REG-LOCUS-2324-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. C	3. D	4. D	5. C
6. D	7. B	8. D	9. C	10. D
11. A	12. B	13. D	14. A	15. D
16. C	17. C	18. A		

1. $\boxed{\text{D}}$

AB is fixed, so the perpendicular distance from P to L is also a constant.
Thus, locus of P is a pair of straight lines, parallel to L and maintain a fixed distance from L.

2. $\boxed{\text{C}}$

y-coordinate of P is 5.
The locus of P is a horizontal line passing through $(-5,\ 5)$.

3. $\boxed{\text{D}}$

Locus of Q should be a pair of straight lines with infinite length, 2 units from L, one above L and one below L.

4. $\boxed{\text{D}}$

Let coordinates of P be $(x,\ y)$.

$$\begin{aligned}\sqrt{(x+3)^2+(y-1)^2} &= 2\sqrt{(x-6)^2+(y+2)^2}\\ (x+3)^2+(y-1)^2 &= 4(x-6)^2+4(y+2)^2\\ 0 &= 3x^2+3y^2-54x+18y+150\\ 0 &= x^2+y^2-18x+6y+50\end{aligned}$$

5. $\boxed{\text{C}}$

Note that MN is fixed and does not vary with P.
Locus of P is a circle with centre M and radius MN.

A. ✗. Not a circle.

B. ✗. Not a circle.

C. ✓.

D. ✗. Centre is at $(6,\ 8)$, not M.

6. D

y-coordinate of $P = 1 \pm 4$

$= 5$ or -3

Required equations are $y = -3$ and $y = 5$.

7. B

Locus of B is a line parallel to AC, slope $= -\frac{2}{3}$.

Only the slope of straight line in option B is $-\frac{2}{3}$.

8. D

Let the coordinates of P be (x, y).

$$\sqrt{(x-2)^2 + (y-0)^2} = \sqrt{[x-(-2)]^2}$$
$$x^2 + y^2 - 4x + 4 = x^2 + 4x + 4$$
$$y^2 = 8x$$

Required equation is $y^2 = 8x$.

9. C

$x^2 + y^2 = 4$ is a circle with centre $(0, 0)$ and radius 2.

The locus of P is a pair of concentric circles with centre $(0, 0)$ and radii 1 and 3 respectively.

The answer is C.

10. D

Let the coordinates of P be (x, y).

$$\sqrt{(x+1)^2 + (y-1)^2} = \sqrt{(x+1)^2 + (y-5)^2}$$
$$x^2 + y^2 + 2x - 2y + 2 = x^2 + y^2 + 2x - 10y + 26$$
$$y = 3$$

Required equation is $y = 3$.

11. A

Locus of P is the perpendicular bisector of AB.

Slope of AB is negative $\Rightarrow$ slope of locus of P is positive.

Positive slope $\Rightarrow$ A, B or C

Locus of P passes through mid-point of AB $(3, 2)$ $\Rightarrow$ A

12. B

Centre $(3, 2)$ satisfies the locus condition, so it lies on the locus $(x + 2y + k = 0)$

$$3 + 2(2) + k = 0$$
$$k = -7$$

13. D

L_1: $12x - 4y + 28 = 0$

Required equation is

$$12x - 4y + \frac{28 + (-11)}{2} = 0$$
$$24x - 8y + 17 = 0$$

14. A

$\angle ABC = 90^\circ$ and BC is parallel to x-axis.

Slope of $BP = \tan 45^\circ = 1$. Equation of BP is

$$y + 4 = 1(x + 4)$$
$$y = x$$

15. D

Since A lies above L, the locus of P is a parabola opening upwards.

16. C

Equidistant from a point and a straight line $\Rightarrow$ locus of P is a parabola.

17. C

Let the coordinates of P be $(x,\ y)$.

$$\sqrt{(x+1)^2 + (y-3)^2} = \sqrt{(y-0)^2}$$
$$x^2 + y^2 + 2x - 6y + 10 = y^2$$
$$x^2 + 2x - 6y + 10 = 0$$
$$y = \frac{1}{6}(x^2 + 2x + 10)$$

Required equation is $y = \frac{1}{6}(x^2 + 2x + 10)$.

18. A

The coordinates of the centre are $(2,\ 1)$.

The line joining mid-point of AB and centre $(2,\ 1)$ is vertical.

So, AB is parallel to the x-axis. The equation of AB is $y = 0$.

$$x^2 + 0^2 - 4x - 2(0) = 0$$
$$x = 0 \quad \text{or} \quad 4$$

The coordinates of A and B are $(0,\ 0)$ and $(4,\ 0)$.

Note that the locus of P is the circle with diameter AB.

Required equation is

$$(x-2)^2 + (y-0)^2 = (0-2)^2 + (0-0)^2$$
$$(x-2)^2 + y^2 = 4$$

Conventional Questions

19. $\sqrt{(x-2)^2+(y-7)^2}=\sqrt{(x+3)^2+(y+6)^2}$ 1M

$x^2+y^2-4x-14y+53=x^2+y^2+6x+12y+45$

$5x+13y-4=0$ 1A

Required equation is $5x+13y-4=0$. 1A

20. Let the coordinates of P be $(x,\ y)$.

$\sqrt{(x-11)^2+(y-1)^2}=y+3$ 1M+1M

$x^2+y^2-22x-2y+122=y^2+6y+9$

$x^2-22x-8y+113=0$ 1A

Required equation is $x^2-22x-8y+113=0$.

21. (a) Let the coordinates of P be $(x,\ y)$.

$\sqrt{(x+15)^2+(y-1)^2}=\sqrt{(x+9)^2+(y+7)^2}$ 1M

$x^2+y^2+30x-2y+226=x^2+y^2+18x+14y+130$

$12x-16y+96=0$

$3x-4y+24=0$

Equation of the locus of P is $3x-4y+24=0$. 1A

(b) Locus of P cuts x-axis and y-axis at $(-8,\ 0)$ and $(0,\ 6)$ respectively. 1A

When the circle is the smallest, centre = mid-point of $AB=(-4,\ 3)$.

Equation of C is

$(x+4)^2+(y-3)^2=(-8+4)^2+(0-3)^2$ 1M

$(x+4)^2+(y-3)^2=25$ 1A

22. (a) Let the radius be r. The coordinates of A are $(0,\ r)$.

$(3-0)^2+(r-9)^2=r^2$ 1M

$-18r+90=0$

$r=5$ 1A

The coordinates of A are $(0,\ 5)$.

(b) $x^2+(y-5)^2=5^2$ 1M

$x^2+y^2-10y=0$ 1A

(c) (i) Γ is a pair of straight lines perpendicular to L and their perpendicular distances from AB are equal to $\frac{BC}{2}$. 1A+1A

(ii) Let the coordinates of C be $(t,\ 0)$.

$$\frac{9-0}{3-t} \times \frac{9-5}{3-0} = -1$$ 1M

$$t = 15$$

Required distance $= \dfrac{BC}{2} - r$ 1M

$= \dfrac{OC}{2} - 5$

$= \dfrac{5}{2}$ 1A

23. (a) (i) $\angle OAD = 90^\circ$ *(given)*

$\angle OBD = 90^\circ$ *(tangent ⊥ radius)*

$= \angle OAD$

Therefore, A, B, O and D are concyclic. *(converse of ∠s in the same segment)*

$\angle OAD = 90^\circ$ *(given)*

$\angle OCE = 90^\circ$ *(tangent ⊥ radius)*

$\angle OAD + \angle OCE = 90^\circ + 90^\circ$

$= 180^\circ$

Therefore, A, O, C and E are concyclic. *(opp. ∠s supp.)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(ii) $\angle OBD = 90^\circ$ *(tangent ⊥ radius)*

$\angle OCE = 90^\circ$ *(tangent ⊥ radius)*

$= \angle OBD$

$OB = OC$ *(radii)*

$\angle ODB = \angle OAB$ *(∠s in the same segment)*

$= \angle OEC$ *(∠s in the same segment)*

$\triangle BDO \cong \triangle CEO$ *(AAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) Since $OC \perp CE$,

$$\frac{8-6}{3-0} \times \frac{0-8}{a-3} = -1$$ 1M

$$a = \frac{25}{3}$$ 1A

Required equation is

$$(x-0)^2+(y-6)^2=\left(\frac{25}{3}\right)^2+(0-6)^2 \qquad \text{1M}$$

$$x^2+(y-6)^2=\frac{949}{9} \qquad \text{1A}$$

(c) Let the coordinates of P be $(x,\ y)$.

$$\sqrt{(x-0)^2+(y-0)^2}=3\sqrt{(x-0)^2+(y-6)^2} \qquad \text{1M}$$

$$x^2+y^2=9x^2+9(y^2-12y+36)$$

$$2x^2+2y^2-27y+81=0 \qquad \text{1A}$$

 END OF PAPER