

REG-DISP-2324-ASM-SET 1-MATH**Suggested solutions****Multiple Choice Questions**

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|-------|-------|-------|-------|-------|
| 1. D | 2. A | 3. A | 4. A | 5. D |
| 6. B | 7. D | 8. C | 9. A | 10. A |
| 11. B | 12. C | 13. B | 14. A | 15. B |
| 16. A | 17. B | 18. D | 19. A | 20. D |
| 21. B | 22. D | 23. A | 24. A | 25. B |
| 26. B | 27. D | 28. C | 29. C | 30. A |

1. D

We have $x = 5$ or $x + 2 = 5$.

When $x + 2 = 5$, $x = 3$ and the mode is 3 and 5, which should be rejected.

Thus, we have $x = 5$, and the numbers are 2, 3, 5, 5, 7, 8.

I. ✓. Mean = $\frac{2 + 3 + 5 + 5 + 7 + 8}{6} = 5$

II. ✓. Range = $8 - 2 = 6$

III. ✓. Inter-quartile range = $7 - 3 = 4$

2. A

Upper quartile = 59 kg and lower quartile = 46 kg

Inter-quartile range = $59 - 46$

$$= 13 \text{ kg}$$

3. A

$$\frac{4 + 4 + 6 + \dots + n}{9} = 6$$

$$m + n = 10$$

I. ✓. Since m and n cannot be both greater than 6, the median (the 5th datum) must be 6.

II. ✓. Range is the greatest when the values of m and n are 1 and 9.

Range is the least when $m = n = 5$.

Thus, $5 \leq y \leq 8$.

III. ✗. When $m = 4$ and $n = 6$, mode = 4 and 6.

4. A

Upper quartile = $(60 + b)$ and lower quartile = $(30 + a)$.

$$(60 + b) - (30 + a) \leq 25 \Rightarrow b - a \leq -5$$

- I. ✓. Since $a \geq 5 + b$, value of a is at least 5, and at most 9 (because it is a single digit integer).
- II. ✓. Since $b \leq a - 5$ and $a \leq 9$, value of b is at most $9 - 5 = 4$, and at least 0 (single digit integer).
- III. ✗. It is possible that $a = 9$ and $b = 1$, such that the interquartile range is 22 (≤ 25), while $a - b = 8$.

5. D

$$\frac{11 + 5 + 17 + 8 + 14 + 9 + 13 + 10 + x}{9} = 12$$

$$x = 3$$

Arrange the data in ascending order:

3 5 8 9 10 11 13 14 17

$$\text{Inter-quartile range} = 13.5 - 6.5 = 7$$

6. B

Denote the data sets $\{x - 2, x - 1, x, x + 2, x + 2, x + 3\}$ and $\{x - 5, x - 4, x - 2, x - 2, x - 1, x\}$ by A and B respectively.

- I. ✓. Range of $A = (x + 3) - (x - 2) = 5$
Range of $B = x - (x - 5) = 5$
- II. ✗. Median of $A = x + 1$
Median of $B = x - 2$
- III. ✓. Inter-quartile range of $A = (x + 2) - (x - 1) = 3$
Inter-quartile range of $B = (x - 1) - (x - 4) = 3$

7. D

Lower quartile = 62 kg.

$$\text{Required probability} = 1 - \frac{4}{24} = \frac{5}{6}.$$

8. C

$$\text{Range} = (40 + k) - (20 + h) \geq 24$$

$$k - h \geq 4$$

- I. ✗. It is possible that $h = 3$ and $k = 9$ such that range = 26 ≥ 24 .
- II. ✓. $k \geq 4 + h \geq 4 + 0 = 4$ and $k \leq 9$ obviously.
- III. ✓. $k - h \geq 4$ as proved above. Also, $k - h \leq 9 - 0 = 9$

9. A

$$a = x - \frac{18}{11}, b = x, c = x \text{ and } d = 10.$$

I. ✓. $a - b = -\frac{18}{11} < 0$

II. ✗. $b = c = x$

III. ✗. $c - d = x - 10$ which can be positive or negative.

10. A

We have $3 \leq m, n \leq 12$.

I. ✓. No matter how large or how small m and n , the median (5th datum) is always 8.

II. ✗. When $m = n = 3$, mean = $\frac{58}{9} \neq 8$.

III. ✗. When $m = n = 3$, mode = 3 \neq 8.

11. B

$$\text{Range} = (40 + k) - (10 + h) \geq 33 \Rightarrow k - h \geq 3.$$

I. ✗. It is possible that $k = 7$ and $h = 4$. The range is 33, satisfying all the conditions.

II. ✓. $k - h \geq 3 \Rightarrow k \geq 3 + h \geq 3$.

III. ✗. It is possible that $k = 9$ and $h = 0$. The range is 39, satisfying all the conditions.

12. C

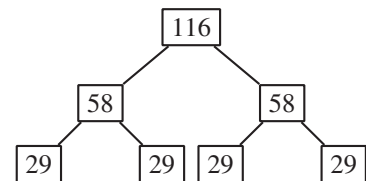
Number of students = 116. We can obtain the median and quartiles using the figure below.

A. ✗. Mode = 7

B. ✗. Median = 7

C. ✓. Lower quartile = 6

D. ✗. Upper quartile = 8



13. B

The upper quartile of the distribution is 210 g.

$$\text{Required probability} = \frac{7}{24}$$

14. A

Arrange the data in ascending order:

16 19 23 24 29 31 33

We have $24 \leq x \leq 29$ and $\frac{x+24}{2} = 25$.

Thus, we have $x = 26$.

Inter-quartile range = $30 - 21 = 9$

15. B

Mean = 5 $\Rightarrow 2 + 3 + 4 + \dots + m + n = 5 \times 9 \Rightarrow m + n = 4$

I. ✗. It is possible that $m = 1$ and $n = 3$. Then mode = $a = 3$.

II. ✓. We have $m < 4$ and $n < 4$. Median = $b = 5$ th datum = 4.

III. ✗. It is possible that $m = 1$ and $n = 3$. Then range = $c = 10 - 1 = 9 \neq 8$.

16. A

Median = $\frac{(20+n)+25}{2} \leq 24$ and Interquartile range = $(30+n) - (10+m) \geq 18$

$$n \leq 3$$

$$n - m \geq -2$$

$$m - n \leq 2$$

I. ✓. $m \leq n + 2 \leq 3 + 2 = 5$ and $m \geq 0$.

II. ✓. From the stem-and-leaf diagram, $n \geq 1$. Combine with $n \leq 3$, we have $1 \leq n \leq 3$.

III. ✗. It is possible that $m = n = 1$ such that all conditions are satisfied.

17. B

Total number of data is 25.

Range = $17 - 13 = 4$

Inter-quartile range = $16.5 - 14.5 = 2$

18. D

A. ✗. Mode of the distribution is 8.

B. ✗. Mean = $\frac{5(3) + 6(4) + 7(23) + 8(50) + 9(40)}{3 + 4 + 23 + 50 + 40}$
= 8

C. ✗. Median of the distribution is 8.

D. ✓. Inter-quartile range = $9 - 7.5$
= 1.5

19. A

By simple calculation, $a = x + 0.875$, $b = x + 0.5$, $c = x$, $d = 4$

I. ✓. $a - c = 0.875 > 0$

II. ✗. $b - d = x - 3.5$, which can be positive or negative.

III. ✗. $d - c = 4 - x$, which can be positive or negative.

20. D

We have $m = n = 5$.

I. ✓.

II. ✓. Mean = $\frac{1 + 2 + 5 + 12 + \dots + 5}{10} = 5.2$

III. ✓. Range = $12 - 1 = 11$

21. B

Upper quartile = \$40

Angle of sector \$10 = $360^\circ - 72^\circ - 36^\circ - 90^\circ - 144^\circ = 18^\circ$

Lower quartile = $\frac{20 + 30}{2} = \$25$

Inter-quartile range = $40 - 25 = \$15$

22. D

A. ✗. Mode = 3

B. ✗. Median = 3

C. ✗. Lower quartile = 2.5

D. ✓.

23. A

There are 28 students. Lower quartile = 5 and upper quartile = 6.

Interquartile range = $6 - 5 = 1$.

24. A

If $x = 0$, IQR = 3; if $x = 8$, IQR = 3.

If $x \geq 9$, IQR < 3. Thus, $0 \leq x \leq 8$.

25. B

I. ✗. m could be 4.

II. ✓.

III. ✗. Maximum value of $x = 6.9 - 3.0 = 3.9 < 34$.

26. B

Inter-quartile range = $82 - 67 = 15$

27. D

$$\text{Range} = 850 - 50 = 800$$

28. C

$$\begin{aligned}\text{Lower quartile} &= \frac{28 + 30}{2} = 29 \text{ and upper quartile} = \frac{52 + (50 + a)}{2} = \frac{102 + a}{2} \\ \frac{102 + a}{2} - 29 &\leq 24 \\ a &\leq 4\end{aligned}$$

According to the stem-and-leaf diagram, $2 \leq a \leq 8$.

Thus, $2 \leq a \leq 4$ and $a = 2, 3$ or 4 .

29. C

Lower quartile = 47. Number of students has a weight exceeding lower quartile = 6.

$$\text{Required probability} = \frac{{}^6C_1 {}^4C_2}{{}^{10}C_3} = \frac{3}{10}$$

30. A

$$\text{Range} = 63.5 - 43.5 = 20 \text{ kg}$$

Conventional Questions

31. (a) $11 = 28 - \frac{(20 + c) + 13}{2}$ 1M
 $c = 1$ 1A
- (b) $(30 + b) - (10 + a) \geq 25$
 $b - a \geq 5$
 $23 = \frac{(10 + a) + 12 + 13 + \dots + (30 + b)}{16}$ 1M
 $a + b = 6$
 We have $a = 0$ and $b = 6$. 1A+1A
32. (a) The median = 31 1A
 The mode = 23 1A
- (b) (i) Since $0 \leq a \leq 5$ and $7 \leq b \leq 9$,
 we have $\begin{cases} a = 0 \\ b = 7 \end{cases}$, $\begin{cases} a = 1 \\ b = 8 \end{cases}$ or $\begin{cases} a = 2 \\ b = 9 \end{cases}$. 1A+1A
- (ii) Required probability = $\frac{3 + 3 + 3 + 3 + 2 + 9 + 9}{20 \times 13}$ 1M
 $= \frac{8}{65}$ 1A
33. (a) $(80 + b) - (30 + a) = 57$
 $b - a = 7$ 1M
 So, $(a, b) = (0, 7)$ or $(1, 8)$. 1A+1A
- (b) Interquartile range = $72.5 - 44 = 28.5$ g 1M+1A
- (c) Proportion of loaves of bread in Bakery A that exceed 70 g = $\frac{4}{17} < \frac{1}{4}$. 1M
 The claim is disagreed. 1A
34. (a) $47 = (50 + c) - (10 + a)$ 1M
 $c - a = 7$
 $33 = \frac{(10 + a) + 14 + 18 + \dots + (50 + c)}{18}$ 1M
 $a + b + c = 10$
 Since $0 \leq a \leq 4$, $0 \leq b \leq 3$ and $7 \leq c \leq 9$, we have
 $(a, b, c) = (0, 3, 7)$ or $(1, 1, 8)$. 1A
- (b) Required probability = $\frac{10}{18}$ 1M
 $= \frac{5}{9}$ 1A