REG-PROB-2324-ASM-SET 3-MATH
Suggested solutions

Conventional Questions

1. (a) Required probability $= \dfrac{C_2^3}{C_2^6}$ 1M

$= \dfrac{1}{5}$ 1A

(b) Expected gain $= 10 \times \dfrac{1}{5} + 5 \times \dfrac{C_1^3 C_1^3}{C_2^6}$ 1M+1A

$= \$5$ 1A

2. (a) Required probability $= \dfrac{3}{8}$ 1A

(b) Required probability $= \dfrac{C_2^5}{C_2^8} \times \dfrac{3}{6}$ 1M

$= \dfrac{5}{28}$ 1A

(c) $P(A \text{ wins}) = \dfrac{3}{8} + \dfrac{5}{28} + \dfrac{C_4^5}{C_4^8} \times \dfrac{3}{4}$ 1M

$= \dfrac{17}{28}$ 1A

$> \dfrac{1}{2}$

Therefore, A is more likely to win the game. 1A

3. (a) Area of the larger circle $= r^2\pi$ 1A

$OP = OB \sin \dfrac{90°}{2}$

$= \dfrac{r\sqrt{2}}{2}$

Area of the smaller circle $= \left(\dfrac{r\sqrt{2}}{2}\right)^2 \pi = \dfrac{r^2\pi}{2}$ 1A

Area of the square $ABCD = (r\sqrt{2})^2 = 2r^2$ 1A

(b) Required probability $= \dfrac{2r^2 - \frac{r^2\pi}{2}}{r^2\pi}$

$= \dfrac{4 - \pi}{2\pi}$

≈ 0.137 1A

(c) Required probability $= \left(1 - \dfrac{4 - \pi}{2\pi}\right)^2 \left(\dfrac{4 - \pi}{2\pi}\right)$ 1M

≈ 0.102 1A

4. (a) Required probability $= \left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2$ 1M

$= \frac{5}{8}$ 1A

(b) (i) Required probability $= \left(1 - \frac{5}{8}\right)^3$ 1M

$= \frac{27}{512}$ 1A

(ii) Required probability $= \left(\frac{5}{8}\right)^3 + 3\left(\frac{5}{8}\right)\left(1 - \frac{5}{8}\right)^2$ 1M

$= \frac{65}{128}$ 1A

5. (a) $1 - (x)(0.9)(0.55) = 0.703$ 1M+1A

$x = 0.6$ 1A

(b) Required probability $= (0.6)(0.1)(0.45) + (0.4)(0.9)(0.45) + (0.4)(0.1)(0.55)$ 1M+1A

$= 0.211$ 1A

6. (a) Required probability $= 3(0.7)(1 - 0.7)^2$ 1M+1A

$= 0.189$ 1A

(b) Required probability $= 3(0.7)^2(1 - 0.7)$ 1M+1A

$= 0.441$ 1A

7. (a) Required probability $= \frac{28 \times 32}{60 \times 70}$ 1M

$= \frac{16}{75}$ 1A

(b) Required probability $= 1 - \frac{14 \times 12}{60 \times 70}$ 1M+1A

$= \frac{24}{25}$ 1A

(c) Required probability $= \frac{14 \times (32 + 26) + (28 + 12) \times 12}{60 \times 70}$ 1M+1A

$= \frac{341}{1050}$ 1A

8. (a) (i) Required probability $= \frac{C_2^8}{C_2^{20}}$ 1M

$= \frac{14}{95}$ 1A

(ii) Required probability $= \frac{C_1^3 C_1^8}{C_2^{20}}$ 1M

$= \frac{12}{95}$ 1A

(iii) Required probability $= \dfrac{C_2^{13}}{C_2^{20}}$ 1M

$= \dfrac{39}{95}$ 1A

(b) Required probability $= \dfrac{14}{95} \times \dfrac{3}{18} + \left(\dfrac{12}{95} + \dfrac{39}{95}\right) \times \dfrac{4}{18}$ 1M+1A

$= \dfrac{41}{285}$ 1A

9. (a) Required probability $= (50\%)[(0.4)^2 + 2(0.5)(0.1)] + (20\%)(0.5)^3$

$= 0.155$ 1A

(b) (i) Required probability $= (30\%)(0.1) + (50\%)[(0.5)^2 + 2(0.5)(0.4)] + 0.155$ 1M

$= 0.51$ 1A

(ii) Required probability $= (20\%)[(0.1)^3 + 3(0.4)(0.1)^2]$ 1M

$= 0.0026$ 1A

(c) Expected value $= 10(0.51) + 100(0.0026) + 20[1 - 0.51 - 0.0026 - (30\%)(0.5 + 0.4)]$ 1M+1M

$= \$9.708$ 1A

10. (a) Number of possible orders $= 5! = 120$ 1A

(b) Required probability $= \dfrac{1}{4!}$ 1M

$= \dfrac{1}{24}$ 1A

(c) Required probability $= \dfrac{C_3^4}{4!}$ 1M

$= \dfrac{1}{6}$ 1A

(d) Required probability $= \dfrac{1}{5} \times \dfrac{C_2^4}{4!} + \dfrac{2}{5} \times \dfrac{1}{6} + \dfrac{2}{5} \times \dfrac{1}{24}$ 1M+1A

$= \dfrac{2}{15}$ 1A

(e) Required probability $= \dfrac{\frac{1}{5} \times \frac{C_2^4}{4!}}{\frac{2}{15}}$ 1M

$= \dfrac{3}{8}$ 1A

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