

M2REG-AOD-2324-ASM-SET 3-MATH

Suggested solutions

1. (a) 4 1A

(b) $y = \frac{x^2 - 5x}{x + 4}$
 $= x - 9 + \frac{36}{x + 4}$ 1M

Oblique asymptote is $y = x - 9$. 1A

(c) $\frac{dy}{dx} = 1 - \frac{36}{(x + 4)^2}$ 1A
 $= \frac{(x + 10)(x - 2)}{(x + 4)^2}$

When $\frac{dy}{dx} = 0$, $x = -10$ or 2 . 1A

x	$x < -10$	$-10 < x < -4$	$-4 < x < 2$	$x > 2$
$\frac{dy}{dx}$	+	-	-	+

1M

There is only one maximum point at $x = -10$.

The claim is agreed. 1A

2. (a) $g'(x) = \frac{-k(2x - 6)}{(x^2 - 6x + 21)^2}$ 1A

When $g'(x) = 0$, $x = 3$.

x	$x < 3$	$x > 3$
$g'(x)$	-	+

1M

Minimum point is $\left(3, \frac{k}{12}\right)$. 1A

- (b) No vertical asymptote.

Horizontal asymptote is $y = 0$. 1A

(c) $\frac{k}{12} = -2$ 1M

$$k = -24$$

$$g'(x) = \frac{48x - 144}{(x^2 - 6x + 21)^2}$$

$$g''(x) = \frac{(x^2 - 6x + 21)^2(48) - (48x - 144)(2)(x^2 - 6x + 21)(2x - 6)}{(x^2 - 6x + 21)^4}$$
 1M

$$= \frac{-144(x - 1)(x - 5)}{(x^2 - 6x + 21)^3}$$

When $g''(x) = 0$, $x = 1$ or 5 .

x	$x < 1$	$1 < x < 5$	$x > 5$
$g''(x)$	-	+	-

1M

Points of inflexion are $\left(1, -\frac{3}{2}\right)$ and $\left(5, -\frac{3}{2}\right)$. 1A+1A

3. (a) (i) $f'(x) = \frac{(2x-4)(x^2+1) - (x^2-4x+1)(2x)}{(x^2+1)^2}$ 1M

$$= \frac{4x^2 - 4}{(x^2 + 1)^2}$$
 1A

(ii) $f'(x) = \frac{4(x+1)(x-1)}{(x^2+1)^2}$.

When $f'(x) = 0$, $x = \pm 1$.

x	$x < -1$	$-1 < x < 1$	$x > 1$
$f'(x)$	+	-	+

$f(-1) = 3$ and $f(1) = -1$.

Thus, the maximum and minimum value of $f(x)$ are 3 and -1 respectively.

(b) $f''(x) = \frac{8x(x^2+1)^2 - (4x^2-4)(2)(x^2+1)(2x)}{(x^2+1)^4}$ 1M

$$= \frac{8x(x^2+1) - 4x(4x^2-4)}{(x^2+1)^3}$$

$$= \frac{-8x^3 + 24x}{(x^2+1)^3}$$
 1A

$$= \frac{-8x(x^2-3)}{(x^2+1)^3}$$

When $f''(x) = 0$, $x = 0$ or $\pm\sqrt{3}$.

x	$x < -\sqrt{3}$	$-\sqrt{3} < x < 0$	$0 < x < \sqrt{3}$	$x > \sqrt{3}$
$f''(x)$	+	-	+	-

$f(-\sqrt{3}) = 1 + \sqrt{3}$, $f(0) = 1$ and $f(\sqrt{3}) = 1 - \sqrt{3}$

The points of inflexion are $(-\sqrt{3}, 1 + \sqrt{3})$, $(0, 1)$ and $(\sqrt{3}, 1 - \sqrt{3})$.

(c) There are no vertical asymptotes.

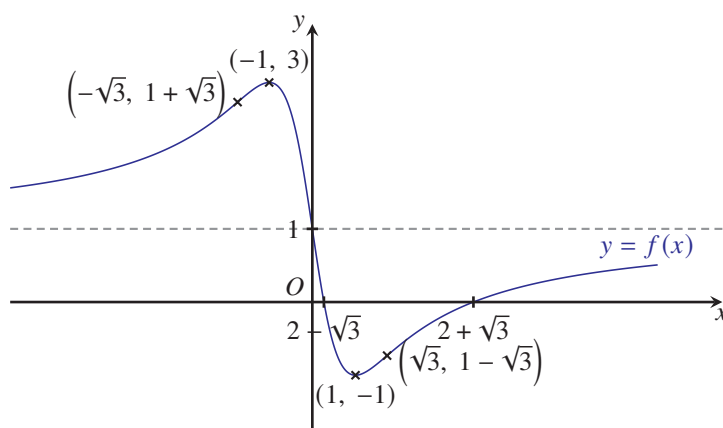
$$f(x) = 1 - \frac{4x}{x^2+1}$$

Horizontal asymptote is $y = 1$.

(d) (Correct shape)

(Correct points and asymptotes)

(All correct)



4. (a) $f'(x) = \frac{(x^2 + 1)(2) - 2x(2x)}{(x^2 + 1)^2}$ 1M

$= \frac{2 - 2x^2}{(x^2 + 1)^2}$ 1A

$f''(x) = \frac{(x^2 + 1)^2(-4x) - (2 - 2x^2)(2)(x^2 + 1)(2x)}{(x^2 + 1)^4}$

$= \frac{-4x(x^2 + 1 + 2 - 2x)}{(x^2 + 1)^3}$

$= \frac{4x(x^2 - 3)}{(x^2 + 1)^3}$ 1A

(b) When $f'(x) = 0$, $x = \pm 1$. 1A

$f''(1) = -1 < 0$ and $f''(-1) = 1 > 0$ 1M

The maximum point is (1, 1) and the minimum point is (-1, -1). 1A

(c) When $f''(x) = 0$, $x = 0$ or $\pm\sqrt{3}$.

x	$x < -\sqrt{3}$	$-\sqrt{3} < x < 0$	$0 < x < \sqrt{3}$	$x > \sqrt{3}$
$f''(x)$	-	+	-	+

1M

The points of inflexion are $\left(-\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$, (0, 0) and $\left(\sqrt{3}, \frac{\sqrt{3}}{2}\right)$.

1A

(d) No vertical asymptote.

Horizontal asymptote is $y = 0$.

1M+1A

(e) (Correct shape)

1A

(All correct)

1A

