

M2REG-AOD-2324-ASM-SET 1-MATH**Suggested solutions**

$$1. \quad \frac{dy}{dx} = e^x \ln x + e^x \left(\frac{1}{x} \right) \quad 1M$$

$$\left. \frac{dy}{dx} \right|_{(1, 0)} = 0 + e(1)$$

$$= e$$

1M

Required equation is

$$y - 0 = e(x - 1)$$

$$ex - y - e = 0$$

1A

$$2. \quad x = y^3 - 3y^2$$

$$1 = 3y^2 \frac{dy}{dx} - 6y \frac{dy}{dx}$$

1M

$$\frac{dy}{dx} = \frac{1}{3y^2 - 6y}$$

$$\left. \frac{dy}{dx} \right|_{(-2, 1)} = \frac{1}{3 - 6} = -\frac{1}{3}$$

1M

Required equation is

$$y - 1 = -\frac{1}{3}(x + 2)$$

$$x + 3y - 1 = 0$$

1A

$$3. \quad x^2 - 2 \sin y = 3$$

$$2x - 2 \cos y \frac{dy}{dx} = 0$$

1M

$$\frac{dy}{dx} = \frac{x}{\cos y}$$

$$\left. \frac{dy}{dx} \right|_{(2, \frac{\pi}{6})} = \frac{2}{\cos \frac{\pi}{6}} = \frac{4}{\sqrt{3}}$$

1M

Required equation is

$$y - \frac{\pi}{6} = \frac{4}{\sqrt{3}}(x - 2)$$

$$6y - \pi = 8\sqrt{3}(x - 2)$$

$$8\sqrt{3}x - 6y + \pi - 16\sqrt{3} = 0$$

1A

$$4. \quad x^2(0) + 3x - 4(0) - 9 = 0$$

$$x = 3$$

The curve cuts the x -axis at $(3, 0)$.

1A

$$x^2y + 3x - 4y - 9 = 0$$

$$2xy + x^2 \frac{dy}{dx} + 3 - 4 \frac{dy}{dx} = 0$$

1M

$$\frac{dy}{dx} = \frac{2xy + 3}{4 - x^2}$$

$$\left. \frac{dy}{dx} \right|_{(3, 0)} = \frac{0 + 3}{4 - 3^2}$$

$$= -\frac{3}{5}$$

1M

Required equation is

$$y - 0 = -\frac{3}{5}(x - 3)$$

$$3x + 5y - 9 = 0$$

1A

$$5. \quad y = \sqrt{10 - (0 + 1)^2}$$

$$= 3$$

The curve cuts the y -axis at $(0, 3)$.

1A

$$\frac{dy}{dx} = \frac{1}{2} [10 - (x + 1)^2]^{-\frac{1}{2}} (-2)(x + 1)$$

1M

$$= -\frac{x + 1}{\sqrt{10 - (x + 1)^2}}$$

$$\left. \frac{dy}{dx} \right|_{(0, 3)} = -\frac{1}{\sqrt{10 - 1}} = -\frac{1}{3}$$

1M

Required equation is

$$y - 3 = -\frac{1}{3}(x - 0)$$

$$x + 3y - 9 = 0$$

1A

$$6. \quad \cos^2 x - 2y + 1 = 0$$

$$2 \cos x (-\sin x) - 2 \frac{dy}{dx} = 0$$

1M

$$\frac{dy}{dx} = -\sin x \cos x$$

Let the coordinates of the point of contact be (a, b) .

$$-\sin a \cos a = -\frac{1}{2}$$

1M

$$-\frac{\sin 2a}{2} = -\frac{1}{2}$$

$$\sin 2a = 1$$

$$2a = \frac{\pi}{2} \quad \text{or} \quad \frac{5\pi}{2}$$

$$a = \frac{\pi}{4} \quad \text{or} \quad \frac{5\pi}{4}$$

We have $(a, b) = \left(\frac{\pi}{4}, \frac{3}{4}\right)$ or $\left(\frac{5\pi}{4}, \frac{3}{4}\right)$.

The equation of tangent to the curve at $\left(\frac{\pi}{4}, \frac{3}{4}\right)$ is

$$y - \frac{3}{4} = -\frac{1}{2} \left(x - \frac{\pi}{4}\right)$$

$$4x + 8y - \pi - 6 = 0$$

1A

The equation of tangent to the curve at $\left(\frac{5\pi}{4}, \frac{3}{4}\right)$ is

$$y - \frac{3}{4} = -\frac{1}{2} \left(x - \frac{5\pi}{4}\right)$$

$$4x + 8y - 5\pi - 6 = 0$$

1A

$$7. \frac{dy}{dx} = \frac{(3x+1) - (x-1)(3)}{(3x+1)^2} \quad 1M$$

$$= \frac{4}{(3x+1)^2}$$

Let the coordinates of the point of contact be (a, b) .

$$\frac{4}{(3a+1)^2} = \frac{1}{4} \quad 1M$$

$$(3a+1)^2 = 16$$

$$3a+1 = \pm 4$$

$$a = 1 \quad \text{or} \quad -\frac{5}{3}$$

We have $(a, b) = (1, 0)$ or $\left(-\frac{5}{3}, \frac{2}{3}\right)$. The equation of tangent to the curve at $(1, 0)$ is

$$y - 0 = \frac{1}{4}(x - 1)$$

$$x - 4y - 1 = 0 \quad 1A$$

The equation of tangent to the curve at $\left(-\frac{5}{3}, \frac{2}{3}\right)$ is

$$y - \frac{2}{3} = \frac{1}{4}\left(x + \frac{5}{3}\right)$$

$$3x - 12y + 13 = 0 \quad 1A$$

$$8. \frac{dy}{dx} = \ln x + x \left(\frac{1}{x}\right) \quad 1M$$

$$= \ln x + 1$$

$$\ln a + 1 = 1 \quad 1M$$

$$\ln a = 0$$

$$a = 1$$

We have $a = 1$ and $b = 0$. 1A

$$9. \quad \frac{dy}{dx} = 6x^2 - 4 \quad 1M$$

$$\left. \frac{dy}{dx} \right|_{(-1, 11)} = 6(-1)^2 - 4 = 2 \quad 1M$$

The equation of tangent to the curve at A is

$$y - 11 = 2(x + 1)$$

$$y = 2x + 13 \quad 1A$$

$$2x + 13 = 2x^3 - 4x + 9 \quad 1M$$

$$0 = 2x^3 - 6x - 4$$

$$0 = 2(x + 1)(x^2 - x - 2)$$

$$0 = 2(x + 1)^2(x - 2)$$

$$x = -1 \quad \text{or} \quad 2$$

The coordinates of B are $(2, 17)$. 1A

$$10. \quad x^2 + y^2 = 16$$

$$2x + 2y \frac{dy}{dx} = 0 \quad 1M$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Let the coordinates of the point of contact be (a, b) .

$$-\frac{a}{b} = \frac{b - 0}{a - 8} \quad 1M$$

$$-a^2 + 8a = b^2$$

$$-a^2 + 8a = 16 - a^2 \quad 1M$$

$$a = 2$$

When $a = 2$, $b = \pm\sqrt{16 - 2^2} = \pm 2\sqrt{3}$.

The equation of tangent to the curve at $(2, 2\sqrt{3})$ is

$$y - 0 = -\frac{2}{2\sqrt{3}}(x - 8)$$

$$x + \sqrt{3}y - 8 = 0 \quad 1A$$

The equation of tangent to the curve at $(2, -2\sqrt{3})$ is

$$y - 0 = -\frac{2}{-2\sqrt{3}}(x - 8)$$

$$x - \sqrt{3}y - 8 = 0 \quad 1A$$

11. $\frac{dy}{dx} = 6x - 2$ 1M
 Let the coordinates of the point of contact be (a, b) .

$$6a - 2 = \frac{b + 1}{a - 2} \quad 1M$$

$$(6a - 2)(a - 2) = b + 1$$

$$6a^2 - 14a + 4 = (3a^2 - 2a + 3) + 1 \quad 1M$$

$$3a^2 - 12a = 0$$

$$a = 0 \quad \text{or} \quad 4$$

We have $(a, b) = (0, 3)$ or $(4, 43)$.

The equation of tangent to the curve at $(0, 3)$ is

$$y - 3 = [6(0) - 2](x - 0)$$

$$2x + y - 3 = 0 \quad 1A$$

The equation of tangent to the curve at $(4, 43)$ is

$$y - 43 = [6(4) - 2](x - 4)$$

$$22x - y - 45 = 0 \quad 1A$$