REG-LOG-2324-ASM-SET 5-MATH
Suggested solutions

Multiple Choice Questions

1. B	2. D	3. B	4. B	5. A
6. A	7. B	8. B	9. C	10. B
11. C	12. C	13. C	14. D	15. D
16. D	17. A	18. C	19. A	20. C
21. B	22. D	23. A	24. C	25. D
26. C	27. A	28. B	29. B	30. C
31. C	32. D	33. C	34. A	35. B
36. B	37. A	38. C		

1. B

$$\begin{aligned}\log\sqrt[4]{x+y} &= \frac{1}{4}\log(x+y)\\ &= \frac{1}{4}\log\frac{x^2-y^2}{x-y}\\ &= \frac{1}{4}[\log\left(x^2-y^2\right)-\log(x-y)]\\ &= \frac{1}{4}(10-2)\\ &= 2\end{aligned}$$

2. D

I. ✗. $\log a+\log b=\log ab\neq\log(a+b)$

II. ✓.

III. ✓. $$\begin{aligned}\log\frac{1}{a}+\log\frac{1}{b} &= -\log a-\log b\\ &= -(\log a+\log b)\\ &= -\log ab\end{aligned}$$

3. B

$$\begin{aligned}\frac{\log\left(a\sqrt{b}\right)}{2\log a+\log b} &= \frac{\log a+\frac{1}{2}\log b}{2\log a+\log b}\\ &= \frac{\frac{1}{2}(2\log a+\log b)}{2\log a+\log b}\\ &= \frac{1}{2}\end{aligned}$$

4. B

$$\log \frac{\sqrt{y^x}}{\sqrt{x^y}} = \log \frac{y^{\frac{x}{2}}}{x^{\frac{y}{2}}}$$
$$= \frac{x}{2}\log y - \frac{y}{2}\log x$$
$$= \frac{1}{2}(x\log y - y\log x)$$

5. A

$$\frac{\log(xy^4) - \log \frac{y}{x^2}}{\log x^2 + \log y^2} = \frac{(\log x + 4\log y) - (\log y - 2\log x)}{2\log x + 2\log y}$$
$$= \frac{3\log x + 3\log y}{2\log x + 2\log y}$$
$$= \frac{3(\log x + \log y)}{2(\log x + \log y)}$$
$$= \frac{3}{2}$$

6. A

$$y = \frac{3^x}{3^x - 3}$$
$$(3^x - 3)y = 3^x$$
$$3^x(y-1) = 3y$$
$$x = \log_3 \frac{3y}{y-1}$$
$$= \log_3 \frac{y}{y-1} + \log_3 3$$
$$= \log_3 \frac{y}{y-1} + 1$$

7. B

$$x = \log \sqrt{\frac{y-1}{y}}$$
$$10^x = \sqrt{\frac{y-1}{y}}$$
$$10^{2x} = 1 - \frac{1}{y}$$
$$\frac{1}{y} = 1 - 10^{2x}$$
$$y = \frac{1}{1 - 10^{2x}}$$

8. B

$4\log x + 3\log y = 0$

$$\log y = -\frac{4}{3}\log x$$
$$\log y = \log x^{\frac{-4}{3}}$$
$$y = x^{-\frac{4}{3}}$$
$$y = \frac{1}{x^{\frac{4}{3}}}$$

9. C

$\log x^2 = \log 5x - 1$

$\log x^2 = \log 5x - \log 10$

$$\log x^2 = \log \frac{5x}{10}$$
$$x^2 = \frac{x}{2}$$
$$x = \frac{1}{2} \quad \text{or} \quad 0 \text{ (rejected)}$$

10. B

$$\frac{1}{3}\log x - 1 = \log 2$$
$$\frac{1}{3}\log x - \log 2 = 1$$
$$\log \frac{x^{\frac{1}{3}}}{2} = 1$$
$$\frac{x^{\frac{1}{3}}}{2} = 10^1$$
$$x^{\frac{1}{3}} = 20$$
$$x = 20^3$$
$$= 8000$$

11. C

$y = 3x^2$

$\log y = 2\log x + \log 3$

I. ✓.

II. ✗. There is no y-intercept indeed, we have only the log y-intercept.

III. ✓. $y = 3x^2$

$\log_3 y = 2\log_3 x + \log_3 3$‘

Slope of the line is also 2.

12. C

From the point $(0,\ 2)$.

$$\begin{aligned} \log_5 x &= 0 \quad \text{and} \quad \log_5 y = 2 \\ x &= 5^0 \qquad\qquad y = 5^2 \\ x &= 1 \qquad\qquad y = 25 \end{aligned}$$

From the point $(-4,\ 0)$.

$$\begin{aligned} \log_5 x &= -4 \quad \text{and} \quad \log_5 y = 0 \\ x &= 5^{-4} \qquad\qquad y = 1 \end{aligned}$$

	$x = 1$ and $y = 25$	$x = 5^{-4}$ and $y = 1$
A.	✓	✗
B.	✗	
C.	✓	✓
D.	✗	

13. C

Slope of the graph $= \dfrac{4-0}{0+2} = 2$. The equation is

$$\begin{aligned} \log_3 y &= 2x + 4 \\ y &= 3^{2x+4} \\ &= 3^{2x} \times 3^4 \\ &= 81 \times 9^x \end{aligned}$$

So, $n = 9$.

14. D

Substitute $\left(\log_4 x,\ \log_4 y\right) = (1,\ 2)$ and $(9,\ 6)$ into $\log_4 y = \log_4 k + a \log_4 x$,

$$\begin{cases} 2 = \log_4 k + a(1) \\ 6 = \log_4 k + a(9) \end{cases}$$

Solving, we have $\log_4 k = \dfrac{3}{2}$ and $a = \dfrac{1}{2}$.

So, $k = 4^{\frac{3}{2}} = 8$.

15. D

For the point $(0,\ 2)$.

$$\log x = 0 \quad \text{and} \quad \log y = 2$$
$$x = 1 \qquad\qquad y = 10^2 = 100$$

For the point $(4,\ 0)$.

$$\log x = 4 \qquad\qquad \text{and} \quad \log y = 0$$
$$x = 10^4 = 10\,000 \qquad\qquad y = 1$$

The answer is D.

16. D

For the point $(0,\ -3)$.

$$\log_9 x = 0 \quad \text{and} \quad \log_3 y = -3$$
$$x = 1 \qquad\qquad y = 3^{-3} = \frac{1}{27}$$

Only option D satisfies this pair of x and y.

17. A

Slope of the graph $= -4$ and $y^3 = -4\log_5 x + 12$.

$$2^3 = -4\log_5 x + 12$$
$$\log_5 x = 1$$
$$x = 5$$

18. C

For the point $(0,\ -2)$.

$$\log x = 0 \quad \text{and} \quad \log y = -2$$
$$x = 1 \qquad\qquad y = 10^{-2} = \frac{1}{100}$$

$$\frac{1}{100} = k(1)^2$$
$$k = \frac{1}{100}$$

19. A

For the point $(0,\ 2)$.

$$\log_3 x = 0 \quad \text{and} \quad \log_3 y = 2$$
$$x = 1 \qquad\qquad y = 3^2 = 9$$

Only option A satisfies this.

20. C

For the point $(0,\ -1)$,

$$\log_7 x = 0 \quad \text{and} \quad \log_7 y = -1$$
$$x = 1 \qquad\qquad y = 7^{-1}$$

We have $7^{-1} = a(1)^b$, and $a = \dfrac{1}{7}$.
For the point $(2,\ 0)$,

$$\log_7 x = 2 \quad \text{and} \quad \log_7 y = 0$$
$$x = 49 \qquad\qquad y = 1$$

We have $1 = \dfrac{1}{7}(49)^b$, and $b = \dfrac{1}{2}$.

21. B

$y = m \log_{\frac{1}{3}} x + c = -m \log_3 x + c$
$\text{Slope} = -m = \dfrac{2}{4}$
$m = -\dfrac{1}{2}$

22. D

Slope of the graph $= \dfrac{0+2}{4-0} = \dfrac{1}{2}$. The equation is

$$\log_9 y = \frac{1}{2}x - 2$$
$$y = 9^{\frac{1}{2}x-2}$$
$$= 9^{-2} \times 3^x$$

So, $b = 3$.

23. A

For the point $\left(\frac{1}{3},\ 0\right)$,

$$\log_8 x = \frac{1}{3} \qquad \text{and} \log_4 y = 0$$
$$x = 8^{\frac{1}{3}} = 2 \qquad y = 1$$

Only option A satisfies this.

24. C

For the point $(0,\ 8)$,

$$\log_4 x = 0 \quad \text{and} \quad \log_8 y = 8$$
$$x = 1 \qquad y = 8^8$$
$$= 2^{24}$$

Only option C satisfies this.

25. D

$$\log_{27} y = \log_{27} a + x \log_{27} b$$
$$\text{Slope} = \log_{27} b = \frac{0+1}{3-0}$$
$$b = 27^{\frac{1}{3}}$$
$$= 3$$

26. C

$$\text{Slope of the line} = \frac{6-0}{0-3} = -2$$

Equation of the graph is

$$\log_a y - 6 = -2(x-0)$$
$$\log_a y = -2x + 6$$
$$y = a^{-2x+6}$$
$$= a^6(a^{-2})^x$$

We have $m = a^6$ and $n = a^{-2}$.

I. ✓.

II. ✗. Take $a = 0.5$, then $n = 4$.

III. ✓. $mn^3 = (a^6)(a^{-2})^3 = 1$.

27. A

For the point $(0,\ 5)$,

$$\log_2 x = 0 \quad \text{and} \quad \log_8 y = 5$$
$$x = 1 \qquad\qquad y = 8^5$$
$$= 2^{15}$$

Only option A satisfies this.

28. B

For the point $(0,\ 7)$.

$$\log_3 x = 0 \quad \text{and} \quad \log_9 y = 7$$
$$x = 1 \qquad\qquad y = 9^7$$

A. ✗. $x^4y^7 = 9^{56} \neq 3^{56}$

B. ✓. $x^7y^4 = 9^{28} = 3^{56}$

C. ✗. $x^7y^8 = 9^{56} \neq 3^{56}$

D. ✗. $x^8y^7 = 9^{49} \neq 3^{56}$

29. B

At the point $(2,\ 1)$,

$$\log_2 x = 2 \quad \text{and} \quad \log_4 y = 1$$
$$x = 4 \qquad\qquad y = 4$$

Put $(x,\ y) = (4,\ 4)$ into $y = kx^a$,

$$4 = k(4^a)$$
$$k = \frac{4}{4^a}$$
$$= 4^{1-a}$$

30. C

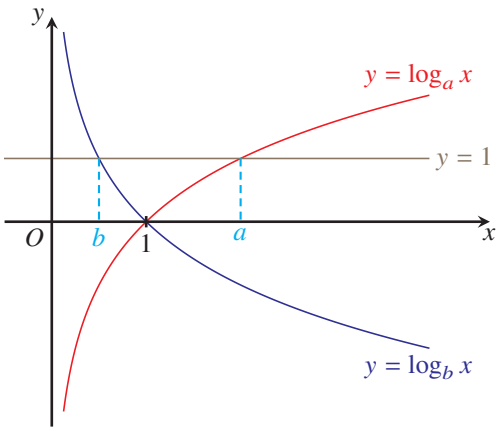
$$y = ab^x$$

$\log y = \log a + x \log b$
Slope $= \log b < 0 \Rightarrow 0 < b < 1$
Vertical intercept $= \log a > 0 \Rightarrow a > 1$

31. C

Draw the line $y = 1$.

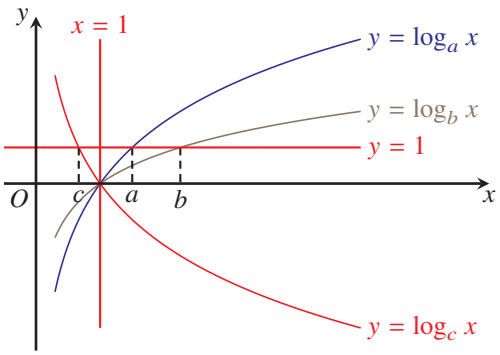
The line intersects the graph at $(b,\ 1)$ and $(a,\ 1)$.
From the graph, we have $0 < b < 1 < a$.
The result follows.



32. D

Draw the line $y = 1$.

The line intersects the graphs at $(a,\ 1)$, $(b,\ 1)$ and $(c,\ 1)$.
From the graph, we have $c < a < b$.



33. C

$0 = a + \log_b x$ and $0 = \log_c x$

$\log_b x = -a$ $\quad$ $x = 1$

$x = b^{-a}$

Thus, $OT : OS = 1 : b^{-a} = b^a : 1$.

34. A

A. ✓.

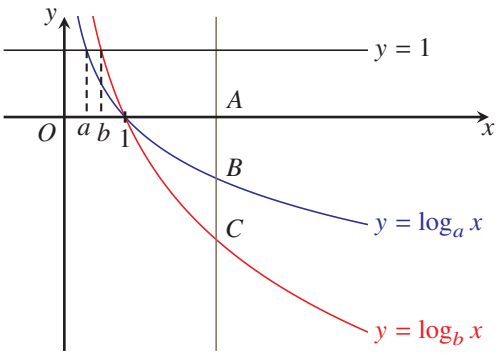
B. ✗. The graph passes through $(1,\ 0)$ but $y = 5^1 = 5 \neq 0$.

C. ✗. The graph passes through $(1,\ 0)$ but $y = 0.2^1 = 0.2 \neq 0$.

D. ✗. When $y = 1$, $x = 0.2^1 = 0.2$ but the graph shown does not pass through $(0.2,\ 1)$ obviously.

35. B

Draw the line $y = 1$. We have $0 < a < b < 1$.



I. ✗.

II. ✗.

III. ✓. Let the equation of ABC be $x = k$.
The coordinates of B and C are $(k,\ \log_a k)$ and $(k,\ \log_b k)$ respectively.

$$\begin{aligned}\frac{BC}{AB} &= \frac{\log_a k - \log_b k}{-\log_a k} \\ &= \frac{\frac{\log k}{\log a} - \frac{\log k}{\log b}}{-\frac{\log k}{\log a}} \\ &= \frac{\log a}{\log b} - 1 \\ &= \log_b a - \log_b b \\ &= \log_b \frac{a}{b}\end{aligned}$$

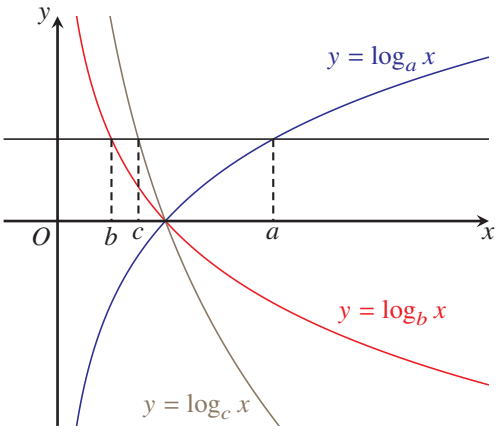
36. B

$$\begin{aligned}\frac{AC}{AB} &= \frac{\log_c k}{\log_b k} \\ &= \frac{\log k}{\log c} \div \frac{\log k}{\log b} \\ &= \frac{\log b}{\log c} \\ &= \log_c b\end{aligned}$$

37. A

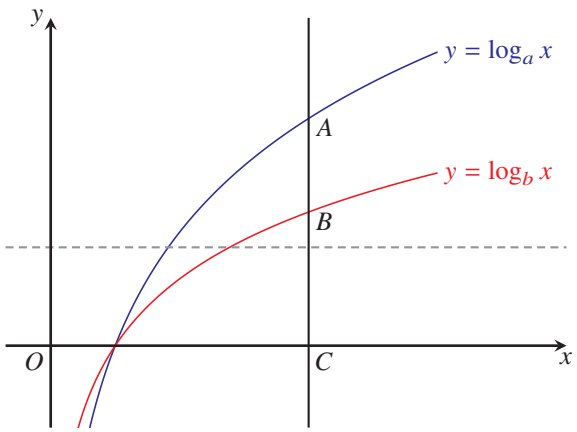
Draw the line $y = 1$.

Note that the x-intercepts of the graphs are 1.
We have $0 < b < c < 1 < a$.



38. C

The x-intercept of the graphs are 1. Draw the line $y = 1$.



I. ✓. $y = \log_a x$ intersects the line $y = 1$ at $(a,\ 1)$.
Compare with the x-intercept, we have $a > 1$.

II. ✗. $y = \log_b x$ intersects the line $y = 1$ at $(b,\ 1)$.
Therefore, $b > a$

III. ✓. Let $x = k$. Then $A\left(k,\ \log_a k\right)$ and $B\left(k,\ \log_b k\right)$.

$$\frac{AB}{BC} = \frac{\log_a k - \log_b k}{\log_b k}$$

$$= \frac{\frac{\log k}{\log a} - \frac{\log k}{\log b}}{\frac{\log k}{\log b}} \times \frac{\log a \log b}{\log a \log b}$$

$$= \frac{\log b - \log a}{\log a}$$

$$= \log_a \frac{b}{a}$$

END OF PAPER