**DAYM2-TRIG-2324-ASM-SET 1-MATH**
**Suggested solutions**

1. (a) $\dfrac{\cos A+\cos B}{\sin A+\sin B}$

$= \dfrac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}$ 1M

$= \dfrac{\cos\frac{A+B}{2}}{\sin\frac{A+B}{2}}$

$= \cot\dfrac{A+B}{2}$ 1

(b) $5\sin A-3\cos A = 3\cos B-5\sin B$

$5(\sin A+\sin B) = 3(\cos A+\cos B)$

$\dfrac{5}{3} = \dfrac{\cos A+\cos B}{\sin A+\sin B}$ $\quad(\sin A+\sin B\neq 0)$

$\cot\dfrac{A+B}{2} = \dfrac{5}{3}$

$\tan\dfrac{A+B}{2} = \dfrac{3}{5}$ 1A

$\cot(A+B) = \dfrac{1-\tan^2\frac{A+B}{2}}{2\tan\frac{A+B}{2}}$ 1M

$= \dfrac{1-\left(\frac{3}{5}\right)^2}{2\left(\frac{3}{5}\right)}$

$= \dfrac{8}{15}$ 1A

2. (a) $\cot 3x = \dfrac{1}{\tan(2x+x)}$

$$= \frac{1-\tan 2x \tan x}{\tan 2x + \tan x} \qquad \text{1M}$$

$$= \frac{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)\tan x}{\frac{2\tan x}{1-\tan^2 x}+\tan x}$$

$$= \frac{(1-\tan^2 x)-2\tan^2 x}{2\tan x + \tan x(1-\tan^2 x)}$$

$$= \frac{1-3\tan^2 x}{3\tan x - \tan^3 x}$$

$$= \frac{1-\frac{3}{\cot^2 x}}{\frac{3}{\cot x}-\frac{1}{\cot^3 x}} \times \frac{\cot^3 x}{\cot^3 x} \qquad \text{1M}$$

$$= \frac{\cot^3 x - 3\cot x}{3\cot^2 x - 1} \qquad \text{1}$$

(b)

$$\cot x - 3\cot 3x = 4$$

$$\cot x - 3\left(\frac{\cot^3 x - 3\cot x}{3\cot^2 x - 1}\right) = 4 \qquad \text{1M}$$

$$(3\cot^3 x - \cot x) - 3(\cot^3 x - 3\cot x) = 4(3\cot^2 x - 1)$$

$$-12\cot^2 x + 8\cot x + 4 = 0 \qquad \text{1M}$$

$$\cot x = 1 \quad \text{or} \quad -\frac{1}{3}$$

$$x = \frac{\pi}{4} \qquad \text{1A}$$

3. (a) $\cos\theta(x+\sec\theta)(x\cos 3\theta - 1)$

$$= (x\cos\theta + 1)(x\cos 3\theta - 1)$$

$$= x^2\cos 3\theta + x(\cos 3\theta - \cos\theta) - 1$$

$$= x^2\cos 3\theta + x\left(-2\sin\frac{3\theta+\theta}{2}\sin\frac{3\theta-\theta}{2}\right) - 1 \qquad \text{1M}$$

$$= x^2\cos 3\theta - 2x\sin 2\theta\sin\theta - 1 \qquad \text{1}$$

(b)

$$\cos 4\theta + \cos 2\theta + 4\sin^2\theta\cos\theta = \frac{1}{2}$$

$$2\cos 3\theta\cos\theta + 2(2\sin\theta\cos\theta)\cos\theta = \frac{1}{2} \qquad \text{1M}$$

$$4\cos 3\theta\cos\theta + 4\sin 2\theta\sin\theta = 1$$

$$(-2)^2\cos 3\theta\cos\theta - 2(-2)\sin 2\theta\sin\theta - 1 = 0$$

$$\cos\theta(-2+\sec\theta)(-2\cos 3\theta - 1) = 0 \qquad \text{1M}$$

$$\cos\theta = 0 \quad \text{or} \quad \sec\theta = 2 \quad \text{or} \quad \cos 3\theta = -\frac{1}{2}$$

$$\theta = \frac{\pi}{2}\ \text{(rejected)} \qquad \cos\theta = \frac{1}{2} \qquad 3\theta = \frac{2\pi}{3} \quad \text{or} \quad \frac{4\pi}{3}$$

$$\theta = \frac{\pi}{3} \qquad \theta = \frac{2\pi}{9} \quad \text{or} \quad \frac{4\pi}{9} \qquad \text{1A+1A}$$

4. (a) $\tan 4x = \dfrac{2\tan 2x}{1-\tan^2 2x}$ 1M

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-(2\tan x)^2}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$ 1

(b) $\cot^4 x + 4\cot^3 x - 6\cot^2 x - 4\cot x + 1 = 0$

$$1 + 4\tan x - 6\tan^2 x - 4\tan^3 x + \tan^4 x = 0$$

$$4\tan x - 4\tan^3 x = -(1 - 6\tan^2 x + \tan^4 x)$$

$$\frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x} = -1$$

$$\tan 4x = -1$$ 1M

$$4x = \frac{7\pi}{4} \quad \text{or} \quad \frac{11\pi}{4}$$

$$x = \frac{7\pi}{16} \quad \text{or} \quad \frac{11\pi}{16}$$ 1A+1A

5. (a) When $n = 1$,

$$\text{L.H.S.} = \sin\theta[\sin\frac{\pi}{4} + \sin\left(\frac{\pi}{4} + 2\theta\right)]$$
$$= \sin\theta\left[2\sin\left(\frac{\pi}{4} + \theta\right)\cos\theta\right]$$
$$= 2\sin\theta\sin\left(\frac{\pi}{4} + \theta\right)\cos\theta$$
$$\text{R.H.S.} = \sin 2\theta\sin\left(\frac{\pi}{4} + \theta\right)$$
$$= 2\sin\theta\sin\left(\frac{\pi}{4} + \theta\right)\cos\theta$$
$$= \text{L.H.S.}$$

It is true for $n = 1$. 1

Assume $\sin\theta\sum_{k=0}^{p}\sin\left(\frac{\pi}{4} + 2k\theta\right) = \sin(p+1)\theta\sin\left(\frac{\pi}{4} + p\theta\right)$, where $p \in \mathbb{Z}^+$. 1

$$\sin\theta\sum_{k=0}^{p+1}\sin\left(\frac{\pi}{4} + 2k\theta\right)$$
$$= \sin\theta\sum_{k=0}^{p}\sin\left(\frac{\pi}{4} + 2k\theta\right) + \sin\theta\sin\left(\frac{\pi}{4} + 2(p+1)\theta\right)$$
$$= \sin(p+1)\theta\sin\left(\frac{\pi}{4} + p\theta\right) + \sin\theta\sin\left(\frac{\pi}{4} + 2(p+1)\theta\right) \quad \text{1M}$$
$$= \frac{1}{2}\left[\cos\left(\frac{\pi}{4} - \theta\right) - \cos\left(\frac{\pi}{4} + (2p+1)\theta\right)\right] + \frac{1}{2}\left[\cos\left(\frac{\pi}{4} + (2p+1)\theta\right) - \cos\left(\frac{\pi}{4} + (2p+3)\theta\right)\right]$$
$$= \frac{1}{2}\left[\cos\left(\frac{\pi}{4} - \theta\right) - \cos\left(\frac{\pi}{4} + (2p+3)\theta\right)\right]$$
$$= \sin(p+2)\theta\sin\left(\frac{\pi}{4} + (p+1)\theta\right)$$

It is true for $n = p + 1$. 1

By M.I., it is true $\forall n \in \mathbb{Z}^+$. 1

(b) Put $\theta = \frac{\pi}{6}$ and $n = 33$.

$$\sin\frac{\pi}{6}\sum_{k=0}^{33}\sin\left[\frac{\pi}{4} + 2k\left(\frac{\pi}{6}\right)\right] = \sin\frac{34\pi}{6}\sin\left(\frac{\pi}{4} + \frac{33\pi}{6}\right) \quad \text{1M}$$
$$\frac{1}{2}\left[\sin\frac{\pi}{4} + \sin\frac{7\pi}{12} + \sin\frac{11\pi}{12} + \ldots + \sin\frac{45\pi}{4}\right] = \sin\frac{17\pi}{3}\sin\frac{23\pi}{4}$$
$$\sin\frac{\pi}{4} + \sin\frac{7\pi}{12} + \sin\frac{11\pi}{12} + \ldots + \sin\frac{45\pi}{4} = 2\sin\left(6\pi - \frac{\pi}{3}\right)\sin\left(6\pi - \frac{\pi}{4}\right)$$
$$= 2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{6}}{2} \quad \text{1A}$$

6. (a) $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta}$

$= \dfrac{\sin 3\theta \cos\theta - \cos 3\theta \sin\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin(3\theta - \theta)}{\frac{1}{2}\sin 2\theta}$ 1M

$= 2$ 1

(b) Put $\theta = \dfrac{\pi}{10}$.

$$\frac{\sin\frac{3\pi}{10}}{\sin\frac{\pi}{10}} - \frac{\cos\frac{3\pi}{10}}{\cos\frac{\pi}{10}} = 2 \qquad \text{1M}$$

$$\frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right)}{\sin\frac{\pi}{10}} - \frac{\cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right)}{\cos\frac{\pi}{10}} = 2$$

$$\frac{1 - 2\sin^2\frac{\pi}{10}}{\sin\frac{\pi}{10}} - \frac{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}{\cos\frac{\pi}{10}} = 2 \qquad \text{1M}$$

$$1 - 2\sin^2\frac{\pi}{10} - 2\sin^2\frac{\pi}{10} = 2\sin\frac{\pi}{10}$$

$$4\sin^2\frac{\pi}{10} + 2\sin\frac{\pi}{10} - 1 = 0$$

Thus, $\sin\dfrac{\pi}{10}$ is a root of the quadratic equation $4x^2 + 2x - 1 = 0$. 1

$$4x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$

Since $0 < \dfrac{\pi}{10} < \dfrac{\pi}{2}$, we have $\sin\dfrac{\pi}{10} > 0$.

Thus, $\sin\dfrac{\pi}{10} = \dfrac{-1 + \sqrt{5}}{4}$. 1A

7. (a) When $n = 1$,

L.H.S. $= 2(1 - \cos x)\sin x = 2\sin x - \sin 2x$

R.H.S. $= 2\sin x - \sin 2x =$ L.H.S.

It is true for $n = 1$. 1

Assume $2(1 - \cos x)\sum_{k=1}^{p} k\sin kx = (p+1)\sin px - p\sin(p+1)x$, where $p \in \mathbb{Z}^+$. 1

$$2(1 - \cos x)\sum_{k=1}^{p+1} k\sin kx$$

$$= 2(1 - \cos x)\sum_{k=1}^{p} k\sin kx + 2(1 - \cos x)(p+1)\sin(p+1)x$$

$$= (p+1)\sin px - p\sin(p+1)x + 2(1 - \cos x)(p+1)\sin(p+1)x \qquad \text{1M}$$

$$= (p+1)\sin px + (p+2)\sin(p+1)x - 2(p+1)\cos x\sin(p+1)x$$

$$= (p+1)\sin px + (p+2)\sin(p+1)x - (p+1)[\sin(p+2)x + \sin px]$$

$$= (p+2)\sin(p+1)x - (p+1)\sin(p+2)x$$

It is true for $n = p + 1$.

By M.I., it is true $\forall n \in \mathbb{Z}^+$. 1

(b) Put $x = \frac{\pi}{2}$ and $n = 2018$.

$$2\left(1 - \cos\frac{\pi}{2}\right)\sum_{k=1}^{2018} k\sin\frac{k\pi}{2} = (2018+1)\sin\frac{2018\pi}{2} - 2018\sin\frac{2019\pi}{2} \qquad \text{1M}$$

$$2\sum_{k=1}^{2018} k\sin\frac{k\pi}{2} = 2018$$

$$4\sum_{k=1}^{2018} k\sin\frac{k\pi}{4}\cos\frac{k\pi}{4} = 2018 \qquad \text{1M}$$

$$\sum_{k=1}^{2018} k\sin\frac{k\pi}{4}\cos\frac{k\pi}{4} = \frac{1009}{2} \qquad \text{1A}$$

8. (a) When $n = 1$,

L.H.S. $= 2\cos\theta\,[-\cos 2\theta + \cos 4\theta]$

$= -(\cos 3\theta \cos\theta) + (\cos 5\theta + \cos 3\theta)$

$= \cos 5\theta - \cos\theta$

R.H.S. $= \cos 5\theta - \cos\theta =$ L.H.S.

It is true for $n = 1$. 1

Assume $2\cos\theta \sum_{k=1}^{2p} (-1)^k \cos 2k\theta = \cos(4p+1)\theta - \cos\theta$, where $p \in \mathbb{Z}^+$. 1

$$2\cos\theta \sum_{k=1}^{2p+2} (-1)^k \cos 2k\theta$$

$$= 2\cos\theta \sum_{k=1}^{2p} (-1)^k \cos 2k\theta + 2\cos\theta\left[(-1)^{2p+1}\cos(4p+2)\theta + (-1)^{2p+2}\cos(4p+4)\theta\right]$$

$$= \cos(4p+1)\theta - \cos\theta + 2\cos\theta[-\cos(4p+2)\theta + \cos(4p+4)\theta] \quad \text{1M}$$

$$= \cos(4p+1)\theta - \cos\theta - [\cos(4p+3)\theta + \cos(4p+1)\theta] + [\cos(4p+5)\theta + \cos(4p+3)\theta] \quad \text{1M}$$

$$= \cos(4p+5)\theta - \cos\theta$$

It is true for $n = p + 1$.

By M.I., it is true $\forall n \in \mathbb{Z}^+$. 1

(b) Put $\theta = \frac{\pi}{14}$ and $n = 64$.

$$2\cos\frac{\pi}{14} \sum_{k=1}^{128} (-1)^k \cos\frac{2k\pi}{14} = \cos\frac{257\pi}{14} - \cos\frac{\pi}{14} \quad \text{1M}$$

$$2\cos\frac{\pi}{14} \sum_{k=1}^{128} (-1)^k \left(2\cos^2\frac{k\pi}{14} - 1\right) = \cos\left(18\pi + \frac{\pi}{2} - \frac{\pi}{7}\right) - \cos\frac{\pi}{14} \quad \text{1M}$$

$$2\cos\frac{\pi}{14}\left[2\sum_{k=1}^{128} (-1)^k \cos^2\frac{k\pi}{14} - \sum_{k=1}^{128} (-1)^k\right] = 2\sin\frac{\pi}{14}\cos\frac{\pi}{14} - \cos\frac{\pi}{14}$$

$$4\sum_{k=1}^{128} (-1)^k \cos^2\frac{k\pi}{14} - 0 = 2\sin\frac{\pi}{14} - 1$$

$$\sum_{k=1}^{128} (-1)^k \cos^2\frac{k\pi}{14} = \frac{1}{2}\sin\frac{\pi}{14} - \frac{1}{4}$$

Thus, $a = \frac{1}{2}$, $b = \frac{1}{14}$ and $c = -\frac{1}{4}$. 1A

9. $\sin(7x - 1) - 3\cos 3x + \sin(x - 1) = 0$

$$2\sin(4x-1)\cos 3x - 3\cos 3x = 0 \quad \text{1M}$$

$$\cos 3x[2\sin(4x-1) - 3] = 0$$

$$\cos 3x = 0 \quad \text{or} \quad \sin(4x-1) = \frac{3}{2} \text{ (rejected)} \quad \text{1M}$$

$$3x = \frac{\pi}{2} \quad \text{1A}$$

$$x = \frac{\pi}{6} \quad \text{1A}$$

 **END OF PAPER**