (0, 6), (4, 2) and (6, 6).

(x, y)	(0, 6)	(4, 2)	(6, 6)
$x - 5y + 10$	-20	4	-14

1M

   P attains its greatest value at (4, 2).   1A

   Greatest value of P is 4.   1A

(ii) $P = x - 5y + 10 \ge -5$

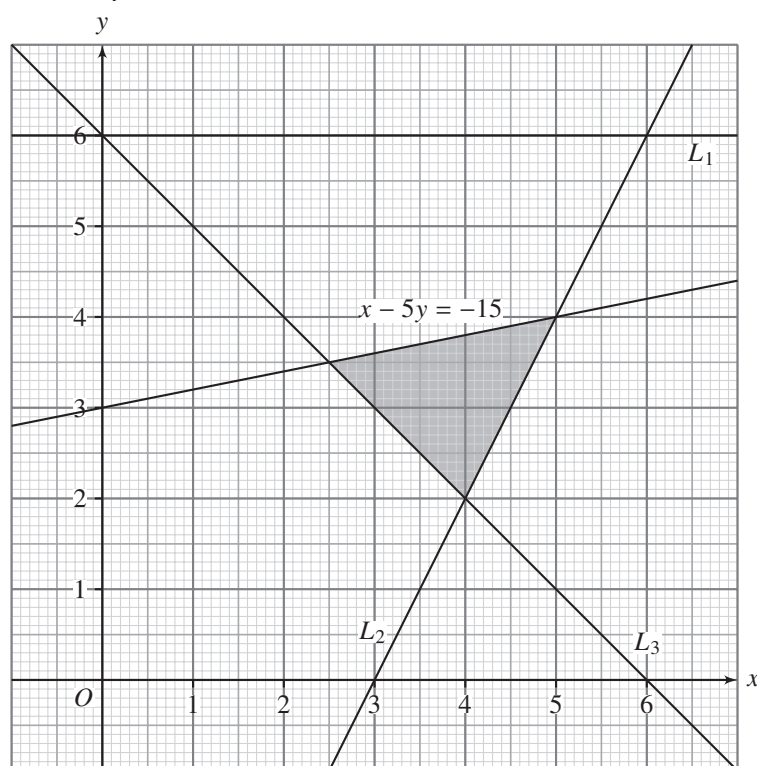 $x - 5y \ge -15$ 1A

   Draw the line $x - 5y = -15$.

1A

   We have $2.5 \le x \le 5$.   1A