**REG-EOC-2324-ASM-SET 7-MATH**
**Suggested solutions**

**Conventional Questions**

1. Let $C$ be the centre of circle. Then $C = \left(\frac{-6}{-2},\ \frac{4}{-2}\right) = (3,\ -2)$. 1M

Slope of $PC = \frac{-6+2}{0-3} = \frac{4}{3}$

Slope of the required tangent $= \frac{-1}{\frac{4}{3}} = -\frac{3}{4}$. 1M

Required equation is

$y + 6 = -\frac{3}{4}(x - 0)$ 1M

$y = -\frac{3}{4}x - 6$ 1A

2. (a) $(3,\ -2)$ 1A

(b) Slope of the radius to $P = \frac{-1+2}{6-3} = \frac{1}{3}$

Slope of the required tangent $= -3$ 1M

Required equation is

$y + 1 = -3(x - 6)$ 1M

$y = -3x + 17$ 1A

3. (a) Slope of $L = \frac{1}{2}$

Slope of $AB = -2$ 1M

The equation of $AB$ is

$y - 0 = -2(x + 2)$

$2x + y + 4 = 0$ 1A

(b) Solve $\begin{cases} 2x + y + 4 = 0 \\ x - 2y + 7 = 0 \end{cases}$, 1M

we have $x = -3$ and $y = 2$.

The coordinates of $B$ are $(-3,\ 2)$. 1A

(c) $(x+2)^2 + y^2 = (-3+2)^2 + 2^2$ 1M

$(x+2)^2 + y^2 = 5$ 1A

4. (a) $D$ is circumcentre of $\triangle ABC$. So, $BC$ is a diameter. 1

$$\angle NAB = \angle ACB \qquad \text{(}\angle\text{ in alt. segment)}$$

$$\angle CBA = \angle NAB \qquad \text{(alt. }\angle\text{s, } MN // BC\text{)}$$

$$= \angle ACB$$

$$AC = AB \qquad \text{(sides opp. equal }\angle\text{s)}$$

$$\angle CAB = 90° \qquad \text{(}\angle\text{ in semicircle)}$$

Thus, $\triangle ABC$ is a right-angled isosceles triangle.

| **Marking Scheme** | | |
|---|---|---|
| **Case 1** | Any correct proof with correct reasons. | 3 |
| **Case 2** | Any correct proof without reasons. | 2 |
| **Case 3** | Incomplete proof with any one correct step with reason. | 1 |

(b) (i) $C(-2,\ 4)$ 1A

$D$ is the mid-point of $BC$, at $(1,\ 3)$.

The equation of circle is

$$(x-1)^2 + (y-3)^2 = (-2-1)^2 + (4-3)^2$$ 1M

$$(x-1)^2 + (y-3)^2 = 10$$ 1A

(ii) Slope of required tangent = slope of $MN$ = slope of $BC = \dfrac{4-2}{-2-4} = -\dfrac{1}{3}$. 1M

$D$ is the mid-point of $A$ and $E$. So, $E(2,\ 6)$.

Required equation is

$$y - 6 = -\frac{1}{3}(x-2)$$ 1M

$$y = -\frac{x}{3} + \frac{20}{3}$$ 1A

5. (a) $CE \perp AB$ *(property of orthocentre)*

$BD \perp AC$ *(property of orthocentre)*

$\angle BEC = \angle BDC = 90°$

Thus, $BCDE$ is a cyclic quadrilateral. *(converse of ∠s in the same segment)*

| **Marking Scheme** | | |
|---|---|---|
| **Case 1** | Any correct proof with correct reasons. | 2 |
| **Case 2** | Any correct proof without reasons. | 1 |

(b) (i) Coordinates of centre $= \left(\dfrac{-6+14}{2},\ \dfrac{-6-6}{2}\right) = (4,\ -6)$ 1A

The equation of the circle is

$$(x-4)^2 + (y+6)^2 = (0-4)^2 + (8+6)^2$$ 1M

$$(x-4)^2 + (y+6)^2 = 100$$ 1A

(ii) Distance between $A$ and centre $= \sqrt{4^2 + (-6-8)^2} = \sqrt{212}$

Radius of circle = 10

Angle between two tangents $= 2 \times \sin^{-1} \dfrac{10}{\sqrt{212}} \approx 86.8° \neq 90°$ 1M+1A

The claim is not agreed. 1A

6. (a) Centre $(-4,\ 3)$ and radius $= \sqrt{4^2 + 3^2 - 5} = 2\sqrt{5}$ 1A

(b) $$(2y-k)^2 + y^2 + 8(2y-k) - 6y + 5 = 0$$ 1M

$$(4+1)y^2 + (-4k+16-6)y + (k^2 - 8k + 5) = 0$$

$$5y^2 + (-4k+10)y + (k^2 - 8k + 5) = 0$$

$y$-coordinate of $M = \dfrac{1}{2}\left(-\dfrac{-4k+10}{5}\right) = \dfrac{2k-5}{5}$ 1M

When $y = \dfrac{2k-5}{5}$, $x = 2\left(\dfrac{2k-5}{5}\right) - k = \dfrac{-k-10}{5}$.

Required coordinates are $\left(\dfrac{-k-10}{5},\ \dfrac{2k-5}{5}\right)$. 1A

(c) If $M$ lies above the $x$-axis, then $2k - 5 > 0$. So, $k > \dfrac{5}{2}$. 1A

Consider the equation $5y^2 + (-4k+10)y + (k^2 - 8k + 5) = 0$.

If $L$ cuts $C$ at two distinct points,

$$\Delta = (-4k+10)^2 - 4(5)(k^2 - 8k + 5) > 0$$ 1M

$$-4k^2 + 80k > 0$$

$$0 < k < 20$$ 1A

Combine $k > \dfrac{5}{2}$ and $0 < k < 20$, we have $\dfrac{5}{2} < k < 20$.

Thus, it is possible. 1A

7. (a) $x^2 + (x+7)^2 - 2x - 6(x+7) - 3 = 0$ 1M

$$2x^2 + 6x + 4 = 0$$ 1A

$$x = -2 \quad \text{or} \quad -1$$

When $x = 2$, $y = -2 + 7 = 5$; when $x = 1$, $y = -1 + 7 = 6$.

Required coordinates are $(-1,\ 6)$ and $(-2,\ 5)$. 1A

(b) Coordinates of centre = $\left(-\frac{3}{2},\ \frac{11}{2}\right)$ 1A

The equation of circle is

$$\left(x + \frac{3}{2}\right)^2 + \left(y - \frac{11}{2}\right)^2 = \left(-1 + \frac{3}{2}\right)^2 + \left(6 - \frac{11}{2}\right)^2$$ 1M

$$\left(x + \frac{3}{2}\right)^2 + \left(y - \frac{11}{2}\right)^2 = \frac{1}{2}$$ 1A

8. (a) $x^2+\left(\frac{1}{3}x\right)^2-4x-4\left(\frac{1}{3}x\right)=0$ 1M

$$\frac{10}{9}x^2-\frac{16}{3}x=0$$

$$x=0 \quad \text{or} \quad \frac{24}{5}$$

When $x=\frac{24}{5}$, $y=\frac{8}{5}$. The coordinates of $P$ are $\left(\frac{24}{5},\ \frac{8}{5}\right)$. 1A

Length of $OP=\sqrt{\left(\frac{24}{5}\right)^2+\left(\frac{8}{5}\right)^2}=\frac{8\sqrt{10}}{5}$ 1A

(b) Let the coordinates of $Q$ be $(a,\ b)$. Since $PQ=OP$,

$$\left(a-\frac{24}{5}\right)^2+\left(b-\frac{8}{5}\right)^2=\left(\frac{8\sqrt{10}}{5}\right)^2$$ 1M

$$a^2+b^2-\frac{48}{5}a-\frac{16}{5}b=0$$

Since $(a,\ b)$ lies on the circle, $a^2+b^2-4a-4b=0$.

$$\left(-\frac{48}{5}a-\frac{16}{5}b\right)-(-4a-4b)=0$$ 1M

$$-\frac{28}{5}a+\frac{4b}{5}=0$$

$$b=7a$$

Substitute $b=7a$ into $a^2+b^2-4a-4b=0$,

$$a^2+(7a)^2-4a-4(7a)=0$$

$$50a^2-32a=0$$

$$a=\frac{16}{25} \quad \text{or} \quad 0$$

When $a=\frac{16}{25}$, $b=\frac{112}{25}$. The coordinates of $Q$ are $\left(\frac{16}{25},\ \frac{112}{25}\right)$. 1A

The coordinates of the centre of circle are $(2,\ 2)$.

Slope of $CQ=\dfrac{\frac{112}{25}-2}{\frac{16}{25}-2}=-\frac{31}{17}$. 1M

Slope of tangent to the circle at $Q=\frac{17}{31}$. 1M

The equation of tangent to the circle at $Q$ is

$$y-\frac{112}{25}=\frac{17}{31}\left(x-\frac{16}{25}\right)$$ 1M

$$775y-3472=425x-272$$

$$17x-31y+128=0$$ 1A

9. (a) $AB^2 = 9^2 + 18^2 = 405$

$AC^2 = 13^2 + 16^2 = 425$

$BC^2 = 4^2 + 2^2 = 20$

Therefore, $AB^2 + BC^2 = AC^2$. 1M

Thus, $\angle ABC = 90°$.

$\triangle ABC$ is a right-angled triangle. 1A

(b) The coordinates of the centre $= \left(\frac{-9+4}{2}, \frac{-8+8}{2}\right) = \left(-\frac{5}{2}, 0\right)$ 1A

The equation of $\Omega$ is

$$\left(x + \frac{5}{2}\right)^2 + y^2 = \left(0 + \frac{5}{2}\right)^2 + 10^2 \quad \text{1M}$$

$$x^2 + y^2 + 5x - 100 = 0 \quad \text{1A}$$

(c) (i) The coordinates of $D$ are $(10, 0)$. 1A

$$(10)^2 + 0^2 + 5(10) - 100 = 50 \neq 0$$

Therefore, $D$ does not lie on the circle passing through $A$, $B$ and $C$. 1M

Thus, $ABCD$ is not a cyclic quadrilateral. 1A

(ii) Let the slope of $L$ be $m$.

The equation of $L$ is $y = m(x - 10)$. 1M

Substitute $L$ into $\Omega$,

$$x^2 + (mx - 10m)^2 + 5x - 100 = 0 \quad \text{1M}$$

$$(1 + m^2)x^2 + (-20m^2 + 5)x + (100m^2 - 100) = 0$$

Since $L$ is tangent, $\Delta = 0$.

$$(20m^2 - 5)^2 - 4(1 + m^2)(100m^2 - 100) = 0 \quad \text{1M}$$

$$-200m^2 + 425 = 0$$

$$m = \pm\frac{\sqrt{34}}{4} \quad \text{1A}$$

The equations of $L$ are $y = \frac{\sqrt{34}}{4}(x - 10)$ and $y = -\frac{\sqrt{34}}{4}(x - 10)$.

10. (a) $\sqrt{(h-6)^2+(9-3)^2}=\sqrt{(h-a)^2+(11-3)^2}$ 1M

$$h^2-12h+72=h^2+a^2-2ah+64$$

$$h=\frac{a^2-8}{2a-12}$$

The coordinates of $G$ are $\left(\frac{a^2-8}{2a-12},\ 3\right)$. 1A

(b) (i) $\dfrac{11-3}{a-\frac{a^2-8}{2a-12}}=\dfrac{4}{3}$ 1M

$$24(2a-12)=4[a(2a-12)-(a^2-8)]$$

$$0=4a^2-96a+320$$

$$a=4 \quad \text{or} \quad 20$$

When $a=4$, $h=-2<0$; when $a=20$, $h=14>0$.

So, $a=20$. 1A

(ii) Coordinates of $G$ are $(14,\ 3)$. The equation of $C$ is

$(x-14)^2+(y-3)^2=(6-14)^2+(9-3)^2$ 1M

$$x^2+y^2-28x-6y+105=0$$

$x^2+(kx)^2-28x-6kx+105=0$ 1M

$$(1+k^2)x^2+(-28-6k)x+105=0$$

$$x\text{-coordinate of } M=\frac{1}{2}\times\frac{28+6k}{1+k^2}$$

$=\dfrac{14+3k}{1+k^2}$ 1

(iii) Since $\angle OMG = 90°$, $OM = 2\sqrt{41}$.

$$\sqrt{\left(\frac{14+3k}{1+k^2}\right)^2 + \left(\frac{k(14+3k)}{1+k^2}\right)^2} = 2\sqrt{41} \qquad \text{1M}$$

$$\frac{(14+3k)^2}{1+k^2} = 164$$

$$-155k^2 + 84k + 32 = 0$$

$$k = \frac{4}{5} \quad \text{or} \quad -\frac{8}{31} \text{ (rejected)}$$

Coordinates of $M$ are $(10,\ 8)$. 1M

$M$, $G$ and $A$ are collinear.

Coordinates of $B$ are $(4,\ 3)$. 1M

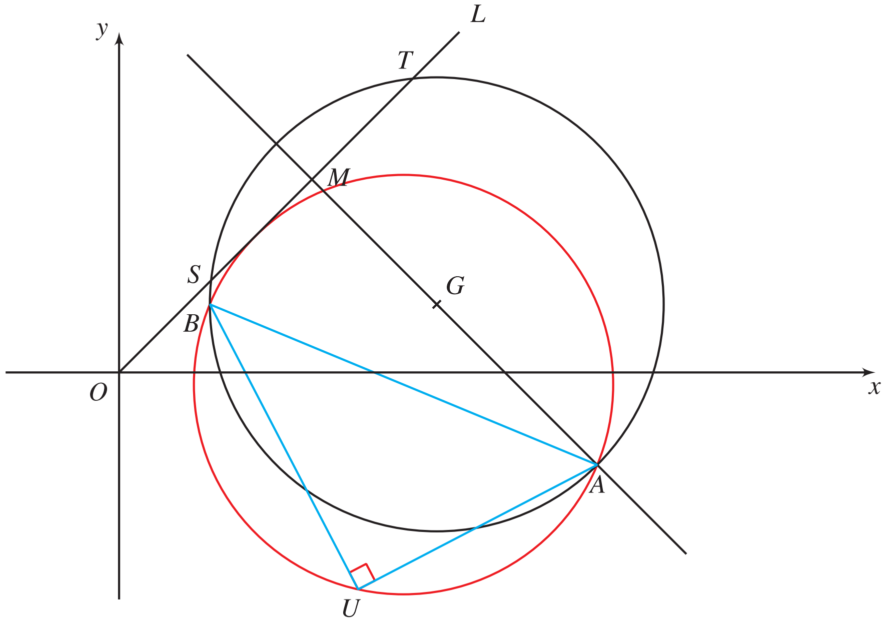
When the area of circle $AUB$ is the least, $\angle AUB = 90°$. 1M

$$\text{slope of } AM \times \text{slope of } BM = \frac{8-3}{10-14} \times \frac{8-3}{10-4} \qquad \text{1M}$$

$$= -\frac{25}{24} \neq -1$$

So, $\angle AMB \neq 90°$ and $\angle AUB + \angle AMB \neq 180°$.

$A$, $M$, $B$ and $U$ are not concyclic. 1A

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