REG-DISP-2324-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. B	3. C	4. B	5. A
6. A	7. C	8. D	9. A	10. D
11. D	12. C	13. A	14. B	15. B
16. C	17. C	18. C	19. C	20. C
21. A	22. B	23. A	24. A	25. B
26. A	27. A	28. B	29. A	30. C

1. $\boxed{\text{C}}$

Let the standard deviation be σ marks.

$$0.25 = \frac{68-65}{\sigma}$$

$$\sigma = 12$$

$$\text{Standard score of Betty} = \frac{80-65}{12} = 1.25$$

2. $\boxed{\text{B}}$

Let x marks be the required score.

$$\frac{x-60}{12} = 2.25$$

$$x = 87$$

3. $\boxed{\text{C}}$

Let $\overline{x}$ and σ be the mean and the standard deviation of the sleeping hours respectively.

$$\begin{cases} 1.75 = \dfrac{9.4-\overline{x}}{\sigma} \\ -0.5 = \dfrac{7.6-\overline{x}}{\sigma} \end{cases}$$

Solving, we have $\overline{x} = 8$ and $\sigma = 0.8$.

Required standard score

$$= \frac{7-8}{0.8}$$

$$= -1.25$$

4. $\boxed{\text{B}}$

Let the mean and standard deviation be $\overline{x}$ marks and σ marks respectively.

$$\begin{cases} \dfrac{78 - \overline{x}}{\sigma} = 1 \\ \dfrac{66 - \overline{x}}{\sigma} = -0.5 \end{cases}$$

Solving, we have $\overline{x} = 70$ and $\sigma = 8$.

5. $\boxed{\text{A}}$

Let the mean of the scores be $\overline{x}$ marks.

$$\frac{96 - \overline{x}}{3} = 4$$
$$\overline{x} = 84$$

$$\text{Required standard score} = \frac{81 - 84}{3} = -1$$

6. $\boxed{\text{A}}$

I. ✓. Normal distribution ⇒ mean at C.R.F. = 0.5

II. ✓. Distribution of M is less dispersed.

III. ✗. C.R.F. equal ⇒ standard score equal

7. $\boxed{\text{C}}$

Let the standard deviation of the scores and the score of Billy be σ and x respectively.

$$\begin{cases} 2a = \dfrac{60 - 74}{\sigma} \\ -a = \dfrac{x - 74}{\sigma} \end{cases}$$

$$\frac{2a}{-a} = \frac{60 - 74}{\sigma} \div \frac{x - 74}{\sigma}$$
$$-2 = \frac{-14}{x - 74}$$
$$x = 81$$

8. $\boxed{\text{D}}$

Let the mean and standard deviation be $\overline{x}$ marks and σ marks respectively.

$$\begin{cases} 55 = \overline{x} + (-3)\sigma \\ 95 = \overline{x} + 2\sigma \end{cases}$$

Solving, we have $\overline{x} = 79$ and $\sigma = 8$.

9. A

Let a marks and b marks be the minimum and maximum test scores respectively.
Denote the mean score by m marks.

$$\begin{aligned}\text{Required difference} &= \frac{b-m}{20} - \frac{a-m}{20} \\ &= \frac{b-a}{20} \\ &= \frac{100}{20} \\ &= 5 \text{ marks}\end{aligned}$$

10. D

Let the mean and standard deviation be $\overline{x}$ marks and σ marks respectively.

$$\begin{cases} \dfrac{26-\overline{x}}{\sigma} = -1 \\ \dfrac{92-\overline{x}}{\sigma} = 0.5 \end{cases}$$

Solving, we have $\overline{x} = 70$ and $\sigma = 44$.

11. D

Let the scores, mean, standard deviation, standard scores be x marks, μ marks, σ marks and z respectively.

$$\begin{aligned} z_2 - z_1 &= \frac{x_2-\mu}{\sigma} - \frac{x_1-\mu}{\sigma} \\ &= \frac{x_2-x_1}{\sigma} \\ 5 &= \frac{20}{\sigma} \\ \sigma &= 4 \end{aligned}$$

12. C

Let the score of Mary be M.

$$\begin{aligned} M - M(1-12.5\%) &= 10 \\ M &= 80 \end{aligned}$$

Peter scores 70 marks in the test.

$$\begin{cases} -1 = \dfrac{70-\mu}{\sigma} \\ 1.5 = \dfrac{80-\mu}{\sigma} \end{cases}$$

Solving, we have $\mu = 74$ and $\sigma = 4$.
The mean score of the test is 74 marks.

13. A

Let the scores be a and b with $a > b$. Let $\overline{x}$ marks and σ marks be the mean and standard deviation respectively.

$$\frac{a-\overline{x}}{\sigma} - \frac{b-\overline{x}}{\sigma} = 6$$
$$\frac{a-b}{\sigma} = 6$$
$$\sigma = \frac{30}{6} = 5$$

14. B

Let the mean and standard deviation be μ marks and σ marks respectively.

$$\begin{cases} \dfrac{16-\mu}{\sigma} = 2 \\ \dfrac{4-\mu}{\sigma} = -1 \end{cases}$$

Solving, we have $\mu = 8$ and $\sigma = 4$.

15. B

Let the mean and standard deviation be μ marks and σ marks respectively.

$$\begin{cases} \dfrac{55-\mu}{\sigma} = -3 \\ \dfrac{95-\mu}{\sigma} = 2 \end{cases} \longrightarrow \begin{cases} 55 = \mu - 3\sigma \\ 95 = \mu + 2\sigma \end{cases}$$

Solving, we have $\mu = 79$ and $\sigma = 8$. The standard deviation is 8 marks.

16. C

Median = $15 \times 2 + 3 = 33$

Interquartile range = $10 \times 2 = 20$

Variance = $40 \times 2^2 = 160$

17. C

New variance = $49 \times 4^2 = 784$

18. C

Required value is equal to the standard deviation of the set $\{-5, -2, 3, 5, 10\} \approx 5.27$

19. C

Score of Victor = $70 + 5 \times 4 = 90$

Standard score of Victor after deduction = $\dfrac{(90-4)-(70-4)}{4} = 5$

Useful fact:

Subtracting a constant from each datum will not change the standard score of each datum.

20. C

There are two steps for the transformation of data set: $\times 2$, $+23$.

New standard deviation = $14 \times 2 = 28$

21. A

I. ✓. A: mean = 0 and B: mean = 0

II. ✓. A: range = $(x+3)-(x-3)=6$ and B: range = $(x+3)-(x-3)=6$

III. ✗. Subtract x form each datum such that the standard deviation does not change.
A: S.D. ≈ 2.28 and B: S.D. ≈ 2.10

22. B

I. ✗. If $x = a_5$, then the new and original median are a_5.

II. ✓. $r_1 = a_9 - a_1$. Since a_1 and a_9 are elements of the new group, the new range is not less than r_1.

III. ✗. We can assign extreme values to x and y such that the new group is more dispersed, and $s_2 > s_1$ in this case.

23. A

I. ✗. Counter example: $A = \{1, 1000, 1001, 1002\}$. Then $B = \{4, 754, 1004, 1005\}$

II. ✓. A: range = $x_4 - x_1$ and B: range = $(x_4+3)-(x_1+3) = x_4 - x_1$

III. ✗. Counter example: $A = \{1, 1000, 1001, 1002\}$. Then $B = \{4, 754, 1004, 1005\}$

24. A

I. ✓.

II. ✓.

III. ✗. The new group of numbers is less dispersed. We have $v_1 \geq v_2$.

25. B

$\mu_2 = \mu_1$ and the data set is less dispersed $\Rightarrow$ standard deviation is decreased.
However, when all data are equal, the new and original standard deviations are both 0.

26. A

I. ✓. Range of $B = (g-4)-(a-4) = g-a$ = range of A

II. ✗. Median of $B = d-4$; median of $A = d$

III. ✗. Take $A = \{0,\ 2,\ 3,\ 4,\ 99999\}$.
Variance of $A \approx 1.5999 \times 10^9$
Variance of B is equal to the variance of the set $\{a, b, d, f, g\}$.
Take $b = 1$. The mean of a, b, d, g is approximately 25 000.
Take $f = 25\,000$ to get a smaller variance for group B.
Variance of $B \approx 1.4999 \times 10^9$
The variance of B can be smaller than that of A.

27. A

I. ✓.

II. ✗. $r' = kr \neq kr + p$.

III. ✗. $v' = k^2 v \neq kv + p$.

28. B

The marks are multiplied by a constant h and then added by a constant k.
Then $32h + k = 52$ and $4h = 8$. Solving, we have $h = 2$ and $k = -12$.
Simon's new mark $= 23(2) - 12 = 34$

29. A

Median $= (18 - 3)4 = 60$
Inter-quartile range $= 8 \times 4 = 32$
Variance $= 16 \times 4^2 = 256$

30. C

$\times 2 \quad \longrightarrow \quad +15$
New variance $= k^2(2^2) = 4k^2$

Conventional Questions

31. (a) Let Ada'a score be x.

$$\frac{x - 90}{\sqrt{64}} = -0.75 \qquad \text{1M}$$

$$x = 84 \qquad \text{1A}$$

Her score is 84.

(b) Ada's score lies between 1 standard deviation below mean and the mean. 1

Score of 1 standard deviation below mean is at the $50 - \frac{68}{2} = 16$th percentile.

Thus, Ada's score is at least the 16th percentile. The claim is agreed. 1A

32. (a) Let $\overline{x}$ be the mean of the scores of the examination.

$$\frac{71 - \overline{x}}{6} = 1.5 \qquad \text{1M}$$

$$\overline{x} = 62 \qquad \text{1A}$$

(b) Score of David $= 62 - 2.5(6) = 47$ 1M

Range of scores $\geq 71 - 47 = 24 > 23$

The claim is disagreed. 1A

33. (a) Let m marks be the mean of the test.

$$\frac{86 - m}{8} = 1.5 \qquad \text{1M}$$

$$m = 74$$

$$\text{Standard score of Ringo} = \frac{68 - 74}{8}$$

$$= -0.75 \qquad \text{1A}$$

(b) (i) Required standard deviation $= 8(1 + 30\%) = 10.4$ marks 1A

(ii) Let z and x be the original standard score and the original score of a student respectively.

$$z = \frac{x - 74}{8}$$

$$\text{New standard score} = \frac{[x(1 + 30\%) + 3] - [74(1 + 30\%) + 3]}{10.4} \qquad \text{1M+1A}$$

$$= \frac{1.3(x - 74)}{10.4}$$

$$= \frac{x - 74}{8}$$

$$= z$$

The claim is agreed. 1A

34. (a) Let the mean score be μ marks.

$$\frac{x-\mu}{4}-\frac{56-\mu}{4}=1.5-(-1)$$ 1M

$$\frac{x-56}{4}=2.5$$

$$x=66$$ 1A

(b) The original mean score is 60 marks.

If David left the group, the mean score remains unchanged and the standard deviation of the score increases. 1A

New standard score of Joan $=\frac{56-60}{\sigma}>\frac{56-60}{4}$

The standard score of Joan increases. 1A

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