REG-LOCUS-2324-ASM-SET 3-MATH
Suggested solutions

Conventional Questions

1. (a) (i) Slope of $L = \dfrac{-2}{3}$.

The equation of ℓ is

$$y - 0 = \frac{3}{2}(x - 14) \quad \text{1M}$$

$$y = \frac{3x}{2} - 21 \quad \text{1A}$$

(ii) Required distance $= \left(\dfrac{-2k}{3} + 5\right) - \left(\dfrac{3k}{2} - 21\right)$ 1M

$$= \frac{-13k}{6} + 26 \quad \text{1A}$$

(b) (i) AB is the radius of Γ. 1A

(ii) $\dfrac{-13k}{6} + 26 = 39$ 1M

$k = -6$

Coordinates of A are $(-6,\ 9)$.

Denote the intersection of L and ℓ by E.

Solving L and ℓ, the coordinate of E are $(12,\ -3)$. 1M

$AE = \sqrt{(12+6)^2 + (9+3)^2} = \sqrt{468}$

$$\frac{r}{1} = \frac{\sqrt{468}}{39 - \sqrt{468}} \quad \text{1M}$$

$r \approx 1.25$ 1A

2. (a) (i) Let $P\,(x,\ y)$.

$$\frac{y+1}{x-5} \times \frac{y-5}{x+3} = -1 \quad \text{1M}$$

$$(x+3)(x-5) + (y+1)(y-5) = 0$$

$$x^2 + y^2 - 2x - 4y - 20 = 0 \quad \text{1A}$$

The equation of locus of P is $x^2 + y^2 - 2x - 4y - 20 = 0$.

(ii) Locus of P is a circle centred at $(1,\ 2)$ with radius 5, 1A+1A
excluding points A and B.

Remarks:

The fact that the locus excludes points A and B can be omitted.

(b) Centre of C is at $(9,\ 8)$.

Distance between centres $= \sqrt{(9-1)^2 + (8-2)^2} = 10$. 1M

Sum of radii $= \sqrt{25} + 5 = 10 =$ distance between centres 1M

The circles touch each other externally.

The claim is disagreed. 1A

3. (a) (0, 7) 1A

(b) (i) Let $P(x,\ y)$.

$$\sqrt{x^2+(y-7)^2}=\sqrt{(x-8)^2+(y-1)^2} \quad \text{1M}$$

$$x^2+y^2-14y+49=x^2+y^2-16x-2y+65$$

$$4x-3y-4=0 \quad \text{1A}$$

The equation of locus of P is $4x-3y-4=0$.

(ii) Let the mid-point of AE be F. Then $F(4,\ 4)$.

$AF=\sqrt{4^2+3^2}=5$ and radius of $C=\sqrt{7^2-40}=3<5$. 1M

Thus, F lies outside the circle and C does not have any intersection with Γ. 1A

(c) Required ratio $= AH : AE$

$= 3 : 2\times 5$

$= 3 : 10$ 1A

4. (a) mid-point of PQ is $(0,\ 2)$.

Equation of C is

$$x^2+(y-2)^2=(8)^2+(8-2)^2 \quad \text{1M}$$

$$x^2+(y-2)^2=100 \quad \text{1A}$$

(b) $GR=\sqrt{(4-0)^2+(5-2)^2}=5<10$ 1A

Thus, R lies inside C. 1

(c) $RU=\sqrt{10^2-5^2}=\sqrt{75}$ 1M

$TR\perp UV$ and $\triangle TUV$ is an isosceles triangle.

$TU=\sqrt{(\sqrt{75})^2+(10+5)^2}=\sqrt{300}$

Perimeter of $\triangle TUV = 2\sqrt{300}+2\sqrt{75}$ 1M

$\approx 52.0 > 50$ 1A

The claim is agreed. 1A

5. (a) (i) $\sqrt{(x-7)^2+(y+3)^2}=\sqrt{(x-1)^2+(y+1)^2}$ 1M+1A

$$-12x+4y+56=0$$

$$3x-y-14=0 \quad \text{1A}$$

The equation of Γ is $3x-y-14=0$.

(ii) Γ is the perpendicular bisector of AB. 1A

(b) (i) 3 1A

(ii) When $\triangle AQB$ is an acute-angled triangle,

$\angle AQB=\dfrac{1}{2}\angle AMB=30^\circ$. 1M+1A

When $\triangle AQB$ is an obtuse-angled triangle,

$\angle AQB=180^\circ-30^\circ=150^\circ$. 1A

6. (a) (i) Slope of $L = \frac{5-4}{5-2} = \frac{1}{3}$

Equation of L is

$$y - 5 = \frac{1}{3}(x-5) \quad \text{1M}$$

$$x - 3y + 10 = 0 \quad \text{1A}$$

(ii) $0 - 3y + 10 = 0$

$$y = \frac{10}{3}$$

The coordinates of C are $\left(0,\ \frac{10}{3}\right)$. 1A

(b) (i) $\sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-5)^2}$ 1M

$x^2 + y^2 - 4x - 8y + 20 = x^2 + y^2 - 10x - 10y + 50$

$$3x + y - 15 = 0 \quad \text{1A}$$

Required equation is $3x + y - 15 = 0$.

(ii) $3x - 0 - 15 = 0$

$$x = 5$$

The coordinates of D are $(5,\ 0)$. 1A

(c) Solve $\begin{cases} x - 3y - 10 = 0 \\ 3x + y - 15 = 0 \end{cases}$, we have $(x,\ y) = \left(\frac{7}{2},\ \frac{9}{2}\right)$. 1A

Area of $OCED = \frac{1}{2}\left(\frac{10}{3}\right)\left(\frac{7}{2}\right) + \frac{1}{2}(5)\left(\frac{9}{2}\right)$ 1M

$$= \frac{205}{12} < 20$$

The claim is agreed. 1A

7. (a) AD is the perpendicular bisector of BC. The coordinates of D are $(-8,\ -6)$. 1A

Since D is the mid-point of BC, the coordinates of C are $(-8,\ -16)$. 1A

mid-point of $AC = (1,\ -11)$ and slope of $AC = \frac{-6+16}{10+8} = \frac{5}{9}$ 1A

Required equation is

$$y + 11 = -\frac{9}{5}(x-1) \quad \text{1M}$$

$$9x + 5y + 46 = 0 \quad \text{1A}$$

(b) Since AD is perpendicular bisector of BC, y-coordinate of circumcentre = −6. 1M

Put $y = -6$ into $9x + 5y + 46 = 0$, we have $x = -\frac{16}{9}$.

Required coordinates are $\left(-\frac{16}{9},\ -6\right)$. 1A

(c) Required equation is

$$\left(x + \frac{16}{9}\right)^2 + (y + 6)^2 = \left(10 + \frac{16}{9}\right)^2 + (-6 + 6)^2$$ 1M

$$\left(x + \frac{16}{9}\right)^2 + (y + 6)^2 = \frac{11\,236}{81}$$ 1A

(d) (i) Locus of P is a parabola (opens rightwards). 1A

(ii) Let the coordinates of P be $(x,\ y)$.

$$\sqrt{(x - 10)^2 + (y + 6)^2} = \sqrt{(x + 8)^2}$$ 1M

$$y^2 - 36x + 12y + 72 = 0$$

Required equation is $y^2 - 36x + 12y + 72 = 0$. 1A

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