

REG-MAE-2324-ASM-SET 3-MATH

建議題解

結構式試題

1. $3^{2x} - 5(3^x) - 24 = 0$

$$(3^x - 8)(3^x + 3) = 0$$

$$3^x = 8 \quad \text{或} \quad -3 \quad (\text{捨去})$$

2A

$$x \log 3 = \log 8$$

$$x \approx 1.89$$

1A

2. $\log_9(x + 5) = \log_3(1 - x)$

$$\frac{\log(x + 5)}{\log 9} = \frac{\log(1 - x)}{\log 3}$$

$$\frac{\log(x + 5)}{2 \log 3} = \frac{\log(1 - x)}{\log 3}$$

1M

$$\log(x + 5) = 2 \log(1 - x)$$

$$x + 5 = (1 - x)^2$$

1M

$$x + 5 = x^2 - 2x + 1$$

$$0 = x^2 - 3x - 4$$

$$x = -1 \quad \text{或} \quad 4 \quad (\text{捨去})$$

2A

3. $49^x - 4(7^x) - 77 = 0$

$$(7^x)^2 - 4(7^x) - 77 = 0$$

1M

$$7^x = 11 \quad \text{或} \quad -7 \quad (\text{捨去})$$

1A

$$x \log 7 = \log 11$$

$$x \approx 1.23$$

1A

4. $3^{\frac{2x}{3}} + 3^{\frac{x+6}{3}} = 36$

$$(3^{\frac{x}{3}})^2 + 9(3^{\frac{x}{3}}) - 36 = 0$$

1M

$$3^{\frac{x}{3}} = 3 \quad \text{或} \quad -12 \quad (\text{捨去})$$

1A

$$\frac{x}{3} = 1$$

$$x = 3$$

1A

$$5. \quad \log_{2x^2-x+1} 4 = \log_{x+1} 2$$

$$\frac{2 \log 2}{\log(2x^2 - x + 1)} = \frac{\log 2}{\log(x + 1)} \quad 1M$$

$$2 \log(x + 1) = \log(2x^2 - x + 1)$$

$$\log(x + 1)^2 = \log(2x^2 - x + 1)$$

$$(x + 1)^2 = 2x^2 - x + 1 \quad 1M$$

$$0 = x^2 - 3x$$

$$x = 3 \quad \text{或} \quad 0 \quad (\text{捨去}) \quad 1A$$

$$6. \quad \log x + \log 2y = \log(x - 4y)^2$$

$$\log(2xy) = \log(x - 4y)^2$$

$$2xy = (x - 4y)^2 \quad 1M$$

$$0 = x^2 - 10xy + 16y^2$$

$$0 = (x - 2y)(x - 8y)$$

$$x = 2y \quad \text{或} \quad 8y \quad 1A$$

$$\frac{x}{y} = 2 \quad \text{或} \quad 8 \log_{\sqrt{2}} \frac{x}{y} = 2 \quad \text{或} \quad 6 \quad 1A$$

7. 設該兩個連續整數為 x 及 $x + 1$ 。

$$\frac{1}{x} + \frac{1}{x + 1} = \frac{7}{12} \quad 1A$$

$$12(x + 1) + 12(x) = 7x(x + 1)$$

$$0 = 7x^2 - 17x - 12 \quad 1A$$

$$x = 3 \quad \text{或} \quad -\frac{4}{7} \quad (\text{捨去}) \quad 1A$$

該兩個整數為 3 及 4。 1A

8. 可得

$$\begin{cases} xy = 8 \\ (10x + y) - (10y + x) = 18 \end{cases} \quad 1A$$

$$9x - 9 \times \frac{8}{x} = 18 \quad 1M$$

$$9x^2 - 18x - 72 = 0 \quad 1A$$

$$x = 4 \quad \text{或} \quad -2 \quad (\text{捨去}) \quad 1A$$

當 $x = 4$, $y = \frac{8}{4} = 2$ 。原來的數為 42。 1A

$$9. \frac{1}{x} + \frac{1}{2x-3} = \frac{1}{2 + \frac{55}{60}} \quad 1M+1M$$

$$\frac{3x-3}{x(2x-3)} = \frac{12}{35}$$

$$35(3x-3) = 12x(2x-3)$$

$$0 = 24x^2 - 141x + 105 \quad 1A$$

$$x = 5 \quad \text{或} \quad \frac{7}{8} \quad (\text{捨去}) \quad 1A$$

10. 設所求速率為 x km/min。

$$\frac{2}{x + \frac{1}{36}} + \frac{2}{x - \frac{1}{36}} = 16 + \frac{12}{60} \quad 1M+1M$$

$$\frac{72}{36x+1} + \frac{72}{36x-1} = \frac{81}{5}$$

$$40(36x-1) + 40(36x+1) = 9(36x-1)(36x+1)$$

$$0 = 11\,664x^2 - 2880x - 9 \quad 1A$$

$$x = \frac{1}{4} \quad \text{或} \quad -\frac{1}{324} \quad (\text{捨去}) \quad 1A$$

所求速率為 $\frac{1}{4}$ km/min。

11. 設每隻碗的成本為 $\$x$ 。

$$\left(\frac{1500}{x} - 12\right)(x+6) = 1500 + 1038 \quad 1M+1M$$

$$(1500 - 12x)(x+6) = 2538x$$

$$-12x^2 - 1110x + 9000 = 0 \quad 1A$$

$$x = 7.5 \quad \text{或} \quad -100 \quad (\text{捨去}) \quad 1A$$

每隻碗的成本為 $\$7.5$ 。

$$12. \quad (a) \quad \frac{BE}{AB} = \frac{y}{x-y} \quad 1A$$

$$(b) \quad \text{設 } u = \frac{x}{y}。$$

$$\frac{y}{x-y} = \frac{x}{y} \quad 1M$$

$$\frac{1}{\frac{x}{y} - 1} = \frac{x}{y}$$

$$\frac{1}{u-1} = u$$

$$0 = u^2 - u - 1$$

$$u = \frac{1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} \quad 1M$$

$$= \frac{1 + \sqrt{5}}{2} \quad \text{或} \quad \frac{1 - \sqrt{5}}{2} \quad (\text{捨去}) \quad 1A$$

13. (a) (i) $FG = \sqrt{a^2 + a^2}$
 $= \sqrt{2}a \text{ cm}$ 1A
- (ii) $EH = \sqrt{b^2 + b^2}$
 $= \sqrt{2}b \text{ cm}$ 1A
- (b) $2(4 - a) + 2(4 - b) + \sqrt{2}a + \sqrt{2}b = 8 + 4\sqrt{2}$ 1M
 $8 - 4\sqrt{2} = (2 - \sqrt{2})(a + b)$
 $a + b = 4$
- $4^2 - \frac{1}{2}a^2 - \frac{1}{2}b^2 = 11$ 1M
 $a^2 + b^2 = 10$
- $(a + b)^2 - 2ab = 10$ 1M
 $4^2 - 2ab = 10$
 $ab = 3$ 1A
 < 4
- 不同意該宣稱。 1A
14. (a) D 的坐標為 $(k + 15, \log(k + 15))$ 。 1A
- (b) $\frac{[\log k + \log(k + 15)]15}{2} = 15$ 1M
 $\log k(k + 15) = 2$
 $k^2 + 15k = 10^2$
 $k^2 + 15k - 100 = 0$ 1A
 $k = 5$ 或 -20 (捨去) 1A
15. (a) $PA = 2\sqrt{x - 1} \text{ cm}$ 1A
 $QB = 4\sqrt{x - 1} \text{ cm}$ 1A
- (b) $2\sqrt{x - 1} + 8 + 4\sqrt{x - 1} = x$ 1M+1A
 $6\sqrt{x - 1} = x - 8$
 $36(x - 1) = (x - 8)^2$
 $x^2 - 52x + 100 = 0$ 1M
 $x = 50$ 或 $x = 2$ (捨去) 1A
- (c) 總成本 $= 50 \times 2\sqrt{50 - 1} \times 0.1 \times 3$ 1M
 $= \$210 > \200 1A
 不足夠製造三個書架。 1A