M2REG-AOD-2324-ASM-SET 2-MATH
Suggested solutions

1. $f'(x) = 2\cos x - 1$ 1A

When $f'(x) = 0$,

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$ 1A

x	$0 < x < \frac{\pi}{3}$	$\frac{\pi}{3} < x < \pi$
$f'(x)$	$+$	$-$

1M

$f(0) = 0$; $f\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3}$ and $f(\pi) = -\pi$.

The greatest value of $f(x)$ is $\sqrt{3} - \frac{\pi}{3}$ 1A

The least value of $f(x)$ is $-\pi$. 1A

2. (a) $\frac{dy}{dx} = 3\sin^2\left(x + \frac{\pi}{3}\right)\cos\left(x + \frac{\pi}{3}\right)$ 1M

When $\frac{dy}{dx} = 0$, $x = \frac{\pi}{6}$.

The stationary point is $\left(\frac{\pi}{6},\ 1 + \pi\right)$. 1A

(b)

x	$0 < x < \frac{\pi}{6}$	$\frac{\pi}{6} < x < \frac{\pi}{2}$
$\frac{dy}{dx}$	$+$	$-$

1M

$\left(\frac{\pi}{6},\ 1 + \pi\right)$ is a maximum point. 1A

3. (a) $\frac{dy}{dx} = e^{x+1} + xe^{x+1}$ 1A

$= e^{x+1}(1 + x)$

When $\frac{dy}{dx} = 0$, $x = -1$.

Stationary point is $(-1,\ -1)$. 1A

(b)

x	$x < -1$	$x > -1$
$\frac{dy}{dx}$	$-$	$+$

1M

$(-1,\ -1)$ is a minimum point. 1A

4. $\dfrac{dy}{dx} = 2x(1-3x^2)^{\frac{1}{2}} + x^2\left(\dfrac{1}{2}\right)(1-3x^2)^{-\frac{1}{2}}(-6x)$ 1M

$= (1-3x^2)^{-\frac{1}{2}}\left[2x(1-3x^2) - 3x^3\right]$

$= (1-3x^2)^{-\frac{1}{2}}(-9x^3 + 2x)$

When $\dfrac{dy}{dx} = 0$, $x = 0$ or $\pm\dfrac{\sqrt{2}}{3}$.

x	$-\dfrac{\sqrt{3}}{3} < x < -\dfrac{\sqrt{2}}{3}$	$-\dfrac{\sqrt{2}}{3} < x < 0$	$0 < x < \dfrac{\sqrt{2}}{3}$	$\dfrac{\sqrt{2}}{3} < x < \dfrac{\sqrt{3}}{3}$
$\dfrac{dy}{dx}$	+	−	+	−

1M

Maximum points are $\left(-\dfrac{\sqrt{2}}{3}, \dfrac{2\sqrt{3}}{27}\right)$ and $\left(\dfrac{\sqrt{2}}{3}, \dfrac{2\sqrt{3}}{27}\right)$. 1A

Minimum point is $(0, 0)$. 1A

5. (a) $\dfrac{dy}{dx} = \lim_{h\to 0}\dfrac{\frac{(x+h)^2}{(x+h)+1} - \frac{x^2}{x+1}}{h}$ 1M

$= \lim_{h\to 0}\dfrac{(x+h)^2(x+1) - x^2(x+h+1)}{h(x+1)(x+h+1)}$

$= \lim_{h\to 0}\dfrac{hx^2 + (h^2+2h)x + h^2}{h(x+1)(x+h+1)}$

$= \lim_{h\to 0}\dfrac{x^2 + (h+2)x + h}{(x+1)(x+h+1)}$ 1M

$= \dfrac{x(x+2)}{(x+1)^2}$ 1A

(b) $\dfrac{x(x+2)}{(x+1)^2} \geq 0$ 1M

$x(x+2) \geq 0$

$x \leq -2$ or $x \geq 0$ 1A

(Accept $x < -2$ or $x > 0$)

6. (a) $\dfrac{dy}{dx} = \lim_{h\to 0}\dfrac{-\frac{6}{(x+h)^2} + \frac{6}{x^2}}{h}$ 1M

$= \lim_{h\to 0}\dfrac{6(x+h)^2 - 6x^2}{hx^2(x+h)^2}$

$= \lim_{h\to 0}\dfrac{12x + 6h}{x^2(x+h)^2}$ 1M

$= \dfrac{12x}{x^4}$

$= \dfrac{12}{x^3}$ 1A

(b) When the curve is increasing, $\dfrac{dy}{dx} = \dfrac{12}{x^3} \geq 0$. 1M

Required range is $x > 0$. 1A

7. (a) $\dfrac{dy}{dx} = 3x^2 - 4kx - 4k^2$ 1M

$= (x - 2k)(3x + 2k)$

$\dfrac{d^2y}{dx^2} = 6x - 4k$

When $\dfrac{dy}{dx} = 0$, $x = 2k$ or $-\dfrac{2k}{3}$.

$\left.\dfrac{d^2y}{dx^2}\right|_{x=2k} = 6(2k) - 4k = 8k > 0$ 1M

Thus, y attains its local minimum at $x = 2k$. 1

(b) $(2k)^3 - 2k(2k)^2 - 4k^2(2k) = -1000$

$-8k^3 = -1000$

$k^3 = 125$

$k = 5$ 1A

8. (a) $f'(x) = 12x^2 + 4kx + 15$ 1A

(b) Since $f(x)$ is continuous and there are no turning points, we have $f'(x) \geq 0$ or $f'(x) \leq 0$ for all $x \in \mathbb{R}$.

$\Delta = (4k)^2 - 4(12)(15) \leq 0$ 1M

$16k^2 - 720 \leq 0$

$-3\sqrt{5} \leq k \leq 3\sqrt{5}$ 1A

9. (a) $\dfrac{dy}{dx} = \dfrac{2x(x+1) - (x^2 + p)}{(x+1)^2}$ 1M

$= \dfrac{x^2 + 2x - p}{(x+1)^2}$ 1A

(b) (i) $\dfrac{1^2 + 2 - p}{(1+1)^2} = 0$ 1M

$p = 3$ 1A

(ii) $x^2 + 2x - 3 = 0$ 1M

$x = -3$ or 1

x	$x < -3$	$-3 < x < -1$	$-1 < x < 1$	$x > 1$
$\dfrac{dy}{dx}$	+	−	−	+

1M

When $x = -3$, $y = -6$; when $x = 1$, $y = 2$.

The local maximum is -6 and the local minimum is 2. 1A+1A

10. (a) $y = 1 + \dfrac{\ln x}{x}$

$\dfrac{dy}{dx} = \dfrac{\frac{1}{x}(x) - \ln x(1)}{x^2}$ 1M

$= \dfrac{1 - \ln x}{x^2}$ 1A

$\dfrac{d^2y}{dx^2} = \dfrac{x^2\left(-\frac{1}{x}\right) - (1 - \ln x)(2x)}{x^4}$

$= \dfrac{-3x + 2x\ln x}{x^4}$

$= \dfrac{2\ln x - 3}{x^3}$ 1A

(b) When $\dfrac{dy}{dx} = 0$,

$1 - \ln x = 0$

$\ln x = 1$

$x = e$

When $x = e$, $y = \dfrac{e + \ln e}{e} = \dfrac{e+1}{e}$.

Stationary point is $\left(e, \dfrac{e+1}{e}\right)$. 1A

$\left.\dfrac{d^2y}{dx^2}\right|_{x=e} = \dfrac{2\ln e - 3}{e^3} = -\dfrac{1}{e^3} < 0$ 1M

$\left(e, \dfrac{e+1}{e}\right)$ is a maximum point. 1A

11. (a) $\dfrac{dy}{dx} = \dfrac{e^{2x} - (x+1)(2e^{2x})}{e^{4x}}$ 1M

$= \dfrac{-2x - 1}{e^{2x}}$ 1A

$\dfrac{d^2y}{dx^2} = \dfrac{-2e^{2x} - (-2x-1)(2e^{2x})}{e^{4x}}$

$= \dfrac{4x}{e^{2x}}$ 1A

(b) $\dfrac{dy}{dx} = \dfrac{-2x-1}{e^{2x}} = 0$ 1M

$x = -\dfrac{1}{2}$

Stationary point is $\left(-\dfrac{1}{2}, \dfrac{e}{2}\right)$. 1A

$\left.\dfrac{d^2y}{dx^2}\right|_{\left(-\frac{1}{2}, \frac{e}{2}\right)} = \dfrac{4\left(\frac{-1}{2}\right)}{e^{-1}}$

$= -2e$

< 0

It is a maximum point. 1A

12. (a) $f'(x) = \frac{1}{x^2} - \frac{\ln x}{x^2}$ 1A

When $f'(x) = 0$,

$$\ln x = 1$$

$$x = e$$ 1M

x	$0 < x < e$	$x > e$
$f'(x)$	$+$	$-$

1M

The maximum value of $f(x)$ is $f(e) = \frac{1}{e}$. 1A

(b) By (a), $f(x) \leq f(e)$.

$$\frac{\ln x}{x} \leq \frac{1}{e}$$ 1M

$$\ln x^e \leq x$$

$$e^x \geq x^e$$ 1

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