DAYCP-AOT-2324-ASM-SET 1-MATH
Suggested solutions

Conventional Questions

1. (a) $BC^2 = 30^2 + 30^2 - 2(30)(30)\cos 40°$ 1M

$BC \approx 20.5\text{ cm}$ 1A

(b) Since $\triangle ABC$ is equilateral, the circumcentre of $\triangle ABC$ coincides with centroid of $\triangle ABC$.

$r = \frac{2}{3} \times BC \sin 60°$ 1M

$\approx 11.8\text{ cm}$ 1A

(c) Required angle $= \cos^{-1} \frac{r}{30}$ 1M

$\approx 66.7°$ 1A

2. (a) Let K be a point on EF such that $CK \perp EF$.
Consider $\triangle CFK$.

$$\cos \angle CFK = \frac{\left(\frac{60-30}{2}\right)}{50}$$

$$\angle CFK \approx 72.5°$$

$\angle DCF = 180° - \angle CFK \approx 107°$ 1A

Consider $\triangle DCF$.

$$DF^2 = 30^2 + 50^2 - 2(30)(50)\cos \angle DCF$$

$DF \approx 65.6\text{ cm}$ 1A

(b) Let J be a point on FH such that $CJ \perp FH$.
$AC = \sqrt{30^2 + 30^2} = 30\sqrt{2}\text{ cm}$
$HF = \sqrt{60^2 + 60^2} = 60\sqrt{2}\text{ cm}$
Consider $\triangle CFJ$.

$CJ = \sqrt{50^2 - \left(\frac{HF - AC}{2}\right)^2}$ 1M

$= 5\sqrt{82}\text{ cm}$

Height of the frustum is $5\sqrt{82}$ cm. 1A

(c) $BD = AC = 30\sqrt{2}\text{ cm}$
$BF = DF \approx 65.6\text{ cm}$
Consider $\triangle BDF$.
Let $s = \frac{BD + BF + DF}{2}$.
Area of $\triangle BDF = \sqrt{s(s - BD)(s - DF)(s - BF)}$ 1M

$\approx 1320\text{ cm}^2$

Let h cm be the required distance.

Consider the volume of the tetrahedron $CBDF$.

$$\frac{1}{3}(\text{area of } \triangle BDF)(h) = \frac{1}{3}\left[\frac{(30)(30)}{2}\right](5\sqrt{82}) \qquad \text{1M}$$

$$h \approx 15.5 \qquad \text{1A}$$

Required distance is 15.5 cm.

3. (a) $AC = \sqrt{78^2 + 104^2}$

$= 130\,\text{cm}$

$AB = \sqrt{130^2 - 120^2}$ 1M

$= 50\,\text{cm}$ 1A

$\angle BAD = \tan^{-1}\dfrac{120}{50} + \tan^{-1}\dfrac{104}{78}$

$\approx 121°$

$BD^2 = 50^2 + 78^2 - 2(50)(78)\cos\angle BAD$ 1M

$BD = 112\,\text{cm}$ 1A

(b) (i) Note that $GA = GB = GC = GD = \dfrac{AC}{2} = 65\,\text{cm}$.

$VG = \sqrt{169^2 - 65^2}$

$= 156\,\text{cm}$ 1A

(ii) Let H be the projection of G on the plane VBD.

Let h cm be the required distance.

Consider the area of $\triangle BDG$.

Let $s = \dfrac{65 + 65 + 112}{2} = 121\,\text{cm}$.

Area $= \sqrt{121(121-65)^2(121-112)}$ 1M

$= 1848\,\text{cm}^2$

Consider the area of $\triangle VBD$.

Let $s = \dfrac{169 + 169 + 112}{2} = 225\,\text{cm}$.

Area $= \sqrt{225(225-169)^2(225-112)}$

$= 840\sqrt{113}\,\text{cm}$

Consider the volume of the tetrahedron $VBGD$.

$$\frac{1}{3}(1848)(156) = \frac{1}{3}(840\sqrt{113})(h) \qquad \text{1M}$$

$$h \approx 32.3 \qquad \text{1A}$$

Required distance is 32.3 cm.

4. (a) (i) $AC = \sqrt{3 + 1^2} = 2$

$AK = \dfrac{1}{2}AC = 1$ 1A

$\tan\angle BAC = \dfrac{1}{\sqrt{3}} = \dfrac{KM}{1}$

$KM = \dfrac{1}{\sqrt{3}}$ 1A

(ii) $\tan\angle BAC = \dfrac{1}{\sqrt{3}}$

$\angle BAC = 30°$ 1M

Required distance $= \sqrt{3}\sin 30°$ 1M

$= \dfrac{\sqrt{3}}{2}$ 1A

(b) (i) Let E be the foot of perpendicular from B to AK. Then $\alpha = \angle BED$. 1M

$BD^2 = BE^2 + DE^2 - 2(BE)(DE)\cos\alpha$ 1M

$= \dfrac{3}{4} + \dfrac{3}{4} - 2\left(\dfrac{3}{4}\right)\cos\alpha$

$= \dfrac{3}{2}(1 - \cos\alpha)$ 1A

(ii) $MN^2 = KM^2 + KN^2 - 2(KM)(KN)\cos\alpha$

$MN^2 = \dfrac{2}{3}(1 - \cos\alpha)$ 1A

$\dfrac{MN^2}{BD^2} = \dfrac{\frac{2}{3}(1 - \cos\alpha)}{\frac{3}{2}(1 - \cos\alpha)}$

$= \dfrac{4}{9}$ 1A

Required ratio is 2 : 3. 1A

5. (a) $\angle BAC = 180° - 56° - 82° = 42°$

$\dfrac{BC}{\sin 42°} = \dfrac{20}{\sin 56°}$ 1M

$BC \approx 16.1\text{ cm}$ 1A

(b) (i) $10^2 = BC^2 + 20^2 - 2(BC)(20)\cos\angle ABC$ 1M

$\angle ABC \approx 29.8°$ 1A

(ii) Let E be a point on BD such that $CE \perp BD$.

Let F be a point on AB such that $EF \perp BD$.

Required angle is $\angle CEF$. 1M

$BF = BC \approx 16.1\text{ cm}$

$CE = FE = BC\sin\dfrac{82°}{2} \approx 10.6\text{ cm}$

$CF^2 = BC^2 + BF^2 - 2(BC)(BF)\cos\angle ABC$ 1M

$CF \approx 8.29\text{ cm}$

$CF^2 = EF^2 + CE^2 - 2(EF)(CE)\cos\angle CEF$ 1M

$\angle CEF \approx 46.1° > 45°$

The claim is agreed. 1A

6. (a) The paper card is symmetric about AC.

So, $\angle BAC = 60°$ and $\angle ACB = 180° - 60° - 70° = 50°$. 1A

$$\frac{AC}{\sin 70°} = \frac{10}{\sin 50°}$$ 1M

$AC \approx 12.3\,\text{cm}$ 1A

$BC = \dfrac{10\sin 60°}{\sin 50°} \approx 11.3\,\text{cm}$ 1A

(b) (i) $AE = \sqrt{AB^2 - BE^2}$ and $CE^2 = \sqrt{BC^2 - BE^2}$

$AC^2 = AE^2 + CE^2$

$= BC^2 + AB^2 - 2BE^2$ 1M

$BE \approx 6.22\,\text{cm}$ 1A

(ii) Let h cm be the required distance.

$AE = \sqrt{10^2 - BE^2} \approx 7.83\,\text{cm}$ and $CE = \sqrt{BC^2 - BE^2} \approx 9.44\,\text{cm}$

$$\frac{1}{3} \times \frac{(AE)(DB)}{2} \times CE = \frac{1}{3} \times \frac{(AD)(AC)\sin 60°}{2} \times h$$ 1M+1M

$h \approx 8.66$ 1A

Required distance is 8.66 cm.

7. (a) $AD = 20\cos 50° \approx 12.9\,\text{cm}$ 1A

$DB = \sqrt{30^2 - (20\sin 50°)^2} \approx 25.8\,\text{cm}$ 1A

(b) (i) $AB^2 = 20^2 + 30^2 - 2(20)(30)\cos 45°$ 1M

$AB \approx 21.2\,\text{cm}$ 1A

(ii) Required angle is $\angle ADB$.

$$AB^2 = AD^2 + DB^2 - 2(AD)(DB)\cos\angle ADB$$ 1M

$\angle ADB \approx 55.1°$ 1A

(iii) $CD = 20\sin 50° \approx 15.3\,\text{cm}$

Volume of tetrahedron $= \dfrac{1}{3} \times \dfrac{1}{2}(AD)(DB)\sin\angle ADB(CD)$ 1M

$\approx 695\,\text{cm}^3 > 650\,\text{cm}^3$

The claim is disagreed. 1A

8. (a) $$\frac{\sin\angle ABD}{19} = \frac{\sin 80°}{30}$$ 1M

$\angle ABD \approx 38.6°$ or $141°$ (rejected) 1A

(b) (i) $25^2 = 19^2 + 30^2 - 2(19)(30)\cos\angle CDA$ 1M

$\angle CDA \approx 56.1°$ 1A

(ii) Let E be a point on BD such that $AE \perp BD$.

Let F be a point on CD such that $EF \perp BD$.

Required angle is $\angle AEF$. 1A

$AE = 19\sin 80° \approx 18.7\,\text{cm}$ 1M

$DE = 19\cos 80° \approx 3.30\,\text{cm}$

$\angle EDF = \angle ABD \approx 38.6°$

$EF = DE \tan EDF \approx 2.63$ cm

$DF = \dfrac{DE}{\cos \angle EDF} \approx 4.22$ cm

$AF^2 = DF^2 + 19^2 - 2(DF)(19)\cos \angle CDA$ 1M

$AF \approx 17.0$ cm

In $\triangle AEF$,

$$AF^2 = AE^2 + EF^2 - 2(AE)(EF)\cos \angle AEF$$

$\angle AEF \approx 46.6°$ 1A

Required angle is $46.6°$.

9. (a) $VC = \sqrt{15^2 + 36^2}$

$= 39$ cm

$AC^2 = 39^2 + 12^2 - 2(39)(12)\cos 100°$ 1M

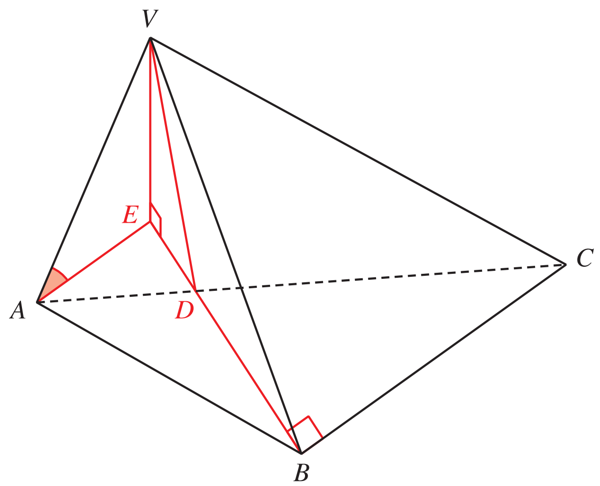
$AC \approx 42.7$ cm 1A

(b) Let D be a point on AC such that $BD \perp BC$.

Let E be a point on the line BD such that $VE \perp BD$.

Note that $VA \perp AB$ and $AE \perp AB$.

Required angle is $\angle VAE$. 1A



$AB = \sqrt{15^2 - 12^2} = 9$ cm

$9^2 = AC^2 + 36^2 - 2(AC)(36)\cos \angle ACB$

$\angle ACB \approx 8.70°$

$BD = 36 \tan \angle ACB \approx 5.51$ cm 1M

$CD = \dfrac{36}{\cos \angle ACB} \approx 36.4$ cm

$12^2 = 39^2 + AC^2 - 2(39)(AC)\cos \angle ACV$

$\angle ACV \approx 16.0°$

$VD^2 = 39^2 + CD^2 - 2(39)(CD)\cos \angle ACV$

$VD \approx 10.8$ cm

$VD^2 = 15^2 + BD^2 - 2(15)(BD)\cos\angle VBD$

$\angle VBD \approx 33.4°$

$VE = 15\sin\angle VBD \approx 8.26\,\text{cm}$ 1M

$\sin\angle VAE = \dfrac{VE}{12}$

$\angle VAE \approx 43.5°$

The claim is agreed. 1A

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