

## REG-DISP-2324-ASM-SET 1-MATH

### Suggested solutions

#### Multiple Choice Questions

1. D	2. A	3. A	4. A	5. D
6. B	7. D	8. C	9. A	10. A
11. B	12. C	13. B	14. A	15. B
16. A	17. B	18. D	19. A	20. D
21. B	22. D	23. A	24. A	25. B
26. B	27. D	28. C	29. C	30. A

1. D

We have  $x = 5$  or  $x + 2 = 5$ .

When  $x + 2 = 5$ ,  $x = 3$  and the mode is 3 and 5, which should be rejected.

Thus, we have  $x = 5$ , and the numbers are 2, 3, 5, 5, 7, 8.

I. ✓. Mean =  $\frac{2 + 3 + 5 + 5 + 7 + 8}{6} = 5$

II. ✓. Range =  $8 - 2 = 6$

III. ✓. Inter-quartile range =  $7 - 3 = 4$

2. A

Upper quartile = 59 kg and lower quartile = 46 kg

Inter-quartile range =  $59 - 46$

$$= 13 \text{ kg}$$

3. A

$$\frac{4 + 4 + 6 + \dots + n}{9} = 6$$

$$m + n = 10$$

I. ✓. Since  $m$  and  $n$  cannot be both greater than 6, the median (the 5th datum) must be 6.

II. ✓. Range is the greatest when the values of  $m$  and  $n$  are 1 and 9.

Range is the least when  $m = n = 5$ .

Thus,  $5 \leq y \leq 8$ .

III. ✗. When  $m = 4$  and  $n = 6$ , mode = 4 and 6.

4.  A

Upper quartile =  $(60 + b)$  and lower quartile =  $(30 + a)$ .

$$(60 + b) - (30 + a) \leq 25 \Rightarrow b - a \leq -5$$

- I. ✓. Since  $a \geq 5 + b$ , value of  $a$  is at least 5, and at most 9 (because it is a single digit integer).
- II. ✓. Since  $b \leq a - 5$  and  $a \leq 9$ , value of  $b$  is at most  $9 - 5 = 4$ , and at least 0 (single digit integer).
- III. ✗. It is possible that  $a = 9$  and  $b = 1$ , such that the interquartile range is 22 ( $\leq 25$ ), while  $a - b = 8$ .

5.  D

$$\frac{11 + 5 + 17 + 8 + 14 + 9 + 13 + 10 + x}{9} = 12$$

$$x = 3$$

Arrange the data in ascending order:

3        5        8        9        10        11        13        14        17

Inter-quartile range =  $13.5 - 6.5 = 7$

6.  B

Denote the data sets  $\{x - 2, x - 1, x, x + 2, x + 2, x + 3\}$  and  $\{x - 5, x - 4, x - 2, x - 2, x - 1, x\}$  by  $A$  and  $B$  respectively.

- I. ✓. Range of  $A = (x + 3) - (x - 2) = 5$   
Range of  $B = x - (x - 5) = 5$
- II. ✗. Median of  $A = x + 1$   
Median of  $B = x - 2$
- III. ✓. Inter-quartile range of  $A = (x + 2) - (x - 1) = 3$   
Inter-quartile range of  $B = (x - 1) - (x - 4) = 3$

7.  D

Lower quartile = 62 kg.

$$\text{Required probability} = 1 - \frac{4}{24} = \frac{5}{6}.$$

8.  C

Range =  $(40 + k) - (20 + h) \geq 24$

$$k - h \geq 4$$

- I. ✗. It is possible that  $h = 3$  and  $k = 9$  such that range =  $26 \geq 24$ .
- II. ✓.  $k \geq 4 + h \geq 4 + 0 = 4$  and  $k \leq 9$  obviously.
- III. ✓.  $k - h \geq 4$  as proved above. Also,  $k - h \leq 9 - 0 = 9$

9.  A

$$a = x - \frac{18}{11}, b = x, c = x \text{ and } d = 10.$$

I. ✓.  $a - b = -\frac{18}{11} < 0$

II. ✗.  $b = c = x$

III. ✗.  $c - d = x - 10$  which can be positive or negative.

10.  A

We have  $3 \leq m, n \leq 12$ .

I. ✓. No matter how large or how small  $m$  and  $n$ , the median (5th datum) is always 8.

II. ✗. When  $m = n = 3$ , mean =  $\frac{58}{9} \neq 8$ .

III. ✗. When  $m = n = 3$ , mode = 3  $\neq 8$ .

11.  B

$$\text{Range} = (40 + k) - (10 + h) \geq 33 \Rightarrow k - h \geq 3.$$

I. ✗. It is possible that  $k = 7$  and  $h = 4$ . The range is 33, satisfying all the conditions.

II. ✓.  $k - h \geq 3 \Rightarrow k \geq 3 + h \geq 3$ .

III. ✗. It is possible that  $k = 9$  and  $h = 0$ . The range is 39, satisfying all the conditions.

12.  C

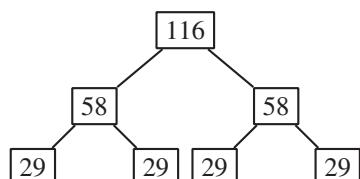
Number of students = 116. We can obtain the median and quartiles using the figure below.

A. ✗. Mode = 7

B. ✗. Median = 7

C. ✓. Lower quartile = 6

D. ✗. Upper quartile = 8



13.  B

The upper quartile of the distribution is 210 g.

$$\text{Required probability} = \frac{7}{24}$$

14. A

Arrange the data in ascending order:

16            19            23            24            29            31            33

We have  $24 \leq x \leq 29$  and  $\frac{x+24}{2} = 25$ .

Thus, we have  $x = 26$ .

Inter-quartile range =  $30 - 21 = 9$

15. B

Mean = 5  $\Rightarrow$   $2 + 3 + 4 + \dots + m + n = 5 \times 9 \Rightarrow m + n = 4$

- I.  $\times$ . It is possible that  $m = 1$  and  $n = 3$ . Then mode =  $a = 3$ .
- II.  $\checkmark$ . We have  $m < 4$  and  $n < 4$ . Median =  $b = 5$ th datum = 4.
- III.  $\times$ . It is possible that  $m = 1$  and  $n = 3$ . Then range =  $c = 10 - 1 = 9 \neq 8$ .

16. A

$$\text{Median} = \frac{(20+n)+25}{2} \leq 24 \quad \text{and} \quad \text{Interquartile range} = (30+n) - (10+m) \geq 18$$
$$n \leq 3 \qquad \qquad \qquad n - m \geq -2$$
$$m - n \leq 2$$

- I.  $\checkmark$ .  $m \leq n + 2 \leq 3 + 2 = 5$  and  $m \geq 0$ .
- II.  $\checkmark$ . From the stem-and-leaf diagram,  $n \geq 1$ . Combine with  $n \leq 3$ , we have  $1 \leq n \leq 3$ .
- III.  $\times$ . It is possible that  $m = n = 1$  such that all conditions are satisfied.

17. B

Total number of data is 25.

Range =  $17 - 13 = 4$

Inter-quartile range =  $16.5 - 14.5 = 2$

18. D

- A.  $\times$ . Mode of the distribution is 8.
- B.  $\times$ . Mean = 
$$\frac{5(3) + 6(4) + 7(23) + 8(50) + 9(40)}{3 + 4 + 23 + 50 + 40}$$
$$= 8$$
- C.  $\times$ . Median of the distribution is 8.
- D.  $\checkmark$ . Inter-quartile range =  $9 - 7.5$ 
$$= 1.5$$

19.  A

By simple calculation,  $a = x + 0.875$ ,  $b = x + 0.5$ ,  $c = x$ ,  $d = 4$

- I. ✓.  $a - c = 0.875 > 0$
- II. ✗.  $b - d = x - 3.5$ , which can be positive or negative.
- III. ✗.  $d - c = 4 - x$ , which can be positive or negative.

20.  D

We have  $m = n = 5$ .

- I. ✓.
- II. ✓. Mean =  $\frac{1+2+5+12+\dots+5}{10} = 5.2$
- III. ✓. Range =  $12 - 1 = 11$

21.  B

Upper quartile = \$40

Angle of sector \$10 =  $360^\circ - 72^\circ - 36^\circ - 90^\circ - 144^\circ = 18^\circ$

Lower quartile =  $\frac{20+30}{2} = \$25$

Inter-quartile range =  $40 - 25 = \$15$

22.  D

- A. ✗. Mode = 3
- B. ✗. Median = 3
- C. ✗. Lower quartile = 2.5
- D. ✓.

23.  A

There are 28 students. Lower quartile = 5 and upper quartile = 6.

Interquartile range =  $6 - 5 = 1$ .

24.  A

If  $x = 0$ , IQR = 3; if  $x = 8$ , IQR = 3.

If  $x \geq 9$ , IQR < 3. Thus,  $0 \leq x \leq 8$ .

25.  B

- I. ✗.  $m$  could be 4.
- II. ✓.
- III. ✗. Maximum value of  $x = 6.9 - 3.0 = 3.9 < 34$ .

26.  B

Inter-quartile range =  $82 - 67 = 15$

27. D

$$\text{Range} = 850 - 50 = 800$$

28. C

$$\text{Lower quartile} = \frac{28 + 30}{2} = 29 \text{ and upper quartile} = \frac{52 + (50 + a)}{2} = \frac{102 + a}{2}$$
$$\frac{102 + a}{2} - 29 \leq 24$$
$$a \leq 4$$

According to the stem-and-leaf diagram,  $2 \leq a \leq 8$ .

Thus,  $2 \leq a \leq 4$  and  $a = 2, 3$  or  $4$ .

29. C

Lower quartile = 47. Number of students has a weight exceeding lower quartile = 6.

$$\text{Required probability} = \frac{C_1^6 C_2^4}{C_3^{10}} = \frac{3}{10}$$

30. A

$$\text{Range} = 63.5 - 43.5 = 20 \text{ kg}$$

### Conventional Questions

31. (a)  $11 = 28 - \frac{(20 + c) + 13}{2}$  1M  
 $c = 1$  1A

(b)  $(30 + b) - (10 + a) \geq 25$   
 $b - a \geq 5$   
 $23 = \frac{(10 + a) + 12 + 13 + \dots + (30 + b)}{16}$  1M  
 $a + b = 6$   
We have  $a = 0$  and  $b = 6$ . 1A+1A

32. (a) The median = 31 1A  
The mode = 23 1A

(b) (i) Since  $0 \leq a \leq 5$  and  $7 \leq b \leq 9$ ,  
we have  $\begin{cases} a = 0 \\ b = 7 \end{cases}$ ,  $\begin{cases} a = 1 \\ b = 8 \end{cases}$  or  $\begin{cases} a = 2 \\ b = 9 \end{cases}$ . 1A+1A  
(ii) Required probability =  $\frac{3 + 3 + 3 + 3 + 2 + 9 + 9}{20 \times 13}$  1M  
 $= \frac{8}{65}$  1A

33. (a)  $(80 + b) - (30 + a) = 57$   
 $b - a = 7$  1M  
So,  $(a, b) = (0, 7)$  or  $(1, 8)$ . 1A+1A

(b) Interquartile range =  $72.5 - 44 = 28.5$  g 1M+1A

(c) Proportion of loaves of bread in Bakery A that exceed 70 g =  $\frac{4}{17} < \frac{1}{4}$ . 1M  
The claim is disagreed. 1A

34. (a)  $47 = (50 + c) - (10 + a)$  1M  
 $c - a = 7$   
 $33 = \frac{(10 + a) + 14 + 18 + \dots + (50 + c)}{18}$  1M  
 $a + b + c = 10$   
Since  $0 \leq a \leq 4$ ,  $0 \leq b \leq 3$  and  $7 \leq c \leq 9$ , we have  
 $(a, b, c) = (0, 3, 7)$  or  $(1, 1, 8)$ . 1A

(b) Required probability =  $\frac{10}{18}$  1M  
 $= \frac{5}{9}$  1A