REG-LOCUS-2324-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. A	3. A	4. D	5. B
6. D	7. B	8. B	9. C	10. C
11. B	12. C	13. D	14. D	15. A
16. A	17. A	18. D	19. A	20. C
21. A	22. D	23. A	24. A	

1. D

   L_1 and L_2 are not parallel lines.
   The locus of P is the angle bisectors of the angles formed by L_1 and L_2, which is a pair of perpendicular lines.

2. A

   Locus of P is a circle with diameter AB.

3. A

   Fixed distance to a fixed point ⇒ circle

4. D

   L_1 and L_2 are parallel.
   Required locus is a straight line parallel to L_1, and is in the middle of L_1 and L_2.

5. B

   AB and BC are not parallel. The locus of P is the angle bisector of $\angle ABC$.

   I. ✗. The locus is not perpendicular to AC unless $AB = BC$.
   II. ✗. The locus does not pass through the mid-point of AC unless $AB = BC$.
   III. ✓.

6. D

   Locus of P is a pair of straight lines parallel to L.

7. B

   The locus of P is the angle bisectors of the angles formed by the x-axis and y-axis.
   The answer is B.

8. [B]

Distance from P to the line AB is the height of $\triangle PAB$, which is a constant.
The locus of P is a pair of straight lines, with a fixed distance to the line AB.

9. [C]

The locus of P is a pair of straight lines, $y = -1$ and $y = 11$.

10. [C]

The locus of P is the angle bisector of $\angle AOF$.
$\angle AOF = 40^\circ + 30^\circ \times 2 + 10^\circ + 50^\circ = 160^\circ$
Note that $\angle AOD = 40^\circ + 30^\circ + 10^\circ = 80^\circ = \dfrac{160^\circ}{2}$.
The locus of P is the line segment OD.

11. [B]

The locus of P passes through B, J and D. It is the line segment BD.

12. [C]

The locus of P is the angle bisectors of the angles formed by the two intersecting lines.
The two angle bisectors are perpendicular to each other.

13. [D]

Let M and N be the mid-points of AB and CD respectively.

I. The locus is the perpendicular bisector of AB.
It is the line segment MN.

II. The locus is the straight line parallel to AD and passes through the mid-point of AB.
It is the line segment MN.

III. Note that $\triangle PCD$ is isosceles. P is equidistant from C and D.
The locus of P is the perpendicular bisector of CD.
It is the line segment MN.

Three conditions will all give the same locus of P.

14. [D]

The locus of Q passes through F, J and H. It is the line segment FH.

15. [A]

Equidistant from two fixed points $\Rightarrow$ perpendicular bisector

16. [A]

P maintains a fixed distance to point A. So, the locus of P is a circle.

17. A

Let $P\ (x,\ y)$.

Then the coordinates of A and B are $(2x,\ 0)$ and $(0,\ 2y)$ respectively.

$$AB = \sqrt{(2x)^2 + (2y)^2}$$
$$x^2 + y^2 = \frac{(AB)^2}{4}$$

Locus of P is a arc (part of circle) with centre $(0,\ 0)$ and radius $\frac{AB}{2}$.

18. D

Locus of P is the angle bisectors of the angles between L_1 and L_2.

Locus of P is therefore a pair of straight lines (perpendicular to each other, passing through intersection of L_1 and L_2).

19. A

Locus of P is the perpendicular bisector of AB, i.e., a straight line.

20. C

Let $P = (x,\ y)$.

$$PX = 2PY$$
$$\sqrt{x^2 + (y-5)^2} = 2\sqrt{(x-1)^2 + y^2}$$
$$x^2 + (y-5)^2 = 4[(x-1)^2 + y^2]$$
$$0 = 3x^2 + 3y^2 - 8x + 10y - 21$$

21. A

The locus of P is a circle with centre $A\ (2,\ -5)$ and radius AB.

A. ✓. Centre $(2,\ -5)$ and $8^2 + 3^2 - 4(8) + 10(3) - 71 = 0$.

B. ✗. Centre $(-2,\ 5)$

C. ✗. Centre $(-2,\ 5)$

D. ✗. Centre $(2,\ -5)$ but $8^2 + 3^2 - 4(8) + 10(3) - 75 = -4 \neq 0$.

22. D

Let the coordinates of P be $(x,\ y)$.

$$\frac{y-4}{x-0} \times \frac{y-2}{x-6} = -1$$
$$(y-4)(y-2) = -x(x-6)$$
$$x^2 + y^2 - 6x - 6y + 8 = 0$$

Required equation is $x^2 + y^2 - 6x - 6y + 8 = 0$.

23. A

It is the perpendicular bisector of AB.

Slope of $AB = \dfrac{5+1}{1+5} = 1$. Slope of locus $= -1$. Only option A has a line with slope -1.

24. A

The coordinates of A and B are $(5,\ 0)$ and $(0,\ -12)$ respectively.

The locus of P is the perpendicular bisector of AB.

The coordinates of mid-point of AB are $\left(\dfrac{5}{2},\ -6\right)$.

Note that $15x + 36y + 179 = 15\left(\dfrac{5}{2}\right) + 36(-6) + 179$ is not an integer, implying it cannot be zero.

The answer is A.

Conventional Questions

25. (a) Let the coordinates of P be (x, y).

$$[(x-8)^2+(y-1)^2]+[(x-3)^2+(y-4)^2]=(8-3)^2+(4-1)^2 \qquad \text{1M}$$
$$2x^2+2y^2-22x-10y+56=0$$
$$x^2+y^2-11x-5y+28=0$$

The equation of the locus of P is $x^2+y^2-11x-5y+28=0$. 1A

(b) Let the coordinates of P be (x, y).

$$[(8-3)^2+(4-1)^2]+[(x-3)^2+(y-4)^2]=(x-8)^2+(y-1)^2 \qquad \text{1M}$$
$$10x-6y-6=0$$
$$5x-3y-3=0$$

The equation of the locus of P is $5x-3y-3=0$. 1A

(c) Let the coordinates of P be (x, y).

$$\sqrt{(x-8)^2+(y-1)^2}=\sqrt{(x-3)^2+(y-4)^2} \qquad \text{1M}$$
$$-10x+6y=40$$
$$5x-3y-20=0$$

The equation of the locus of P is $5x-3y-20=0$. 1A

26. (a) Let the coordinates of P be (x, y). Then the coordinates of F are $(x, 3)$.

$$[(x-2)^2+y^2]+[x^2+(y+2)^2]=2(y-3)^2 \qquad \text{1M}$$
$$2x^2+2y^2-4x+4y+8=2y^2-12y+18$$
$$y=-\frac{x^2}{8}+\frac{x}{4}+\frac{5}{8}$$

The equation of the locus of P is $y=-\frac{x^2}{8}+\frac{x}{4}+\frac{5}{8}$. 1A

(b) The locus of P is a parabola that opens downwards. 1A

27. (a) Length of $AB = 4$. Distance from P to $AB=\frac{6\times 2}{4}=3$ 1A

The locus of P is a pair of vertical straight lines 3 units on the left and right of AB respectively.

1A

(b) The equations of the locus of P are $x=-1$ and $x=5$. 1M+1A

28. (a) Let the coordinates of B be (p, q).

$$\frac{q-4}{p-0}\times(-1)=-1 \qquad \text{1M}$$
$$q-4=p$$

mid-point of AB is at $\left(\frac{p}{2}, \frac{q+4}{2}\right)$.

So, $\dfrac{q+4}{2} = -\dfrac{p}{2}$. 1M

Solving, we have $p = -4$ and $q = 0$. The coordinates of B are $(-4,\ 0)$. 1A

(b) $\dfrac{y-4}{x-0} \times \dfrac{y-0}{x+4} = -1$ 1M

$y(y-4) + x(x+4) = 0$

$x^2 + y^2 + 4x - 4y = 0$ 1

(c) Radius of $\Gamma = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$\pi r^2 - \pi(2\sqrt{2}))^2 = \pi(2\sqrt{2})^2$ 1M

$r^2 = 16$

$r = 4$ 1A