REG-EOC-2324-ASM-SET 8-MATH
Suggested solutions

Conventional Questions

1. (a) (i)

$CE = EB$ *(given)*

$OA = OB$ *(radii)*

$OE /\!/ AC$ *(mid-pt. theorem)*

$\angle BOE = \angle OAD$ *(corr. ∠s, AC//OE)*

$\angle DOE + \angle BOE = 2\angle OAD$ *(∠ at centre twice ∠ at ⊙ce)*

$\angle DOE + \angle BOE = 2\angle BOE$

$\angle DOE = \angle BOE$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) $\angle DOE = \angle BOE$ *(proved)*

$OE = OE$ *(common side)*

$OD = OB$ *(radii)*

$\triangle DOE \cong \triangle BOE$ *(SAS)*$\angle OBE = 90°$ *(tangent ⊥ radius)*

$\angle ODE = \angle OBE$ *(corr. ∠s, ≅ △s)*

$= 90°$

Thus, DE is the tangent to the circle at D. *(converse of tangent ⊥ radius)*.

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $\angle ODE + \angle OBE = 90° + 90° = 180°$ 1M

Thus, O, B, E, D are concyclic.

Since $\angle OBE = 90°$, OE is a diameter of the circle $OBED$. 1M

The coordinates of O are $(0,\ 8)$. 1A

Slope of $OE = \dfrac{8-0}{0+6} = \dfrac{4}{3}$

Required equation is

$$y - 0 = -\frac{3}{4}(x+6)$$ 1M

$$3x + 4y + 18 = 0$$ 1A

(ii) The circumcentre of $\triangle BDE$ is the mid-point of OE.

The coordinates of the centre are $(-3,\ 4)$.

Required equation is

$$(x+3)^2+(y-4)^2=(-6+3)^2+(0-4)^2 \qquad \text{1M}$$

$$(x+3)^2+(y-4)^2=25 \qquad \text{1A}$$

2. (a) $\angle CAB = \angle BAD$ *(common ∠)*

$\angle ABC = \angle ADB$ *(∠ in alt. segment)*

$\triangle ABC \sim \triangle ADB$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $$\frac{AC}{AB}=\frac{AB}{AD} \qquad \text{1M}$$

$$\frac{\sqrt{9^2+12^2}}{36+9}=\frac{36+9}{\sqrt{9^2+12^2}+CD}$$

$$CD=120$$

Radius of Γ is 60.

Note that $\angle EBA = 90°$.

The coordinates of E are $(60,\ 36)$. 1M

Required equation is

$$(x-60)^2+(y-36)^2=60^2$$

$$(x-60)^2+(y-36)^2=3600 \qquad \text{1A}$$

(ii) E is the mid-point of CD.

The coordinates of D are $(108,\ 72)$. 1A

Let $x^2+y^2+dx+ey+f=0$ be the equation of the circumcircle of $\triangle BED$.

$$\begin{cases} 0^2+36^2+0+36e+f=0 \\ 60^2+36^2+60d+36e+f=0 \\ 108^2+72^2+108d+72e+f=0 \end{cases} \qquad \text{1M}$$

Solving, we have $d=-60$, $e=-252$ and $f=7776$.

Required equation is $x^2+y^2-60x-252y+7776=0$. 1A

> E is the mid-point of CD.
> The coordinates of D are $(108,\ 72)$. 1A
> Note that the centre of the circumcircle lies on the perpendicular bisector of BE, which is $x=30$.
> Let the coordinates of the centre of the circumcentre of $\triangle BED$ be $(30,\ k)$.
>
> $$\sqrt{(30-0)^2+(k-36)^2}=\sqrt{(30-108)^2+(k-72)^2} \qquad \text{1M}$$
>
> $$k^2-72k+2196=k^2-144k+11\,268$$
>
> $$k=126$$

Required equation is

$$(x-30)^2+(y-126)^2=(0-30)^2+(36-126)^2$$

$$(x-30)^2+(y-126)^2=9000$$ 1A

(iii) Area of the circumcircle of $\triangle BED$

$=(\sqrt{30^2+126^2-7776})^2\pi$ 1M

$\approx 28\,300$

Let r be the radius of the inscribed circle of $\triangle BED$.

Consider the area of $\triangle BED$.

$$\frac{(36+9)(108)}{2}=\frac{(AB)(r)}{2}+\frac{(BD)(r)}{2}+\frac{(AD)(r)}{2}$$ 1M

$$r\approx 16.5$$

(Area of the inscribed circle of $\triangle BED$) $\times 30$

$=\pi r^2\times 30$

$\approx 25\,800$

$<$ area of the circumcircle of $\triangle BED$

The claim is agreed. 1

3. (a) The coordinates of the centre are $(4,\ 10)$. 1M

Required equation is

$$(x-4)^2+(y-10)^2=(-6-4)^2+(0-10)^2$$ 1M

$$(x-4)^2+(y-10)^2=200$$ 1A

(b) Slope of $L_1=\frac{20-0}{14+6}=1$

Equation of L_1 is

$$y-0=1(x+6)$$ 1M

$$y=x+6$$

L_1 intersects L_2 and the y-axis at $(k-6,\ k)$ and $(0,\ 6)$ respectively.

$$\frac{(k-6)(k-6)}{2}=200$$ 1M

$$(k-6)^2=400$$

$$k=26 \quad \text{or} \quad -14 \text{ (rejected)}$$ 1A

4. (a) Let the coordinates of G be $(h,\ 26)$.

Note that G lies on the perpendicular bisector of AB.

$$h = \frac{5 + 13}{2}$$ 1M

$$= 9$$

The equation of C is

$$(x - 9)^2 + (y - 26)^2 = (5 - 9)^2 + (23 - 26)^2$$ 1M

$$(x - 9)^2 + (y - 26)^2 = 25$$ 1A

(b) $\sqrt{(k - 9)^2 + (38 - 26)^2} = 15$ 1M

$$k^2 - 18k = 0$$

$$k = 18 \quad \text{or} \quad 0 \text{ (rejected)}$$ 1A

(c) (i) T, P and G are collinear. 1A

(ii) Radius of C is 5.

Required ratio $= GP : PT$ 1M

$= 5 : (15 - 5)$

$= 1 : 2$ 1A

5. (a) (i) $x^2 + (mx)^2 - 400x - 300mx + 40\,000 = 0$ 1M

$(1 + m^2)x^2 - (300m + 400)x + 40\,000 = 0$

$\Delta = (300m + 400)^2 - 4(1 + m^2)(40\,000) > 0$ 1M

$10\,000(-7m^2 + 24m) > 0$

$0 < m < \frac{24}{7}$ 1A

(ii) x-coordinate of $M = \frac{1}{2}\left[\frac{300m + 400}{1 + m^2}\right]$

$= \frac{50(3m + 4)}{1 + m^2}$

y-coordinate of $M = m \cdot \frac{50(3m + 4)}{1 + m^2}$

$= \frac{50m(3m + 4)}{1 + m^2}$ 1

(b) (i) Perpendicular bisector of AB is the line passing through O and the centre of C. 1M

The coordinates of the centre of C are $(200,\ 150)$.

Required equation is

$y - 0 = \frac{150 - 0}{200 - 0}(x - 0)$

$3x - 4y = 0$ 1A

(ii) Note that L touches C when $m = 0$ or $\frac{24}{7}$.

When $m = 0$, the coordinates of the intersection of L and C are $(200,\ 0)$.

The coordinates of B are $(200,\ 0)$. 1A

When $m = \frac{24}{7}$, we have the coordinates of A are $(56,\ 192)$.

Denote the centre of C by G $(200,\ 150)$.

Suppose G and M are distinct points.

Note that $\angle GMO = 90°$ and $\angle OBG = 90°$.

We have O, B, G, M are concyclic. 1M

Since $\angle OAG = 90°$ and $\angle GMO = 90°$, we have O, A, G, M are concyclic.

Thus, O, A, M, G and B are concyclic.

If G and M coincides, O, A, G, B are also concyclic.

Note that OG is a diameter of the circle.

The coordinates of the centre of the required circle are $(100,\ 75)$. 1M

Required equation is

$(x - 100)^2 + (y - 75)^2 = (0 - 100)^2 + (0 - 75)^2$

$(x - 100)^2 + (y - 75)^2 = 15\,625$ 1A

(iii) Denote the centre of the circle AMB by D.

$$\tan \angle BOG = \frac{150}{200}$$

$$\angle BOG \approx 36.9^\circ$$

$\angle ADB = 2\angle AOB = 2(2\angle BOG) \approx 147^\circ$ 1M

Length of $\Gamma \le 2\pi(125) \times \dfrac{\angle ADB}{360^\circ}$ 1M

$\approx 322 < 330$

The claim is disagreed. 1A

6. (a) (i) $\angle BEC = \angle BDC$ *($\angle$s in the same segment)*

$\angle ECG = \angle BEC$ *(alt. $\angle$s, $BE // CG$)*

$= \angle BDC$

$\angle BCE = \angle BDE$ *($\angle$s in the same segment)*

$\angle BCG = \angle BCE + \angle ECG$

$= \angle BDE + \angle BDC$ *(proved)*

$= \angle CDG$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) $\angle CBD = \angle DBE$ *(equal arcs, equal $\angle$s)*

$\angle BHC = \angle DBE$ *(alt. $\angle$s, $BE // CG$)*

$= \angle CBD$

$BC = HC$ *(sides opp. equal $\angle$s)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $BH : DH = \frac{144}{25} : 8$ 1M

$= 18 : 7$ 1A

x-coordinate of $H = \frac{18}{25} \times (6) = \frac{108}{25}$ 1M

The coordinates of H are $\left(\frac{108}{25}, -\frac{144}{25}\right)$. 1A

(ii) Slope of $BD = \frac{-8}{6} = -\frac{4}{3}$

Slope of $CE = \frac{0 + \frac{144}{25}}{6 + \frac{42}{25}} = \frac{3}{4}$ 1M

Since $\frac{-4}{3} \times \frac{3}{4} = -1$, $BD \perp CE$ and so $\angle CFD = 90° = \angle CGD$. 1A

C, D, G and F are concyclic. *(converse of ∠s in the same segment)* 1

(iii) CD is a diameter of the circle $CDGF$.

Slope of $CD = \frac{-8 + \frac{144}{25}}{6 + \frac{42}{25}} = -\frac{7}{24}$

Slope of tangent required $= \frac{24}{7}$ 1M

Required equation is

$$y + 8 = \frac{24}{7}(x - 6)$$

$24x - 7y - 200 = 0$ 1A

7. (a) $(x-2)^2+(y-6)^2=r^2$ 1A

(b) (i) Let G be the centre of C'. Then $G(-2,\ 6-c)$. 1A

Since $AG \perp PQ$,

$$\frac{6-c-6}{-2-2}\times\left(-\frac{1}{2}\right)=-1$$ 1M

$$c=8$$ 1A

(ii) mid-point of AG lies on PQ, i.e., $(0,\ 2)$ lies on PQ. 1M

The equation of PQ is $y=-\frac{x}{2}+2$. 1A

(iii) $(x-2)^2+\left(-\frac{x}{2}+2-6\right)^2=r^2$ 1M

$$\frac{5}{4}x^2+20-r^2=0$$

a and d are roots of the equation.

So, $a+d=0$ and $ad=\frac{4(20-r^2)}{5}$. 1M

$(a-d)^2=(a+d)^2-4ad$

$$=\frac{16(r^2-20)}{5}$$ 1A

(c) $PQ^2=(a-d)^2+\left[\left(-\frac{a}{2}+2\right)-\left(-\frac{d}{2}+2\right)\right]^2$

$80=\frac{5}{4}(a-d)^2$

$=4(r^2-20)$ 1M

$r^2=40$

$r=2\sqrt{10}$ or $-2\sqrt{10}$ (rejected)

$AB=\sqrt{(2+1)^2+(6-1)^2}=\sqrt{34}<r$

So, B lies inside C.

The claim is agreed. 1A

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