REV-AOT-2324-ASM-SET 1-MATH
Suggested solutions

Conventional Questions

1. (a) $\dfrac{\sin \angle AVB}{36} = \dfrac{\sin 110^\circ}{60}$ 1M

$\angle AVB \approx 34.3^\circ$

$\angle VBA = 180^\circ - 110^\circ - \angle AVB \approx 35.7^\circ$ 1A

(b) $AM^2 = 36^2 + 30^2 - 2(36)(30)\cos \angle VBA$ 1M

$AM \approx 21.0\,\text{cm}$

$MN = \dfrac{BC}{2} = 10\,\text{cm}$ 1M

Let h cm be the height of the trapezium $ADNM$.

$$h = \sqrt{AM^2 - \left(\frac{20-10}{2}\right)^2} \qquad 1\text{M}$$

$\approx 20.4\,\text{cm}$

Area of trapezium $ADNM = \dfrac{h(20+10)}{2}$ 1M

$\approx 306\,\text{cm}^2$

$> 300\,\text{cm}^2$

The claim is disagreed. 1A

2. (a) (i) $QR^2 = 25^2 + 30^2 - 2(25)(30)\cos 95^\circ$ 1M

$QR \approx 40.7\,\text{cm}$ 1A

(ii) $25^2 = 30^2 + QR^2 - 2(30)(QR)\cos \angle PQR$ 1M

$\angle PQR \approx 37.7^\circ$ 1A

(b) Let R' and M' be the feet of perpendicular of R and M on the horizontal ground respectively.

$RR' = RP\sin 70^\circ \approx 23.5\,\text{cm}$

Since $\triangle QMM' \sim \triangle QRR'$, $MM' = \dfrac{RR'}{2} \approx 11.7\,\text{cm}$ 1M

$$PM^2 = 30^2 + \left(\frac{QR}{2}\right)^2 - 2(30)\left(\frac{QR}{2}\right)\cos \angle PQR$$

$PM \approx 18.7\,\text{cm}$

Angle between PM and the horizontal ground is $\angle MPM'$.

$\sin \angle MPM' = \dfrac{MM'}{PM}$ 1M

$\angle MPM' \approx 39.0^\circ < 40^\circ$

The claim is not correct. 1A

3. (a) Area of $\triangle ACD = \dfrac{1}{2}(20)(24)\sin 60^\circ$ 1M

$= 120\sqrt{3}\,\text{cm}^2$ 1A

(b) Let M be the foot of perpendicular from A to CD.

Required angle is $\angle AMB$. 1A

In $\triangle ACD$,

$$CD^2 = 20^2 + 24^2 - 2(20)(24)\cos 60° \quad \text{1M}$$

$$CD \approx 22.3\,\text{cm}$$

$$20^2 = CD^2 + 24^2 - 2(CD)(24)\cos\angle ADC$$

$$\angle ADC \approx 51.1°$$

In $\triangle ADM$, $AM = AD\sin\angle ADC \approx 18.7\,\text{cm}$ 1M

In $\triangle ABM$,

$$\sin\angle AMB = \frac{18}{AM}$$

$$\angle AMB \approx 74.7° \quad \text{1A}$$

Required angle is $74.7°$.

(c) AB is the shortest distance from A to the plane BCD.

Let h be the shortest distance from B to the plane ACD.

By considering volume of the tetrahedron,

$$\frac{1}{3}(\text{area of }\triangle ACD)(h) = \frac{1}{3}(\text{area of }\triangle BCD)(AB)$$

$$\frac{1}{3}\times\frac{1}{2}(CD)(AM)(h) = \frac{1}{3}\times\frac{1}{2}(CD)(BM)(AB)$$

$$h = \frac{BM}{AM}(AB) \quad \text{1M}$$

Since $AM > BM$, $h < AB$.

The claim is agreed. 1A

4. (a) Let D be the mid-point of AB.

In $\triangle ABC$,

$$CD = \sqrt{10^2 - 5^2}$$

$$= \sqrt{75}\,\text{cm}$$

In $\triangle VAB$,

$$VD = 5\tan 70°$$

$$\approx 13.7\,\text{cm}$$

$$VA = \frac{5}{\cos 70°}$$

$$\approx 14.6\,\text{cm} \quad \text{1A}$$

In $\triangle VCD$,

$$VC^2 = VD^2 + CD^2 - 2(VD)(CD)\cos\angle VDC \quad \text{1M}$$

$$\angle VDC \approx 77.9°$$

Height of the pyramid $= VD \sin \angle VDC$ 1M

$\approx 13.4\text{ cm}$ 1A

(b) (i) In $\triangle MAB$,

$$\frac{MA}{\sin 50^\circ} = \frac{10}{\sin(180^\circ - 70^\circ - 50^\circ)}$$ 1M

$MA \approx 8.85\text{ cm}$ 1A

(ii) Let h cm and H cm be the perpendicular distance from M to $\triangle ABC$ and the height of pyramid $VABC$ respectively.

$$\frac{h}{H} = \frac{AM}{VA}$$ 1M

$h \approx 8.13$ 1A

In $\triangle AMB$,

$$MB^2 = AM^2 + 10^2 - 2(AM)(10)\cos 70^\circ$$

$MB \approx 10.9\text{ cm}$ 1A

Therefore,

$$\sin\alpha = \frac{h}{MB}$$ 1M

$\alpha \approx 48.5^\circ$ 1A

5. (a) Let x be the side of the square cardboard.

Let P be the projection of D on the plane Π.

The angle between BD and the plane Π is $\angle DBP$. 1A

$DP = CD\sin 30^\circ = \dfrac{x}{2}$ 1M

$BD = \sqrt{x^2 + x^2} = \sqrt{2}x$

$$\sin\angle DBP = \frac{\left(\frac{x}{2}\right)}{\sqrt{2}x}$$ 1M

$\approx 20.7^\circ$ 1A

Required angle is 20.7°.

(b) (i) $AF = x \times \dfrac{1}{1+2} = \dfrac{x}{3}$ and $BF = x - \dfrac{x}{3} = \dfrac{2x}{3}$ 1A

In $\triangle BFM$,

$$\frac{\sin\angle FMB}{\left(\frac{2x}{3}\right)} = \frac{\sin 30^\circ}{\left(\frac{x}{3}\right)}$$ 1M

$\angle FMB = 90^\circ$

The angle between the plane $FEMN$ and the plane Π is $\angle FMB$, which is 90°.

The claim is agreed. 1

(ii) $BG : GN = DE : FB = 1 : 2$.

So, $BG = \sqrt{2}x \times \dfrac{2}{3} = \dfrac{2\sqrt{2}x}{3}$ and $GN = \dfrac{\sqrt{2}x}{3}$. 1M

$BM = BF\cos 30^\circ = \dfrac{\sqrt{3}x}{3}$ 1M

$$BN = \sqrt{BM^2 + MN^2} = \sqrt{\left(\frac{\sqrt{3}x)}{3}\right)^2 + x^2} = \frac{2x}{\sqrt{3}}$$ 1M

In $\triangle BGN$,

$$BN^2 = BG^2 + GN^2 - 2(BG)(GN)\cos\angle BGN$$ 1M

$$\frac{4x^2}{3} = \frac{8x^2}{9} + \frac{2x^2}{9} - \frac{16x^2}{9}\cos\angle BGN$$

$$\angle BGN \approx 104^\circ$$ 1A

Required angle is 104°.

END OF PAPER