REG-MAE-2324-ASM-SET 3-MATH
Suggested solutions

Conventional Questions

1. $3^{2x} - 5(3^x) - 24 = 0$

$(3^x - 8)(3^x + 3) = 0$

$3^x = 8 \quad \text{or} \quad -3 \text{ (rejected)}$ 2A

$x \log 3 = \log 8$

$x \approx 1.89$ 1A

2. $\log_9(x+5) = \log_3(1-x)$

$\frac{\log(x+5)}{\log 9} = \frac{\log(1-x)}{\log 3}$

$\frac{\log(x+5)}{2\log 3} = \frac{\log(1-x)}{\log 3}$ 1M

$\log(x+5) = 2\log(1-x)$

$x + 5 = (1-x)^2$ 1M

$x + 5 = x^2 - 2x + 1$

$0 = x^2 - 3x - 4$

$x = -1 \quad \text{or} \quad 4 \text{ (rejected)}$ 2A

3. $49^x - 4(7^x) - 77 = 0$

$(7^x)^2 - 4(7^x) - 77 = 0$ 1M

$7^x = 11 \quad \text{or} \quad -7 \text{ (rejected)}$ 1A

$x \log 7 = \log 11$

$x \approx 1.23$ 1A

4. $3^{\frac{2x}{3}} + 3^{\frac{x+6}{3}} = 36$

$(3^{\frac{x}{3}})^2 + 9(3^{\frac{x}{3}}) - 36 = 0$ 1M

$3^{\frac{x}{3}} = 3 \quad \text{or} \quad -12 \text{ (rejected)}$ 1A

$\frac{x}{3} = 1$

$x = 3$ 1A

5. $\log_{2x^2-x+1} 4 = \log_{x+1} 2$

$$\frac{2\log 2}{\log(2x^2 - x + 1)} = \frac{\log 2}{\log(x+1)}$$ 1M

$$2\log(x+1) = \log\left(2x^2 - x + 1\right)$$

$$\log(x+1)^2 = \log\left(2x^2 - x + 1\right)$$

$$(x+1)^2 = 2x^2 - x + 1$$ 1M

$$0 = x^2 - 3x$$

$x = 3$ or 0 (rejected) 1A

6. $\log x + \log 2y = \log(x - 4y)^2$

$$\log(2xy) = \log(x - 4y)^2$$

$$2xy = (x - 4y)^2$$ 1M

$$0 = x^2 - 10xy + 16y^2$$

$$0 = (x - 2y)(x - 8y)$$

$x = 2y$ or $8y$ 1A

$\frac{x}{y} = 2$ or 8 1A

$\log_{\sqrt{2}} \frac{x}{y} = 2$ or 6 1A

7. Let the two consecutive integers be x and $x + 1$.

$$\frac{1}{x} + \frac{1}{x+1} = \frac{7}{12}$$ 1A

$$12(x+1) + 12(x) = 7x(x+1)$$

$$0 = 7x^2 - 17x - 12$$ 1A

$x = 3$ or $-\frac{4}{7}$ (rejected) 1A

The two integers are 3 and 4. 1A

8. We have

$$\begin{cases} xy = 8 \\ (10x + y) - (10y + x) = 18 \end{cases}$$ 1A

$$9x - 9 \times \frac{8}{x} = 18$$ 1M

$$9x^2 - 18x - 72 = 0$$ 1A

$x = 4$ or -2 (rejected) 1A

When $x = 4$, $y = \frac{8}{4} = 2$. The original number is 42. 1A

9. $\dfrac{1}{x}+\dfrac{1}{2x-3}=\dfrac{1}{2+\frac{55}{60}}$ 1M+1M

$\dfrac{3x-3}{x(2x-3)}=\dfrac{12}{35}$

$35(3x-3)=12x(2x-3)$

$0=24x^2-141x+105$ 1A

$x=5$ or $\dfrac{7}{8}$ (rejected) 1A

10. Let the required speed be x km/min.

$\dfrac{2}{x+\frac{1}{36}}+\dfrac{2}{x-\frac{1}{36}}=16+\dfrac{12}{60}$ 1M+1M

$\dfrac{72}{36x+1}+\dfrac{72}{36x-1}=\dfrac{81}{5}$

$40(36x-1)+40(36x+1)=9(36x-1)(36x+1)$

$0=11\,664x^2-2880x-9$ 1A

$x=\dfrac{1}{4}$ or $-\dfrac{1}{324}$ (rejected) 1A

Required speed is $\dfrac{1}{4}$ km/min.

11. Let the cost of each bowl be $\$x$.

$\left(\dfrac{1500}{x}-12\right)(x+6)=1500+1038$ 1M+1M

$(1500-12x)(x+6)=2538x$

$-12x^2-1110x+9000=0$ 1A

$x=7.5$ or -100 (rejected) 1A

The cost of each bowl is \$7.5.

12. (a) $\dfrac{BE}{AB}=\dfrac{y}{x-y}$ 1A

(b) Let $u=\dfrac{x}{y}$.

$\dfrac{y}{x-y}=\dfrac{x}{y}$ 1M

$\dfrac{1}{\frac{x}{y}-1}=\dfrac{x}{y}$

$\dfrac{1}{u-1}=u$

$0=u^2-u-1$

$u=\dfrac{1\pm\sqrt{1^1-4(1)(-1)}}{2(1)}$ 1M

$=\dfrac{1+\sqrt{5}}{2}$ or $\dfrac{1-\sqrt{5}}{2}$ (rejected) 1A

13. (a) (i) $FG = \sqrt{a^2 + a^2}$

$= \sqrt{2}a$ cm 1A

(ii) $EH = \sqrt{b^2 + b^2}$

$= \sqrt{2}b$ cm 1A

(b) $2(4 - a) + 2(4 - b) + \sqrt{2}a + \sqrt{2}b = 8 + 4\sqrt{2}$ 1M

$8 - 4\sqrt{2} = (2 - \sqrt{2})(a + b)$

$a + b = 4$

$4^2 - \frac{1}{2}a^2 - \frac{1}{2}b^2 = 11$ 1M

$a^2 + b^2 = 10$

$(a + b)^2 - 2ab = 10$ 1M

$4^2 - 2ab = 10$

$ab = 3$ 1A

< 4

The claim is disagreed. 1A

14. (a) The coordinates of D are $(k + 15,\ \log(k + 15))$. 1A

(b) $\frac{[\log k + \log(k + 15)]15}{2} = 15$ 1M

$\log k(k + 15) = 2$

$k^2 + 15k = 10^2$

$k^2 + 15k - 100 = 0$ 1A

$k = 5$ or -20 (rejected) 1A

15. (a) $PA = 2\sqrt{x - 1}$ cm 1A

$QB = 4\sqrt{x - 1}$ cm 1A

(b) $2\sqrt{x - 1} + 8 + 4\sqrt{x - 1} = x$ 1M+1A

$6\sqrt{x - 1} = x - 8$

$36(x - 1) = (x - 8)^2$

$x^2 - 52x + 100 = 0$ 1M

$x = 50$ or $x = 2$ (rejected) 1A

(c) Total cost $= 50 \times 2\sqrt{50 - 1} \times 0.1 \times 3$ 1M

$= \$210 > \$200$ 1A

The amount is not sufficient for a student to make three book racks. 1A

 END OF PAPER